Economics 701: Macroeconomics II

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Lecture 5: Real Business Cycles

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8. Criticisms and Extensions

(e) Cooper and Johri (JME, 2002)

- Learning-by-doing (LBD)
  - LBD: productivity acquired through experience
  - Organizational capital (OC): accumulated institution-specific knowledge. LBD is a form of OC accumulation.
  - This paper: LBD as a propagation mechanism.
- Micro data show:
  - Significant LBD: 1% increase in cumulative output $\Rightarrow$ 0.2% increase in productivity.
  - Significant depreciation of OC $\Rightarrow$ OC can vary with the business cycle.

7. (e) Cooper and Johri (JME, 2002)

- Technologies

$$\begin{aligned} K_{t+1} &= (1-\delta) K_t + I_t, \\ I_t &= Y_t - C_t, \\ H_{t+1} &= \phi(H_t, Y_t) = H_t^{\gamma} Y_t^{\eta}, \\ Y_t &= A_t f(H_t, K_t, L_t) = A_t H_t^{\varepsilon} K_t^{\theta} L_t^{\alpha}, \end{aligned}$$

where

$$\begin{aligned} H_t &= \text{organizational capital}, \\ A_t &= \text{exogenous stochastic technology}. \end{aligned}$$

Combine to get:

$$H_{t+1} = \Lambda(A_t, K_t, H_t, L_t) = A_t^{\eta} K_t^{\eta\theta} H_t^{\gamma+\eta\varepsilon} L_t^{\eta\alpha}.$$

7. (e) Cooper and Johri (JME, 2002)

- Preferences

$$E_0\left(\sum_{t=0}^{\infty}\beta^t u\left(C_t, 1-L_t\right)\right),$$

$$u(C_t, 1-N_t) = \ln\left(C_t\right) + \chi\left(1-L_t\right).$$

- Social Planner's Problem

$$V(A_t, K_t, H_t) =$$

$$\max_{\{K_{t+1}, L_t\}} u\left(A_t f\left(H_t, K_t, L_t\right) + \left(1-\delta\right)K_t - K_{t+1}, 1-L_t\right)$$

$$+ \beta E_t\left\{V\left(A_{t+1}, K_{t+1}, \Lambda\left(A_t, K_t, H_t, L_t\right)\right)\right\}.$$

Note: Social planner accounts for returns to LBD (probably not important for results).

7. (e) Cooper and Johri (JME, 2002)

- Calibration
  - Production function estimated from industry-level data.
  - "Preferred" Specification: IRS-PF with $\alpha = 0.6$, $\theta = 0.4$, $\varepsilon = 0.27$, $\gamma = 0.5$, $\eta = 0.5$. Significant LBD and significant depreciation of OC.
- Results
  - Hump-shaped IRF for output.
  - Volatility of output significantly increases.
  - Corr(ℓ, y/ℓ) decreases.
  - Output growth has a positive autocorrelation.
  - Changes in Solow residual: Changes in OC "look like" changes in labor effort.

7. Criticisms and Extensions

(f) Boldrin, Christiano and Fisher (AER, 2001)

- Motivation: Equity premium puzzle
  - Suppose flow utility is

$$N_t \left(\frac{c_t^{1-\theta} - 1}{1-\theta} \right), \quad c_t \equiv \frac{C_t}{N_t}.$$

  - Euler eqn for asset i:

$$c_t^{-\theta} = \beta E_t \left(\left[1 + r_{t+1}^i \right] c_{t+1}^{-\theta} \right)$$

$$\Leftrightarrow 1 + b = E_t \left(\left[1 + g_{t+1}^c \right]^{-\theta} \left[1 + r_{t+1}^i \right] \right), \quad \text{(EE)}$$

$$1 + b \equiv \beta^{-1}, \quad g_{t+1}^c \equiv \frac{c_{t+1}}{c_t} - 1.$$

7. (f) Boldrin, Christiano and Fisher (AER, 2001)

- Motivation: Equity premium puzzle
  - A Second-order Taylor expansion around $\overline{g}^c = 0$, $\overline{r}^i = 0$ shows:

$$
\begin{aligned}
&(1+g^c)^{-\theta}\left(1+r^i\right) \approx \\
&\qquad (1+\overline{g}^c)^{-\theta}\left(1+\overline{r}^i\right) + (1+\overline{g}^c)^{-\theta}\left(r^i-\overline{r}^i\right) \\
&\qquad -\theta\,(1+\overline{g}^c)^{-\theta-1}\left(1+\overline{r}^i\right)(g^c-\overline{g}^c) \\
&\qquad +\frac{1}{2}\Big[(\theta+1)\,\theta\,(1+\overline{g}^c)^{-\theta-2}\left(1+\overline{r}^i\right)(g^c-\overline{g}^c)^2 \\
&\qquad\qquad -2\theta\,(1+\overline{g}^c)^{-\theta-1}(g^c-\overline{g}^c)\left(r^i-\overline{r}^i\right)\Big] \\
&= \; 1+r^i-\theta g^c+\frac{1}{2}\Big[(\theta+1)\,\theta\,(g^c)^2-2\theta g^c r^i\Big].
\end{aligned}
$$

7. (f) Boldrin, Christiano and Fisher (AER, 2001)

- Motivation: Equity premium puzzle
  - Insert this approximation into (EE):

$$1+b \approx E_t\Bigg(1+r^i_{t+1}-\theta g^c_{t+1}+ \frac{(\theta+1)\,\theta}{2}\left(g^c_{t+1}\right)^2-\theta g^c_{t+1}r^i_{t+1}\Bigg),$$

$$E_t\left(r^i_{t+1}\right) \approx b+\theta E_t\left(g^c_{t+1}\right)-\frac{(\theta+1)\,\theta}{2}Var_t\left(g^c_{t+1}\right) + \ \theta Cov_t\left(g^c_{t+1},r^i_{t+1}\right).$$

7. (f) Boldrin, Christiano and Fisher (AER, 2001)

- Motivation: Equity premium puzzle
  - The return on a risk-free asset is

$$\begin{aligned} r_{t+1}^f &= E_t\left(r_{t+1}^f\right) \\ &= b + \theta E_t\left(g_{t+1}^c\right) - \frac{(\theta+1)\,\theta}{2} Var_t\left(g_{t+1}^c\right). \end{aligned}$$

  - The equity premium is

$$E_t\left(r_{t+1}^e\right) - r_{t+1}^f = \theta Cov_t\left(g_{t+1}^c, r_{t+1}^e\right). \qquad \text{(EP)}$$

7. (f) Boldrin, Christiano and Fisher (AER, 2001)

- Motivation: Equity premium puzzle
  - Data show

$$\begin{aligned} \sigma_r &= 0.167, \\ \sigma_g &= 0.036, \\ Corr(g,r) &= 0.4, \\ \Rightarrow cov\,(g,r) &= 0.0024, \\ E\,(g^c) &= 0.02, \\ E_t\left(r^e_{t+1}\right) - r^f_{t+1} &= 0.06, \end{aligned}$$

  - It then follows from equation (EP) that $\theta = 25$!

7. (f) Boldrin, Christiano and Fisher (AER, 2001)

- Motivation: Equity premium puzzle
  - Solving equation (EP) for θ with implies that $\theta = 25$!
  - Moreover,

$$r^f_{t+1} = b + 25 \times 0.02 - \frac{25 \times 26}{2} 0.036^2.$$

    Then $b \geq 0 \Rightarrow r^f_{t+1} \gg 0$. But $r^f_{t+1} \approx 0.01$.
  - The model either underpredicts the equity premium or overpredicts the risk-free rate.
- BCF solution: habit persistence and factor market rigidities.

7. (f) Boldrin, Christiano and Fisher (AER, 2001)

- Preferences

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t \left[\ln \left(C_t - bC_{t-1} \right) - \chi L_t \right] \right),$$

$b \geq 0$ measures the persistence of habit.

7. (f) Boldrin, Christiano and Fisher (AER, 2001)

- Technology
  - Two-sector model

$$
\begin{aligned}
C_t &= K_{Ct}^{\alpha}\left(Z_t L_{Ct}\right)^{1-\alpha}, \\
K_{Ct+1}+K_{It+1} &= K_{It}^{\alpha}\left(Z_t L_{It}\right)^{1-\alpha}+ \\
&\quad (1-\delta)\left(K_{Ct}+K_{It}\right), \\
L_t &= L_{It}+L_{Ct}, \\
Z_t &= \exp\left(x_t\right) Z_{t-1}, \\
x_t &\sim \mathcal{N}\left(\overline{x}, \sigma^2\right), \text{ i.i.d.},
\end{aligned}
$$

  - L_{Ct}, L_{It}, K_{Ct}, K_{It} chosen before technology shock Z_t is observed.

7. (f) Boldrin, Christiano and Fisher (AER, 2001)

- Social planner's problem:

$$\mathcal{L} = \min_{\{\lambda_{It},\lambda_{Ct}\}} \max_{\{L_{Ct+1},L_{It+1},K_{Ct+1},K_{It+1},C_t\}}$$

$$E_0\left(\sum_{t=0}^{\infty}\beta^t\Big[\ln\left(C_t - bC_{t-1}\right) - \chi\left(L_{Ct}+L_{It}\right)\right.$$

$$+\lambda_{It}\Big(K_{It}^{\alpha}\left(Z_tL_{It}\right)^{1-\alpha} + (1-\delta)\left(K_{Ct}+K_{It}\right)$$

$$-K_{Ct+1}-K_{It+1}\Big)$$

$$\left.+\lambda_{Ct}\Big(K_{Ct}^{\alpha}\left(Z_tL_{Ct}\right)^{1-\alpha} - C_t\Big)\Big]\right).$$

7. (f) Boldrin, Christiano and Fisher (AER, 2001)

- Risk-free Return

$$
\begin{aligned}
1 + r^f_{t+1} &= \frac{\lambda_{Ct}}{\beta E_t \left(\lambda_{Ct+1}\right)} \\
&= \frac{\left[\dfrac{1}{C_t - bC_{t-1}} - E_t\left(\dfrac{\beta b}{C_{t+1} - bC_t}\right)\right]}{\beta E_t\left(\dfrac{1}{C_{t+1} - bC_t} - \dfrac{\beta b}{C_{t+2} - bC_{t+1}}\right)}.
\end{aligned}
$$

7. (f) Boldrin, Christiano and Fisher (AER, 2001)

- Return on equity:

$$
\begin{aligned}
P_{Kt} &= \frac{\lambda_{It}}{\lambda_{Ct}} = \text{ value of capital goods,} \\
1 + r^{eC}_{t+1} &= \frac{1}{P_{Kt}} \left[\alpha K^{\alpha-1}_{Ct+1} \left(Z_{t+1} L_{Ct+1} \right)^{1-\alpha} + (1-\delta) P_{Kt+1} \right], \\
1 + r^{eI}_{t+1} &= \frac{1}{P_{Kt}} \left[\alpha K^{\alpha-1}_{It+1} \left(Z_{t+1} L_{It+1} \right)^{1-\alpha} + 1 - \delta \right] P_{Kt+1}, \\
r^{e}_{t+1} &= r^{eC}_{t+1} \frac{K_{Ct+1}}{K_{t+1}} + r^{eI}_{t+1} \frac{K_{It+1}}{K_{t+1}}.
\end{aligned}
$$

7. (f) Boldrin, Christiano and Fisher (AER, 2001)

- Results for asset prices
  - Can match both risk-free rate and equity premium.
  - Habit formation: Increases risk-aversion while retaining log (balanced-growth) specification.
  - Factor market rigidity: Increases volatility of P_{Kt}, increasing volatility of r^e_{t+1}.
  - In baseline RBC model P_{Kt} is identically 1. If L_{Ct}, L_{It}, K_{Ct}, K_{It} adjusted to x_t, P_{Kt} would equal 1 here as well.

7. (f) Boldrin, Christiano and Fisher (AER, 2001)

- Business cycle results
  - Positive autocorrelation in output growth, due to lag in setting hours.
  - Both L_{Ct} and L_{It} are positively correlated with output.
  - Contrast with "mobile" labor where L_t is set in advance, but L_{Ct} and L_{It} are flexible. Under mobile labor, $x_t \uparrow \Rightarrow L_{It} \uparrow,\ L_{Ct} \downarrow$, as labor shifts to investment sector.
  - With "rigid" labor, L_{Ct} initially fixed, so that $x_t \uparrow \Rightarrow C_t \uparrow$. Then with habit persistence, C_{t+j} and L_{Ct+j} stay high.

7. (f) Boldrin, Christiano and Fisher (AER, 2001)

- Question: Do we need a 2-sector model or just capital frictions?
  - Consider two 1-sector models.
  - Model 1: Time-to-plan
    - K_t chosen 2 periods in advance.
    - Matches asset prices, but consumption too volatile.
    - Why? When x_t changes, initial impact is entirely on C_t. In the 2-sector model, I_t changes as well.

7. (f) Boldrin, Christiano and Fisher (AER, 2001)

- Question: Do we need a 2-sector model or just capital frictions?
  - Model 2: Adjustment cost

$$K_{t+1} = (1-\delta) K_t + \phi\left(\frac{I_t}{K_t}\right) K_t,$$

$$\phi'(\cdot) > 0, \quad \phi''(\cdot) < 0.$$

    - Matches asset prices, but hours are countercyclical.
    - Why? Workers are paid in consumption goods. $x_t \uparrow \Rightarrow i_t \uparrow \Rightarrow P_{Kt} \uparrow \Rightarrow P_{Ct} \downarrow \Rightarrow$ effective wage$\downarrow$.
- Issue: No solid micro evidence of habits.

8. Evaluation

(a) Successes

- A simple dynamic optimizing model can explain many features of the economy.
- Allows precise use of economic theory.
- Has brought numerical methods and simulation into the macroeconomic mainstream.

(b) Failures

- Numerous counterfactual implications.
- Has failed many statistical tests.
- “Fixing” the model often makes it harder to interpret.

8. Evaluation

(c) Philosophical issue 1: Calibration & simulation vs. econometrics & prediction

- A model that is statistically rejected can still be useful: "It takes a model to beat a model."
- Calibration and simulation are here to stay.
- But DGE models are not widely used for (short-term) forecasting—what are we learning?
- Smets and Wouters (2003) argue that DGE models with frictions—both nominal and real—*do* successfully forecast. (Are they interpretable?)

8. Evaluation

(d) Philosophical issue 2: Are representative agent models meaningful?

- The representative agent approach is valid only under extremely strong conditions.
- If the representative agent approach is in error, are the RBC model's insights valid?
- Heterogeneity is an active field.
- Complexity, multiple equilibria theory suggest whole is bigger than the sum of its parts.
- Advantage of "top-down" IS-LM approach: start with aggregate relationships.

8. Evaluation

(e) Philosophical issue 3: can we model the economy without money?

- Can do better than expected.
- But empirical evidence overwhelmingly shows that money has real effects.
- Most scholars not convinced by “reverse causality” arguments.
- Large literature addressing money in DGE models.
- Many of these models are *Neo-Keynesian* models that include price and/or information frictions.
- Lucas (1994) argues that the RBC model fits the data so well *because* monetary policy has real effects: Postwar monetary policy has neutralized financial shocks and distortions.

8. Evaluation

   (f) Philosophical issue 4: is the fully-rational, perfect-market framework reasonable?

   - Essential to questions of welfare: RBC model implies that government intervention is unnecessary.
   - Neo-Keynesians: Use RBC modelling approach to consider market imperfections; derive Keynesian-type behavior from microeconomic foundations.
   - Most DGE forecasting models have Neo-Keynesian elements.

8. Evaluation

(g) Evolution

- The baseline RBC model is not a “true” model of the economy, but a starting point for other analyses: money, imperfect competition, etc.
- Vote on a new name: no longer “RBC,” now “DGE.”
- DGE models of business cycles are now the dominant paradigm.