Economics 701: Macroeconomics II
Spring 2005
Professor Jones

Final Examination
May 9, 2005

Instructions. Answer all the questions in your bluebook. You have 120 minutes to complete the exam. Good luck!

Consider the following simplified version of a stochastic growth model with non-separable preference. There is a representative price-taking firm that solves

$$\max_{L_t \geq 0} \Pi_t = Y_t - W_t L_t,$$

$$Y_t = Z_t L_t^{1-\alpha}, \quad 0 \leq \alpha < 1, \tag{PRF}$$

Π_t is profit; L_t is labor; Z_t is the exogenous technology level; and W_t is the real wage. The population, number of firms and steady state technology level are constant and normalized to 1, so that upper case letters denote intensive as well as aggregate quantities.

Ownership of the firm resides in stocks. Each period, a share entitles its owner to a share of the firm's profits—the firm retains no earnings. We will normalize the number of shares to 1 and impose the equilibrium result that no shares are traded.

The preferences of the representative household over consumption, C_t, and labor, L_t, are given by

$$E_0 \left(\sum\nolimits_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \left(C_t^{\nu} (1-L_t)^{1-\nu} \right)^{1-\gamma} \right),$$

$$0 < \beta < 1, \quad \gamma \geq 0, \quad 0 < \nu < 1,$$

Households receive labor income and profits from firms. Households earn a gross return of $(1+r) K_t$ on their assets, K_t, with $\beta(1+r) = 1$. As usual, assume that assets held at the beginning of period $t+1$, K_{t+1}, are chosen in period t. Note that in this economy, capital is used only as a storage device, and not as a factor of production. Households spend their income on consumption and investment in capital. Households also face the usual initial, non-negativity and No-Ponzi-Game conditions.

The log of technology follows an AR(1) process:

$$\widehat{z}_t \equiv \ln(Z_t) = \phi \widehat{z}_{t-1} + \varepsilon_t, \quad 0 \leq \phi < 1. \tag{TS}$$

where $\{\varepsilon_t\}$ is an exogenous stationary martingale difference sequence.

1. (4 points.) Find the first order conditions for profit maximization.

2. (14 points.) Write down the consumer's problem in recursive form and find the first order conditions.

3. (5 points.) In their 1988 JME article, King, Plosser and Rebelo point out that the preference specification used here is consistent with labor supply being constant along a balanced growth path. Using your answer from question 2, show that this is in fact the case.

4. (12 points.) Low (2005) shows that if the derivative $-\dfrac{\partial^2 U(C,L)}{\partial C \partial L}$ is positive, consumption and **leisure** (note the negative sign on the derivative) are complements; if it is negative, consumption and leisure are substitutes.

   (a) Show that consumption and leisure will be substitutes when $\gamma > 1$.

   (b) Give one brief intuitive argument for why consumption and leisure might be substitutes.

   (c) Give one brief intuitive argument for why consumption and leisure might be complements.

5. (9 points.) Imposing equilibrium, find: the labor allocation condition; the Euler equation for capital; and the capital accumulation equation/resource constraint.

6. (10 points.) Let lower-case letters with carats "$\widehat{\ }$" denote deviations of logged variables around their steady state values. Show that log-linear approximations for labor and output are

$$\begin{aligned} \widehat{\ell}_t &= \theta \left[\widehat{z}_t - \widehat{c}_t \right], \\ \widehat{y}_t &= (1+\lambda)\, \widehat{z}_t - \lambda \widehat{c}_t. \end{aligned} \tag{PRF$'$}$$

7. (20 points.) Let's consider the Euler equation.

   (a) Log-linearize the Euler equation, and insert your results from question 6, to show that

$$\widehat{c}_t + \omega_3 (1-\phi)\, \widehat{z}_t \approx E_t\left(\widehat{c}_{t+1}\right). \tag{EE$'$}$$

   Hint: At low levels of variance, we have:

$$\ln\left(E_t\left(X_t\right)\right) \approx E_t\left(\ln\left(X_t\right)\right).$$

   (b) What is the sign of ω_3 for $\gamma > 1$? In light of your answer to question 4, does this make sense to you? Briefly explain. (**Hint:** How would an increase in $\widehat{z}_t$ affect equilibrium labor/leisure hours?)

8. (26 points.) Suppose that the steady state consumption-to-capital ratio, C/K, is ψ, with $\psi > r$. It is then straightforward to show that the log-linearized capital accumulation equation is:

$$\widehat{k}_{t+1} \approx (1+r)\,\widehat{k}_t + \omega_1 \widehat{z}_t - \omega_2 \widehat{c}_t, \tag{CA$'$}$$

with $\omega_2 > \omega_1 > 0$. (Take this result as given.) Together, equations (EE$'$), (CA$'$) and (TS) form a trivariate linear expectational difference equation. Solve this system with the method of undetermined coefficients. In particular, assume that consumption can be written as a function of capital and technology:

$$\widehat{c}_t = \eta \widehat{k}_t + \mu \widehat{z}_t, \tag{CF}$$

and proceed as follows:

(a) Modify equation (CA$'$) to express $\widehat{k}_{t+1}$ as a function of $\widehat{k}_t$ and $\widehat{z}_t$:

$$\widehat{k}_{t+1} = \pi_1 \widehat{k}_t + \pi_2 \widehat{z}_t. \tag{CA$''$}$$

(b) Modify the *right*-hand-side of equation (EE$'$) to express $E_t\left(\widehat{c}_{t+1}\right)$ as a function of $\widehat{k}_t$ and $\widehat{z}_t$.

(c) Modify the *left*-hand-side of equation (EE$'$) to express $E_t\left(\widehat{c}_{t+1}\right)$ as a function of $E_t\left(\widehat{k}_{t+1}\right)$ and $E_t\left(\widehat{z}_{t+1}\right)$. Substitute for $\widehat{k}_{t+1}$ and $\widehat{z}_{t+1}$ to express $E_t\left(\widehat{c}_{t+1}\right)$ as a function of $\widehat{k}_t$ and $\widehat{z}_t$.

(d) Compare the two expressions for $E_t\left(\widehat{c}_{t+1}\right)$. If the two expressions are to be identical, what must be the values of η and μ?

(e) What is the sign of μ for $\gamma > 1$? Is this sign consistent with observed output-consumption cross correlations? Put differently, if $\widehat{z}_t$ increases, will output and consumption respond in a way consistent with the data? Briefly explain. (**Hint:** You can assume that the output response is the standard one.)