Economics 701: Macroeconomics II
Spring 2006
Professor Jones

Final Examination
May 17, 2006

Instructions. Answer all the questions in your bluebook. You have 120 minutes to complete the exam. Good luck!

(Following Wang and Wen, 2006.) Consider the following simplified version of a stochastic growth model. The population and steady state technology level are constant and normalized to 1, so that upper case letters denote intensive as well as aggregate quantities.

Letting $i \in [0,1]$ index intermediate goods, final output is given by

$$Y = \left(\int_0^1 Y(i)^{(\sigma-1)/\sigma} \, di \right)^{\sigma/(\sigma-1)}, \quad \sigma > 1. \tag{FPRF}$$

(When possible, time subscripts are omitted.) Final goods are produced under perfect competition, with each producer solving

$$\max_{[Y(i)]_0^1} Y - \int_0^1 P(i) Y(i) \, di,$$

subject to equation (FPRF).

Intermediate goods producers, on the other hand, are price setters. The production function for intermediate goods is given by

$$Y(i) = L(i)^{\alpha}, \quad \alpha \in (0,1), \tag{PRF}$$

where L is labor. Letting W denote the real wage, equation (PRF) then implies that the total cost of producing good i is

$$\Phi(Y(i)) = WL(i) = WY(i)^{1/\alpha}. \tag{TC}$$

Each intermediate goods producer operates under uncertainty, in that it chooses its price before it observes the choices of other producers. Producer i thus solves

$$\max_{P_t(i)} E_{t-1}\{\Pi_t(i)\} = E_{t-1}\{P_t(i) \times Y_t[P_t(i)] - \Phi_t(Y_t[P_t(i)])\},$$

where $\Pi_t(i)$ is producer i's profits, and $Y_t[P_t(i)]$ is the demand function for good i.

1. (8 points.) Solving the maximization problem for a final goods producer, show that the intermediate goods quantity $Y(i)$ can be written as a function of the intermediate price $P(i)$ and the aggregate quantity Y.

2. (8 points) Using your answer to question 1, solve the maximization problem for an intermediate goods producer. In particular, show that

$$P_t(i) = \mu \frac{E_{t-1}(\phi_t(i) Y_t)}{E_{t-1}(Y_t)},$$

where $\phi(i) \equiv \frac{\partial \Phi(i)}{\partial Y(i)}$ is firm i's marginal cost, and $\mu > 1$ is the markup ratio.

3. (18 points.) Now consider a symmetric equilibrium, where $Y_t(i) = Y_t(j)$, $\forall i, j, t$.

   (a) Show that in a symmetric equilibrium

   $$Y_t(i) = Y_t.$$

   (b) Since final output is the numeraire, one can show that

   $$P_t(i) = P_t = 1.$$

   Using this result, show that your answer to question 2 can be written as

   $$\frac{E_{t-1}(\phi_t Y_t)}{E_{t-1}(Y_t)} = \frac{\sigma - 1}{\sigma}.$$

   Note that ϕ_t is also the marginal cost of final output. What is the steady-state marginal cost, ϕ_{SS}? Is this cost bigger or smaller than the the price of final output, 1? What is the economic reason for this?

   (c) Show that in a symmetric equilibrium:

   $$\phi_t = W_t \left(\alpha \frac{Y_t}{L_t}\right)^{-1}. \qquad \text{(MC)}$$

Now let's consider consumers. The preferences of the representative household over consumption, C_t, and labor are given by

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \left[\ln(C_t) - \chi \frac{1}{1+\gamma} L_t^{1+\gamma} \right] \right),$$

$$0 < \beta < 1, \quad \gamma \geq 0, \quad \chi > 0,$$

Households receive labor income and profits from firms. Households earn a gross return of $(1+r) K_t$ on their assets, K_t, with $\beta(1+r) = 1$. As usual, assume that assets held at the beginning of period $t+1$, K_{t+1}, are chosen in period t. Note that in this economy, capital is used only as a storage device, and not as a factor of production. Households spend their income on consumption and investment in capital. Households also face the usual initial, non-negativity and No-Ponzi-Game conditions.

4. (8 points.) Find the first order conditions for utility maximization.

5. (6 points.) Imposing equilibrium, find: the Euler equation for capital; and the capital accumulation equation/resource constraint.

6. (8 points) Imposing equilibrium, eliminate wages and labor to express output as a function of consumption and marginal cost.

7. (12 points.) Let lower-case letters with carats "$\widehat{\ }$" denote deviations of logged variables around their steady state values. Recalling your answer to question 3, show that the deviation of marginal cost, $\widehat{\phi}_t$, must obey:

$$E_{t-1}\left(\widehat{\phi}_t\right) \approx 0. \qquad \text{(MC')}$$

Hint: At low levels of variance, we have:

$$\ln\left(E_t\left(X_t\right)\right) \approx E_t\left(\ln\left(X_t\right)\right).$$

Looking ahead, will this condition strengthen or weaken the model's ability to produce realistic economic fluctuations? Briefly explain.

8. (6 pts.) Using your answer to question 6, show that output can be approximated as

$$\widehat{y}_t = \lambda\left[\widehat{\phi}_t - \widehat{c}_t\right]. \qquad \text{(PRF}')$$

9. (18 points.) Suppose that the steady state consumption-to-capital ratio, C/K, is ψ, with $\psi > r$. It is straightforward to show that the steady state output-to-capital ratio, Y/K, is $\psi - r$, and that the log-linearized capital accumulation equation is:

$$\widehat{k}_{t+1} = (1+r)\,\widehat{k}_t + \omega_1\widehat{\phi}_t - \omega_2\widehat{c}_t, \qquad \text{(CA}')$$

with $\omega_2 > \omega_1 > 0$. Similarly, one can show that the log-linearized Euler equation is:

$$E_t\left(\widehat{c}_{t+1}\right) = \widehat{c}_t. \qquad \text{(EE}')$$

(Take all these results as given.) Let's solve the system given by (MC′), (CA′) and (EE′) with the method of undetermined coefficients. In particular, assume that consumption can be written as a function of capital and technology:

$$\widehat{c}_t = \eta\widehat{k}_t + \theta\widehat{\phi}_t, \qquad \text{(CF)}$$

and proceed as follows:

(a) Insert equation (CF) into equation (CA′) to express $\widehat{k}_{t+1}$ as a function of $\widehat{k}_t$ and $\widehat{\phi}_t$:

$$\widehat{k}_{t+1} = \pi_1\widehat{k}_t + \pi_2\widehat{\phi}_t. \qquad \text{(CA}'')$$

(b) Insert equation (CF) into the *right*-hand-side of equation (EE′) to express $E_t\left(\widehat{c}_{t+1}\right)$ as a function of $\widehat{k}_t$ and $\widehat{\phi}_t$.

(c) Insert equation (CF) into the *left*-hand-side of equation (EE′) to express $E_t\left(\widehat{c}_{t+1}\right)$ as a function of $E_t\left(\widehat{k}_{t+1}\right)$ and $E_t\left(\widehat{\phi}_{t+1}\right)$. Then insert (CA″) and (MC′) to express $E_t\left(\widehat{c}_{t+1}\right)$ as a function of $\widehat{k}_t$ and $\widehat{\phi}_t$.

(d) Compare the two expressions for $E_t\left(\widehat{c}_{t+1}\right)$. If the two expressions are to be identical, what must be the values of η and θ?

10. (8 points.) Note that the shocks to $\widehat{\phi}_t$ are *extrinsic*, in that they do not reflect "fundamental" shocks such as shocks to the Solow residual. What are the features of the model that make these sunspot shocks feasible? Briefly explain, using the language of Cooper and John (1988). (**Hint:** Recall your answer to question 1.)