Economics 701: Macroeconomics II
Spring 2006
Professor Jones

Final Examination
Answer Key

We are considering a version of the stochastic growth model with intermediate goods. Final output is given by

$$Y = \left(\int_0^1 Y(i)^{(\sigma-1)/\sigma} \, di \right)^{\sigma/(\sigma-1)}, \quad \sigma > 1.. \tag{FPRF}$$

The production function for intermediate goods is given by

$$Y(i) = L(i)^{\alpha}, \quad \alpha \in (0,1), \tag{PRF}$$

while the total cost of producing good i is

$$\Phi(Y(i)) = WY(i)^{1/\alpha}. \tag{TC}$$

1. (8 points.) A final goods producer operates under perfect competition, solving

$$\max_{[Y(i)]_0^1} \left(\int_0^1 Y(i)^{(\sigma-1)/\sigma} \, di \right)^{\sigma/(\sigma-1)} - \int_0^1 P(i) Y(i) \, di.$$

The first order condition is

$$\frac{\sigma}{\sigma-1} \left(\int_0^1 Y(i)^{(\sigma-1)/\sigma} \, di \right)^{\sigma/(\sigma-1)-1} \frac{\sigma-1}{\sigma} Y(i)^{(\sigma-1)/\sigma-1} = P(i).$$

Noting that $\sigma/(\sigma-1) - 1 = 1/(\sigma-1)$, this reduces to

$$P(i) = Y^{1/\sigma} Y(i)^{-1/\sigma} \Rightarrow Y(i) = P(i)^{-\sigma} Y.$$

2. (8 points.) Intermediate goods producers are price-setters. Each intermediate goods producer operates under uncertainty, in that it must choose its price before it observes the choices of other producers. Inserting the demand function derived in question 1, it follows that producer i solves

$$\max_{P_t(i)} E_{t-1} \left\{ P_t(i) \times P_t(i)^{-\sigma} Y_t - \Phi_t\left(P_t(i)^{-\sigma} Y_t\right) \right\}$$

The first order condition is

$$(1-\sigma) E_{t-1} \left\{ Y_t P_t(i)^{-\sigma} \right\} = E_{t-1} \left\{ \frac{\partial}{\partial Y_t(i)} \Phi_t\left(P_t(i)^{-\sigma} Y_t\right) \times (-\sigma) Y_t P_t(i)^{-\sigma-1} \right\}.$$

Since $P_t(i)$ is known at time $t-1$ (as it is being chosen), this reduces to

$$P_t(i) = \mu \frac{E_{t-1}(\phi_t(i) Y_t)}{E_{t-1}(Y_t)}$$

where $\mu \equiv \sigma/(\sigma-1)$ is the markup ratio.

3. (18 points.) Now consider a symmetric equilibrium, where $Y_t(i) = Y_t(j)$, $\forall i, j, t$.

   (a) In a symmetric equilibrium

$$Y_t = \left(\int_0^1 Y_t(j)^{(\sigma-1)/\sigma} \, di \right)^{\sigma/(\sigma-1)} = \left(Y_t(j)^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)} = Y_t(j), \quad \forall j, t.$$

   (Note the different indices!)

   (b) Since final output is the numeraire, one can show that

$$P_t(i) = P_t = 1.$$

   It follows immediately from question 2 that

$$\frac{E_{t-1}(\phi_t Y_t)}{E_{t-1}(Y_t)} = \frac{\sigma - 1}{\sigma}. \tag{EMC}$$

   Given that $Y_t = Y_t(j)$, ϕ_t is also the marginal cost of aggregate output. In a steady state, there is no uncertainty, and (EMC) simplifies to

$$\phi_{SS} = \frac{\sigma - 1}{\sigma}.$$

   With $\sigma > 1$, we see immediately that the marginal cost is less than 1, the price of final output. This gap is just the markup re-expressed, and thus reflects the market power of intermediate goods producers: they are depressing output below the efficient level.

   (c) It follows from equations (TC) and (PRF) that in a symmetic equilibrium:

$$\begin{aligned} \phi_t(i) &= \frac{\partial}{\partial Y_t(i)} \Phi_t(Y_t(i)) = W_t \frac{1}{\alpha} Y_t(i)^{1/\alpha - 1} = W_t \frac{1}{\alpha} \left(L_t(i)^{\alpha} \right)^{1/\alpha} Y_t(i)^{-1} \\ &= W_t \left(\alpha \frac{Y_t}{L_t} \right)^{-1}. \end{aligned} \tag{MC}$$

4. (8 points.) The consumer's problem can be written as a Lagrangean

$$\begin{aligned} \mathcal{L} = & \left\{ E_0 \sum\nolimits_{t=0}^{\infty} \beta^t \left[\ln(C_t) - \chi \frac{1}{1+\gamma} L_t^{1+\gamma} \right. \right. \\ & \left. \left. + \lambda_t \left((1+r) K_t + W_t L_t + \int_0^1 \Pi_t(i) \, di - C_t - K_{t+1} \right) \right] \right\}. \end{aligned}$$

The first-order conditions are

$$\begin{aligned} \frac{1}{C_t} &= \lambda_t, \\ \lambda_t W_t &= \chi L_t^{\gamma}, \\ \lambda_t &= \beta E_t \left((1+r) \lambda_{t+1} \right). \end{aligned}$$

Substituting for λ_t, and imposing $\beta(1+r) = 1$, we get

$$\frac{1}{C_t} = E_t \left(\frac{1}{C_{t+1}} \right), \tag{EE}$$

$$W_t \frac{1}{C_t} = \chi L_t^{\gamma}. \tag{LL}$$

5. (6 points.) The Euler equation for capital was given by equation (EE) above. The capital accumulation equation can be written as

$$K_{t+1} = (1+r)\,K_t + Y_t - C_t, \tag{CA}$$

with Y_t following equation (PRF). This constraint can either be derived directly as a resource constraint, or found by combining the household's budget constraint with the definition of profits. In particular, note that in a symmetic equilibrium

$$\Pi_t(i) = P_t(i)\,Y_t(i) - \Phi_t(Y_t(i)) = P_t(i)\,Y_t - W_t L_t = Y_t - W_t L_t.$$

Inserting this result into the household's budget constraint yields (CA).

6. (8 points) Combining equations (MC) and (LL) shows that .

$$W_t = \phi_t \alpha \frac{Y_t}{L_t} = \chi L_t^{\gamma} C_t,$$

Rearranging, and inserting equation (PRF), we get

$$\begin{aligned} L_t^{1+\gamma} Y_t^{-1} &= Y_t^{(1+\gamma)/\alpha} Y_t^{-1} \\ &= \frac{\alpha}{\chi}\phi_t C_t^{-1}, \\ \Rightarrow Y_t &= \left[\frac{\alpha}{\chi}\phi_t C_t^{-1}\right]^{\lambda}, \qquad\qquad \text{(LL}'\text{)} \\ \lambda &\equiv \frac{\alpha}{1-\alpha+\gamma}. \end{aligned}$$

7. (12 points.) At low levels of variance, we can write equation (EMC) as:

$$\begin{aligned} \ln\left(\frac{\sigma-1}{\sigma}\right) &= \ln\left(E_{t-1}(\phi_t Y_t)\right) - \ln\left(E_{t-1}(Y_t)\right) \\ &\approx E_{t-1}\left(\ln(\phi_t) + \ln(Y_t)\right) - E_{t-1}\left(\ln(Y_t)\right) \\ &= E_{t-1}\left(\ln(\phi_t)\right), \end{aligned}$$

It immediately follows that

$$\begin{aligned} E_{t-1}\left(\widehat{\phi}_t\right) &= E_{t-1}\left(\ln\left(\frac{\phi_t}{\phi_{SS}}\right)\right) = E_{t-1}\left(\ln(\phi_t) - \ln(\phi_{SS})\right) \\ &\approx \ln\left(\frac{\sigma-1}{\sigma}\right) - \ln\left(\frac{\sigma-1}{\sigma}\right) = 0. \qquad\qquad \text{(MC}'\text{)} \end{aligned}$$

Note that (MC′) restricts $\widehat{\phi}_t$ to be a martingale difference sequence, meaning that it cannot be persistent. This means that shocks to $\widehat{\phi}_t$ are unlikely to generate persistent output fluctuations of the type actually observed.

8. (6 pts.) It follows from equation (LL′) that

$$\begin{aligned}\exp\left(\widehat{y}_t\right) &\equiv \frac{Y_t}{Y_{ss}} \\ &= \left(\frac{\alpha}{\chi}\right)^{\lambda}\phi_t^{\lambda}C_t^{-\lambda}\Big/\left[\left(\frac{\alpha}{\chi}\right)^{\lambda}\phi_{SS}^{\lambda}C_{SS}^{-\lambda}\right] \\ &= \exp\left(\lambda\widehat{\phi}_t\right)\exp\left(-\lambda\widehat{c}_t\right).\end{aligned}$$

Logging both sides yields

$$\widehat{y}_t = \lambda\left[\widehat{\phi}_t - \widehat{c}_t\right]. \qquad \text{(PRF}'\text{)}$$

9. 18 points.) We are given the log-linearized capital accumulation and Euler equations

$$\begin{aligned}E_t\left(\widehat{c}_{t+1}\right) &= \widehat{c}_t && \text{(EE}'\text{)} \\ \widehat{k}_{t+1} &= (1+r)\,\widehat{k}_t + \omega_1\widehat{\phi}_t - \omega_2\widehat{c}_t, && \text{(CA}'\text{)}\end{aligned}$$

with $\omega_2 > \omega_1 > 0$.

To solve the system given by equations (CA′), (EE′) and (MC′), we begin by assuming that consumption can be written as a function of capital and preferences:

$$\widehat{c}_t = \eta\widehat{k}_t + \theta\widehat{\phi}_t, \qquad \text{(CF)}$$

(a) Combining equations (CF) and equation (CA′) yields

$$\widehat{k}_{t+1} = (1+r-\omega_2\eta)\,\widehat{k}_t + (\omega_1 - \omega_2\theta)\,\widehat{\phi}_t. \qquad \text{(CA}''\text{)}$$

(b) Inserting equation (CF) into the *right*-hand-side of equation (EE′) yields

$$E_t\left(\widehat{c}_{t+1}\right) = \eta\widehat{k}_t + \theta\widehat{\phi}_t.$$

(c) Inserting equation (CF) into the *left*-hand-side of equation (EE′) yields

$$E_t\left(\widehat{c}_{t+1}\right) = E_t\left(\eta\widehat{k}_{t+1} + \theta\widehat{\phi}_{t+1}\right).$$

Inserting (CA″) and (MC′) into this expression yields

$$\begin{aligned}E_t\left(\widehat{c}_{t+1}\right) &= \eta\left[(1+r-\omega_2\eta)\,\widehat{k}_t + (\omega_1-\omega_2\theta)\,\widehat{\phi}_t\right] + 0, \\ &= \eta\,(1+r-\omega_2\eta)\,\widehat{k}_t + \eta\,(\omega_1-\omega_2\theta)\,\widehat{\phi}_t.\end{aligned}$$

(d) If the expressions for $E_t\left(\widehat{c}_{t+1}\right)$ in parts (b) and (c) are to be equal,

$$\begin{aligned}\eta\,(1+r-\omega_2\eta) &= \eta, \\ \eta\,(\omega_1-\omega_2\theta) &= \theta.\end{aligned}$$

The first of these two equations has the non-zero solution

$$\eta = \frac{r}{\omega_2},$$

while the second equation implies that

$$\eta\omega_1 = \theta\,(1+\eta\omega_2) = \theta\,(1+r)\,,$$

so that

$$\begin{aligned}\theta &= \eta\frac{\omega_1}{(1+r)} \\ &= \frac{r\omega_1}{\omega_2\,(1+r)}.\end{aligned}$$

10. (8 points.) The shocks to $\widehat{\phi}_t$ are *extrinsic* sunspot shocks, in that they do not reflect "fundamental" shocks such as shocks to the Solow residual. These shocks arise because there are strategic complementaries in the production of intermediate goods. In particular, the demand function that you derived in question 1 shows that amount of good i that producer i can sell at the predetermined price $P\,(i)$ increases in aggregate output (Y). This means that increased output by other producers increases the optimal amount of production by producer i. Because producers commit to their prices before they observe the output choices of the other producers, at the time of production they can coordinate on different levels of intermediate output, leading to multiple equilibria. The variable $\widehat{\phi}_t$ is simply a coordination device that moves producers between these equilibria.