Economics 701: Macroeconomics II
Spring 2005
Professor Jones

Midterm Examination
March 31, 2005

Instructions. Answer all the questions in your bluebook. There are two questions, and they are weighted equally. You have 90 minutes to complete the exam. Good luck!

1. Consider the following variant of the Lucas tree model. There are two types of trees: apple trees, which will be indexed by "a"; and banana trees, which will be indexed by "b". Each tree gives off d_t^i, $i \in \{a, b\}$, units of non-storable fruit (dividends). The preferences of the representative consumer are

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \left(f c_t - \frac{g}{2} c_t^2 \right) \right), \quad 0 < \beta < 1, \quad f, g > 0,$$

where c_t denotes total consumption of apples and bananas; consumers are indifferent between consuming one apple fruit or one banana. You can assume that $0 < c_t < f/g$.

The dividend processes for the two trees follow

$$\begin{aligned} d_t^a &= \widehat{d}_t + \varepsilon_t^a, \\ d_t^b &= \widehat{d}_t + \varepsilon_t^b, \end{aligned}$$

where $\widehat{d}_t$ follows a time-invariant Markov process. The idiosyncratic shocks ε_t^a and ε_t^b share the same zero-mean i.i.d. distribution, but are mutually independent of each other and $\widehat{d}_t$ at all leads and lags. As a matter of notation, let

$$V\left(\varepsilon_t^a\right) = V\left(\varepsilon_t^b\right) = \sigma^2, \quad \forall t,$$

let $\varepsilon_t \equiv \frac{1}{2}[\varepsilon_t^a + \varepsilon_t^b]$ denote the average idiosyncratic shock, and let $d_t \equiv \frac{1}{2}[d_t^a + d_t^b] = \widehat{d}_t + \varepsilon_t$ denote the average dividend. The economy starts off with half of the households owning one apple tree apiece, and the other half owning one banana tree. In addition, initial dividends just happen to be equivalent: $d_0^a = d_0^b$.

   (a) Solve the social planner's problem for this economy. You can assume that the social planner cares equally about all consumers. What is the optimal allocation of consumption? What distribution of trees would achieve this allocation in a competitive economy?

   (b) Under the equilibrium allocation, the state of the aggregate economy is characterized by two variables. What are they? Let p_t^a (p_t^b) denote the price at time t of a title to all future dividends from an apple (banana) tree, and let s_t^a (s_t^b) be the number of apple (banana) trees that the consumer owns at the beginning of time t. Letting x_t denote the consumer's financial resources, write down Bellman's functional equation for the consumer's problem, and derive the Euler equations associated with stocks.

(c) Derive the equilibrium pricing functions for apple and banana stocks, p_t^i, $i \in \{a, b\}$. (This function should not include any expected future prices.) Simplify your answer, by eliminating any terms with an "i" superscript, to show that $p_t^a = p_t^b$.

(d) Suppose the consumer could also purchase shares of a "mutual fund", with one share of the mutual fund giving the consumer 1/2 share apiece of apple and banana stocks. Let p_t denote the price of a share of the mutual fund; it is easy to see that d_t is the associated dividend. Using (preferably simple) arguments, show that $p_t = p_t^a = p_t^b$.

(e) Let $R_t^a \equiv \left(p_{t+1}^a + d_{t+1}^a\right)/p_t^a$ denote the realized rate of return on apple stocks, and let R_t^b and R_t denote returns on banana stocks and mutual funds. Compare the time-t expected rates of return, $E_t\left(R_t^i\right)$. It is immediate that d_t is less volatile than d_t^a or d_t^b. Does the mutual fund have a higher price, or offer a lower expected return, in exchange for having safer dividends? Why or why not?

2. Let y_t, m_t and p_t denote the logarithms of output, the money stock and the price level, respectively, and let π_t denote inflation. Consider the following ad hoc macroeconomic model:

$$y_t = p_t - p_t^e, \qquad \text{(AS)}$$

$$y_t = m_t - p_t + \eta_t, \qquad \text{(AD)}$$

where p_t^e denotes the private sector's subjective expectations of the future (logged) price level. p_t^e might not be the statistically optimal forecast, i.e., private agents might not have rational expectations. However, the information set used to construct p_t^e cannot exceed the information set I_{t-1}, where

$$I_{t-1} = \left\{y_{t-j}, p_{t-j}, m_{t-j}, \pi_{t-j}, p_{t-j}^e, \eta_{t-j},\right\}_{j=1}^{\infty},$$

The demand shock η_t is a zero-mean exogenous i.i.d. variable, with $V(\eta_t) = \sigma_\eta^2$.

(a) Express output and prices as functions of m_t, η_t and p_t^e.

(b) Suppose that agents have adaptive expectations

$$p_t^e = \phi p_{t-1}, \quad \phi \in (0, 1),$$

and that the money supply rule is

$$\begin{aligned} m_t &= \gamma p_{t-1}, \\ \gamma &= \arg\min_{\gamma_0} \alpha V(y_t) + V(p_t), \end{aligned}$$

with $\alpha \geq 0$.

i. Taking γ as given, express output and prices as functions of η_t and p_{t-1}.

ii. Find γ.

iii. Interpret γ for $\alpha = 0$, $\alpha = \infty$ (or the limit as $\alpha \to \infty$), and $\alpha = 1$.

(c) Now suppose that agents have rational expectations

$$p_t^e = E\left(p_t \middle| I_{t-1}\right),$$

where $E\left(\cdot \middle| I_{t-1}\right)$ denotes mathematical expectations conditioned on the information set I_{t-1}. Suppose further that the central bank uses the money supply rule described in part (b), but revises γ to reflect the new private sector forecasting rule.

i. Taking γ as given, express output and prices as functions of η_t and p_{t-1}.

ii. Find γ.

iii. Interpret γ.

(d) It is often thought that central banks "lean against the wind" (in terms of inflation), by reducing the money supply when prices have been rising. Given your answers to parts (b) and (c) above, is "leaning against the wind" a better policy when expectations are adaptive, or when expectations are fully rational?