

Economics 701: Macroeconomics II
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**Lecture 7: Introduction to New Keynesian
Models of Money**

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1. The MIU Model

(a) The Enhanced Sidrauski (1967) model

- Consider a “yeoman farmer” that solves

$$\begin{aligned} \max_{\{C_t, L_t, M_t, K_{t+1}\}_{t=0}^{\infty}} & \sum_{t=0}^{\infty} \tilde{\beta}^t N_t U \left(\frac{C_t}{N_t}, \frac{M_t}{P_t N_t}, 1 - \frac{L_t}{N_t} \right) \\ \text{s.t.} & M_t + P_t [C_t + K_{t+1}] = M_{t-1} + \\ & P_t \left[\tau_t N_t + A_t F(K_t, L_t) + (1 - \tilde{\delta}) K_t \right], \\ & L_t \in [0, N_t], \end{aligned}$$

and other boundary conditions.

- M_t denotes money, τ_t denotes lump-sum taxes.
- Including money in the utility function is a simple way to capture its transaction benefits.

1. The MIU Model

(a) The Enhanced Sidrauski (1967) model

- The production function $F(\cdot, \cdot)$ has constant returns and satisfies the Inada conditions.
- Assume that:

$$N_t = N_0 N^t, \quad N_0, N > 0, \quad \tilde{\beta}N < 1,$$

and that $\{A_t\}$ is perfectly predictable.

- Define:

$$k_{t+1} = \frac{K_{t+1}}{N_t}, \quad y_t = \frac{Y_t}{N_t}, \quad c_t = \frac{C_t}{N_t}, \quad \ell_t = \frac{L_t}{N_t},$$
$$m_t = \frac{M_t}{N_t P_t}, \quad 1 + \pi_t = \frac{P_t}{P_{t-1}}.$$

1. (a) The model

- In intensive terms, the yeoman farmer solves

$$\begin{aligned} \max_{\{c_t, l_t, m_t, k_{t+1}\}_{t=0}^{\infty}} & \sum_{t=0}^{\infty} \beta^t U(c_t, m_t, 1 - l_t) \\ \text{s.t.} \quad & m_t + c_t + k_{t+1} = \frac{m_{t-1}}{(1 + \pi_t)N} \quad (\text{FBC}) \\ & + \tau_t + A_t F(N^{-1}k_t, l_t) + (1 - \delta)k_t, \\ & L_t \in [0, N_t], \\ & \beta \equiv \tilde{\beta}N, \quad \delta \equiv 1 - \frac{1 - \tilde{\delta}}{N}. \end{aligned}$$

and other boundary conditions.

1. (a) The model

- Letting λ_t be the multiplier on (FBC), the F.O.C. are

$$\begin{aligned}U_c[t] &= \lambda_t, \\U_{1-\ell}[t] &= \lambda_t A_t F_\ell[t], \\ \lambda_t &= U_m[t] + \beta \lambda_{t+1} \frac{1}{(1 + \pi_{t+1})N}, \\ \lambda_t &= \beta \lambda_{t+1} (A_{t+1} N^{-1} F_k[t + 1] + 1 - \delta).\end{aligned}$$

- These simplify to:

$$U_{1-\ell}[t] = U_c[t] A_t F_\ell[t], \quad (\text{LL})$$

$$U_c[t] = U_m[t] + \beta U_c[t + 1] \frac{1}{(1 + \pi_{t+1})N}, \quad (\text{EEM})$$

$$U_c[t] = \beta U_c[t + 1] (1 + A_{t+1} N^{-1} F_k[t + 1] - \delta). \quad (\text{EEK})$$

1. (a) The model

- Assume that lump-sum taxes exactly offset seigniorage revenue

$$\begin{aligned} N_t \tau_t &= \frac{M_t - M_{t-1}}{P_t}, \\ \Rightarrow \tau_t &= m_t - \frac{m_{t-1}}{(1 + \pi_t)N} \quad (\text{GBC}) \\ &= m_t - m_{t-1} + m_{t-1} \frac{(1 + \pi_t)N - 1}{(1 + \pi_t)N}. \end{aligned}$$

- N.B. Seigniorage includes revenues from new (real) currency issuances and capital losses from inflation on existing currency.

1. (a) The model

- Note that

$$1 + \theta_t \equiv \frac{M_t}{M_{t-1}}$$
$$\Rightarrow m_t = \frac{1 + \theta_t}{1 + \pi_t} m_{t-1}. \quad (\text{MS})$$

- Assume that $\{\theta_t\}$ is perfectly predictable.

1. (a) The model

- The equations that characterize an equilibrium are

$$U_{1-\ell}[t] = U_c[t]A_tF_\ell[t], \quad (\text{LL})$$

$$U_c[t] = U_m[t] + \beta U_c[t + 1] \frac{1}{(1 + \pi_{t+1})N}, \quad (\text{EEM})$$

$$U_c[t] = \beta U_c[t + 1](1 + A_{t+1}N^{-1}F_k[t + 1] - \delta), \quad (\text{EEK})$$

$$k_{t+1} = A_tF(N^{-1}k_t, \ell_t) - c_t + (1 - \delta)k_t, \quad (\text{RC})$$

$$m_t = \frac{1 + \theta_t}{1 + \pi_t} m_{t-1}, \quad (\text{MS})$$

- And
 - The processes for A_t and θ_t
 - The usual boundary conditions

1. The MIU Model

(b) Monetary neutrality and superneutrality

- If utility is separable in consumption-leisure and money, that is

$$U(c_t, m_t, 1 - \ell_t) = U(c_t, 1 - \ell_t) + U(m_t),$$

we have the **classical dichotomy**:

- Equations (LL), (EEK), (RC), and the process for $\{A_t\}$, determine the behavior of real quantities.
- Equations (EEM), (MS), and the process for $\{\theta_t\}$, determine the behavior of the money supply and prices (inflation).
- Neither the level nor the growth rate of money have any real effects.

1. The MIU Model

(b) Monetary neutrality and superneutrality

- If $U_{c,m} \neq 0$ or $U_{1-l,m} \neq 0$, there is **no** dichotomy:
 - Changes in m_t affect equilibrium conditions in equations (LL) and (EEK).
- Money can be neutral
 - Suppose M_0 changes, but $\{\theta_t\}_{t=1}^{\infty}$ does not.
 - Then $\{P_t\}_{t=0}^{\infty}$ changes proportionally, $\{\pi_t\}_{t=1}^{\infty}$ is unchanged, and all real quantities (including $\{m_t\}$) are unchanged.
- Money will not be superneutral: changing $\{\theta_t\}$ will affect $\{\pi_t\}$, $\{m_t\}$ and other real quantities.

1. The MIU Model

(c) Optimal money growth

- Insert equation (EEK) into equation (EEM):

$$\begin{aligned}\frac{U_m[t]}{U_c[t]} &= 1 - \beta \left(\frac{U_c[t+1]}{U_c[t]} \right) \frac{1}{(1 + \pi_{t+1})N}, \\ &= 1 - \beta \left(\frac{1}{\beta(1 + r_t)} \right) \frac{1}{(1 + \pi_{t+1})N},\end{aligned}$$

$$r_t \equiv A_{t+1}N^{-1}F_k[t+1] - \delta, \quad (\text{RR})$$

$$\Rightarrow \frac{U_m[t]}{U_c[t]} = 1 - \frac{1}{(1 + i_t)N},$$

$$i_t \equiv (1 + r_t)(1 + \pi_{t+1}) - 1, \quad (\text{IR})$$

$$\Rightarrow U_m[t] = U_c[t] \frac{(1 + i_t)N - 1}{(1 + i_t)N}. \quad (\text{CM0})$$

1. The MIU Model

(c) Optimal money growth

- When $N = 1$, equation (CM0) becomes:

$$U_m[t] = U_c[t] \frac{i_t}{1 + i_t}. \quad (\text{CM})$$

- Agent equates marginal benefit of renting money to its marginal cost – the nominal interest rate $\times MU_c$.
- Suppose utility is separable
 - Money is costless to manufacture $\Rightarrow m_t$ should be increased until $U_m[t] = 0$.
 - Equation (CM) $\Rightarrow U_m[t] = 0$ when $i_t = 0$.

1. (c) Optimal money growth

- When $N = 1$ and utility is separable, the efficient nominal interest rate is $i_t = 0$.
- However,

$$i_t = 0,$$

$$\Rightarrow (1 + r_t)(1 + \pi_{t+1}) - 1 = 0,$$

$$\Rightarrow 1 + \pi_{t+1} = \frac{1}{1 + r_t},$$

$$\Rightarrow \pi_{t+1} \approx -r_t.$$

- This is the **Friedman Rule** for the optimal rate of inflation: set the opportunity cost of holding money to zero.

1. The MIU Model

(d) Measuring money's effects

- Consider a stochastic version of the model with

$$A_t = \exp(z_t), \quad (\text{TFP})$$

$$z_t = \rho z_{t-1} + \varepsilon_t, \quad (\text{TS1})$$

$$\theta_t = \theta_{ss} + u_t, \quad (\text{MG})$$

$$u_t = \rho_m u_{t-1} + \phi z_t + \varphi_t, \quad (\text{TS2})$$

$$0 \leq \rho_z < 1, \quad 0 \leq \rho_m < 1,$$

$$E_t(\varepsilon_{t+1}) = 0, \quad E_t(\varphi_{t+1}) = 0.$$

- $\phi \neq 0$ means that money supply “shocks” may actually be responses to real shocks

1. The MIU Model

(d) Measuring money's effects

- Assume a Cobb-Douglas production function:

$$y_t = e^{z_t} k_t^\alpha \ell_t^{1-\alpha}. \quad (\text{PRF})$$

- The flow utility function is

$$U(c_t, m_t, 1 - \ell_t) = \frac{1}{1 - \zeta} \left(c_t m_t^b \right)^{1-\zeta} + \frac{\chi}{1 - \eta} (1 - \ell)^{1-\eta}, \quad (\text{UT})$$

$$b, \eta, \zeta, \chi > 0, \quad b(1 - \zeta) < 1.$$

- $\zeta = 1 \Rightarrow$ preferences over (c, m) are logarithmic and separable $\Rightarrow U_{cm} = 0 \Rightarrow$ superneutrality.
- $\zeta > 1 \Rightarrow U_{cm} < 0 \Rightarrow m$ and c are substitutes.

1. (d) Measuring money's effects

- Calibration
 - $\zeta = 2, b = 0.005$
 - Consider different values of ρ_m and ϕ
 - $\eta = 1, \theta_{ss} = 0.0125, \sigma_\varepsilon = 0.007, \sigma_\varphi = 0.0089$.
 - Other values standard
- Log-linearize and simulate
- Findings
 - Values of ρ_m and ϕ determine cyclicity of inflation.
 - Money has little real effect.

2. The Simplified and Linearized Model

(a) Log-linearization

- Consider a simplified model with $N = 1$ and $\zeta = 1$ (log-separability).
- As before, let “ $\hat{\cdot}$ ” denote logged deviations from the steady state.
- The following equations are the same as in the RBC model:

$$\hat{y}_t = z_t + \alpha \hat{k}_t + (1 - \alpha) \hat{l}_t. \quad (\text{PRF}')$$

$$\hat{k}_{t+1} \approx (1 - \delta) \hat{k}_t + \frac{y_{ss}}{k_{ss}} \hat{y}_t - \frac{c_{ss}}{k_{ss}} \hat{c}_t, \quad (\text{RC}')$$

$$\hat{y}_t - \hat{c}_t \approx d_1 \hat{l}_t, \quad (\text{LL}')$$

$$d_1 \equiv 1 + \eta \frac{l_{ss}}{1 - l_{ss}}.$$

2. The Simplified and Linearized Model

(a) Log-linearization

- Under uncertainty, and with logarithmic preferences, the Euler Equation (EEK) becomes

$$\frac{1}{c_t} = \beta E_t \left(\frac{1}{c_{t+1}} (1 + r_t) \right),$$
$$\frac{c_{ss}}{c_t} = \left(\frac{\beta}{\beta} \right) E_t \left(\left(\frac{c_{ss}}{c_{t+1}} \right) \left(\frac{1 + r_t}{1 + r_{ss}} \right) \right).$$

- Log both sides to get

$$-\hat{c}_t = \ln \left(E_t(\exp[-\hat{c}_{t+1} + \hat{r}_t]) \right),$$
$$\Rightarrow \hat{c}_t \approx E_t(\hat{c}_{t+1} - \hat{r}_t). \quad (\text{EEK}')$$

2. (a) Log-linearization

- With a Cobb-Douglas production function, equation (RR) becomes

$$1 + r_t = \alpha \frac{y_{t+1}}{k_{t+1}} + 1 - \delta.$$

- Log-linearize this as

$$\begin{aligned} \exp(\ln(1 + r_t)) &= \alpha \exp\left(\ln\left(\frac{y_{t+1}}{k_{t+1}}\right)\right) + 1 - \delta, \\ \exp(\ln(1 + r_t)) d \ln(1 + r_t) &= \\ \alpha \exp\left(\ln\left(\frac{y_{t+1}}{k_{t+1}}\right)\right) &\times [d \ln(y_{t+1}) - d \ln(k_{t+1})], \\ \Rightarrow R_{ss} \hat{r}_t &\approx \alpha \frac{y_{ss}}{k_{ss}} [\hat{y}_{t+1} - \hat{k}_{t+1}]. \quad (\text{RR}') \end{aligned}$$

2. (a) Log-linearization

- With logarithmic preferences, the Euler Equation (EEM) becomes

$$1 = b \frac{c_t}{m_t} + \beta E_t \left(\left(\frac{c_t}{c_{t+1}} \right) \left(\frac{1}{1 + \pi_{t+1}} \right) \right),$$

- Log-linearize this as

$$\begin{aligned} b \exp(\ln(c_t/m_t)) [d \ln(c_t) - d \ln(m_t)] = \\ -\beta E_t(\exp(\ln(c_{t+1}/c_t) - \ln(1 + \pi_{t+1})) \\ \times [d \ln(c_t) - d \ln(c_{t+1}) - d \ln(1 + \pi_{t+1})]) \\ \Rightarrow b \frac{c_{ss}}{m_{ss}} [\hat{c}_t - \hat{m}_t] \approx \frac{1}{1 + \pi_{ss}} \beta E_t(\hat{c}_{t+1} - \hat{c}_t + \hat{\pi}_{t+1}). \end{aligned}$$

2. (a) Log-linearization

- The log-linearized Euler Equation (EEM) is

$$b \frac{c_{ss}}{m_{ss}} [\hat{c}_t - \hat{m}_t] \approx \frac{1}{1 + \pi_{ss}} \beta E_t(\hat{c}_{t+1} - \hat{c}_t + \hat{\pi}_{t+1}).$$

- Inserting equation (EEK') yields

$$d_2(\hat{c}_t - \hat{m}_t) \approx E_t(\hat{r}_{t+1} + \hat{\pi}_{t+1}) \approx E_t(\hat{i}_t), \quad (\text{EEM}')$$

$$\begin{aligned} d_2 &\equiv b \left(\frac{c_{ss}}{m_{ss}} \right) \left(\frac{1 + \pi_{ss}}{\beta} \right) \\ &= \left(1 - \frac{\beta}{1 + \pi_{ss}} \right) \left(\frac{1 + \pi_{ss}}{\beta} \right) = i_{ss}, \end{aligned}$$

$$\text{as } 1 + r_{ss} = \beta^{-1} \Rightarrow 1 + i_{ss} = \beta^{-1}(1 + \pi_{ss}).$$

2. (a) Log-linearization

- The equations that approximate an equilibrium are

$$\hat{y}_t - \hat{c}_t \approx d_1 \hat{\ell}_t, \quad (\text{LL}')$$

$$d_2(\hat{c}_t - \hat{m}_t) \approx E_t(\hat{r}_{t+1} + \hat{\pi}_{t+1}), \quad (\text{EEM}')$$

$$\hat{c}_t \approx E_t(\hat{c}_{t+1} - \hat{r}_t), \quad (\text{EEK}')$$

$$R_{ss} \hat{r}_t \approx \alpha \frac{y_{ss}}{k_{ss}} [\hat{y}_{t+1} - \hat{k}_{t+1}], \quad (\text{RR}')$$

$$\hat{y}_t = z_t + \alpha \hat{k}_t + (1 - \alpha) \hat{\ell}_t, \quad (\text{PRF}')$$

$$\hat{k}_{t+1} \approx (1 - \delta) \hat{k}_t + \frac{y_{ss}}{k_{ss}} \hat{y}_t - \frac{c_{ss}}{k_{ss}} \hat{c}_t, \quad (\text{RC}')$$

$$\hat{m}_t = \hat{m}_{t-1} + \hat{\theta}_t - \hat{\pi}_t, \quad (\text{MS}')$$

and the processes for z_t and $\hat{\theta}_t$.

2. The Simplified and Linearized Model

(b) Wage Stickiness

- Under perfect competition, the real wage is given by

$$\frac{W_t}{P_t} = (1 - \alpha) \frac{y_t}{\ell_t},$$
$$\Rightarrow \hat{w}_t - \hat{p}_t = \hat{y}_t - \hat{\ell}_t.$$

- Now introduce uncertainty.
- Suppose wages are set a period in advance, and are based on the efficient wage:

$$\hat{w}_t^c = E_{t-1}(\hat{y}_t - \hat{\ell}_t + \hat{p}_t). \quad (\text{WC})$$

2. The Simplified and Linearized Model

(b) Wage Stickiness

- With sticky wages, the first-order condition for hiring becomes

$$\begin{aligned}\widehat{\ell}_t &= \widehat{y}_t + \widehat{p}_t - \widehat{w}_t^c, & \text{(MPL)} \\ &= \widehat{y}_t + \widehat{p}_t - E_{t-1}(\widehat{y}_t + \widehat{p}_t) + E_{t-1}(\widehat{\ell}_t).\end{aligned}$$

- Insert this into equation (PRF')

$$\begin{aligned}\widehat{y}_t &= z_t + \alpha \widehat{k}_t + (1 - \alpha) \\ &\quad \times \left[\widehat{y}_t + \widehat{p}_t - E_{t-1}(\widehat{y}_t + \widehat{p}_t) + E_{t-1}(\widehat{\ell}_t) \right], \\ \Rightarrow E_{t-1}(\widehat{y}_t) &= E_{t-1}(z_t) + \alpha \widehat{k}_t + (1 - \alpha) E_{t-1}(\widehat{\ell}_t).\end{aligned}\tag{EY}$$

2. (b) Wage Stickiness

- Combining the previous two equations yields

$$\begin{aligned}\hat{y}_t &= z_t + \alpha \hat{k}_t + (1 - \alpha) [\hat{y}_t - E_{t-1}(\hat{y}_t)] \\ &\quad + (1 - \alpha) [\hat{p}_t - E_{t-1}(\hat{p}_t)] \\ &\quad + E_{t-1}(\hat{y}_t) - E_{t-1}(z_t) - \alpha \hat{k}_t.\end{aligned}$$

- This simplifies to

$$\hat{y}_t - E_{t-1}(\hat{y}_t) = a (\hat{p}_t - E_{t-1}(\hat{p}_t)) + \varepsilon_t, \quad (\text{AS})$$

$$a \equiv \frac{1 - \alpha}{\alpha},$$

$$\varepsilon_t \equiv \frac{1}{\alpha} (\hat{z}_t - E_{t-1}(\hat{z}_t)).$$

2. (b) Wage Stickiness

- Proceeding as before, we have:

$$\hat{y}_t = z_t + \alpha \hat{k}_t + (1 - \alpha) \times \left[\hat{y}_t + \hat{p}_t - E_{t-1}(\hat{y}_t + \hat{p}_t) + E_{t-1}(\hat{\ell}_t) \right].$$

- Equations (LL'), (EEK') and (RR') imply

$$E_{t-1}(\hat{\ell}_t) \approx d_1^{-1} E_{t-1}(\hat{y}_t - \hat{c}_t), \quad (\text{LL}'')$$

$$E_{t-1}(\hat{c}_t) \approx \hat{c}_{t-1} + \hat{r}_{t-1},$$

$$\hat{r}_{t-1} \approx \alpha R_{ss}^{-1} \frac{y_{ss}}{k_{ss}} E_{t-1}(\hat{y}_t - \hat{k}_t).$$

2. (b) Wage Stickiness

- Combine the preceding equations to get

$$\begin{aligned}\hat{y}_t &= a (\hat{p}_t - E_{t-1}(\hat{p}_t)) + \phi_k \hat{k}_t - \phi_c \hat{c}_{t-1} \\ &\quad + \phi_z E_{t-1}(z_t) + \varepsilon_t,\end{aligned}$$

$$\phi_k, \phi_c, \phi_k > 0.$$

- Because only price surprises matter, the direct effects of sticky wages last for only one period.
- The propagation mechanisms – the intertemporal substitution of labor and capital accumulation – are the same as in the RBC model, and are thus weak.

2. The Simplified and Linearized Model

(c) Simplified model

- Impose the following simplifications
 - Eliminate capital \Rightarrow equation (RR') vanishes; and $\hat{y}_t = \hat{c}_t$.
 - Ignore labor supply \Rightarrow equation (LL') vanishes ($d_1^{-1} \rightarrow 0$).
 - Assume that r_t and i_t are known at time t .
- Notation changes
 - Let lower-case variables without “hats” ($\hat{}$'s) denote logged deviations
 - Let m denote deviations of the nominal (not real) money supply \Rightarrow replace \hat{m} with $m - p$.

2. The Simplified and Linearized Model

(c) Simplified model

- The equations in the simplified system are equation (AS) and modified versions of equations (EEK') and (EEM')

$$y_t = E_{t-1}(y_t) + a(p_t - E_{t-1}(p_t)) + \varepsilon_t, \quad (\text{AS})$$

$$y_t = E_t(y_{t+1}) - r_t + u_t, \quad (\text{IS})$$

$$m_t - p_t = y_t - d_2^{-1}(r_t + E_t(\pi_{t+1})) + v_t. \quad (\text{LM})$$

and the processes for ε_t (z_t), and u_t and v_t .

- This is a “general equilibrium” IS-LM-AS model.
- This model differs from the traditional IS-LM-AS model because of the expectations in equations (AS) and (IS).

2. (c) Simplified Model

- The modified versions of equations (LL'') and (EY) are

$$E_{t-1}(\ell_t) = 0,$$

$$E_{t-1}(y_t) = E_{t-1}(z_t). \quad (\text{EY})$$

- Using equation (EY) we get

$$y_t = a(p_t - E_{t-1}(p_t)) + E_{t-1}(z_t) + \varepsilon_t, \quad (\text{AS}')$$

$$y_t = E_t(z_{t+1}) - r_t + u_t. \quad (\text{IS}')$$

- Combine equations (IS') and (LM):

$$y_t = \frac{1}{1 + d_2} [d_2(m_t - p_t - v_t) + E_t(\pi_{t+1}) + E_t(z_{t+1}) + u_t].$$

(AD)

2. The Simplified and Linearized Model

(d) Interest rate targeting

- Suppose the central bank adjusts the money supply to set i_t , where i_t follows a **Taylor Rule**:

$$i_t = b_y y_t + b_\pi \pi_t, \quad b_y, b_\pi > 0. \quad (\text{TR})$$

- Taylor found that $b_y = 0.5$, $b_\pi = 1.5$ fits U.S. data.
- We can ignore equation (LM) for now.
- Combine equations (IS') and (TR) to get

$$\begin{aligned} y_t &= E_t(z_{t+1}) - (r_t + E_t(\pi_{t+1})) + E_t(\pi_{t+1}) + u_t \\ &= E_t(z_{t+1}) + E_t(\pi_{t+1}) - (b_y y_t + b_\pi \pi_t) + u_t \\ &= \frac{1}{1 - b_y} (E_t(z_{t+1}) + E_t(\pi_{t+1}) - b_\pi \pi_t + u_t). \end{aligned}$$

(AD')

2. (d) Interest rate targeting

- Assume that z_t is i.i.d., so that $E_{t-1}(z_t) = 0$.
- Rewrite equation (AS') as

$$y_t = a(\pi_t - E_{t-1}(\pi_t)) + \varepsilon_t. \quad (\text{PC})$$

- Combine equations (AD') and (AS'):

$$(a_y + b_\pi)\pi_t = a_y E_{t-1}(\pi_t) + E_t(\pi_{t+1}) + e_t^\pi, \quad (\text{EQY})$$

$$a_y \equiv a(1 - b_y),$$

$$e_t^\pi \equiv u_t - (1 - b_y)\varepsilon_t.$$

2. (d) Interest rate targeting

- Inflation must satisfy

$$(a_y + b_\pi)\pi_t = a_y E_{t-1}(\pi_t) + E_t(\pi_{t+1}) + e_t^\pi. \quad (\text{EQY})$$

- Apply McCallum's minimum state solution method: guess that

$$\pi_t = \kappa e_t^\pi,$$

$$\Rightarrow E_{t-1}(\pi_t) = E_t(\pi_{t+1}) = 0.$$

- Equation (EQY) becomes

$$(a_y + b_\pi)\kappa e_t^\pi = e_t^\pi,$$

$$\Rightarrow \kappa = \frac{1}{a_y + b_\pi} > 0.$$

2. (d) Interest rate targeting

- Inserting our result for π_t into equation (AS') shows that

$$\begin{aligned}\pi_t &= \kappa e_t^\pi, \\ \Rightarrow y_t &= \kappa(a u_t + b_\pi \varepsilon_t).\end{aligned}$$

- Recall:

$$m_t - p_t = y_t - d_2^{-1}(r_t + E_t(\pi_{t+1})) + v_t. \quad (\text{LM})$$

- Price Indeterminacy: if $\{y_t, p_t, m_t\}$ are an equilibrium, then so is $\{y_t, p_t + \xi, m_t + \xi\}$.
- Prices can be pinned down if the central bank commits to m_0 .

2. The Simplified and Linearized Model

(e) Price persistence 1: Taylor (1979, 1980)

- Suppose nominal wages are set for two periods, with 1/2 of the workforce adjusting their wages each period.
- Loosely following equation (WC), this suggests that the wage negotiated at time t depends on average expected prices:

$$w_t^c = \frac{1}{2}(p_t + E_t(p_{t+1})) + ky_t, \quad k > 0.$$

- The price level at time t depends on the average wage:

$$p_t = \frac{1}{2}(w_{t-1}^c + w_t^c).$$

2. The Simplified and Linearized Model

(e) Price persistence 1: Taylor (1979, 1980)

● Combine the two preceding equations:

$$p_t = \frac{1}{4}(p_{t-1} + E_{t-1}(p_t)) + \frac{1}{2}ky_{t-1} \\ + \frac{1}{4}(p_t + E_t(p_{t+1})) + \frac{1}{2}ky_t,$$

$$\Rightarrow 3p_t = p_{t-1} + E_{t-1}(p_t) + E_t(p_{t+1}) + 2k(y_{t-1} + y_t),$$

$$\Rightarrow E_t(p_{t+1}) = 2p_t - p_{t-1} - 2k(y_{t-1} + y_t) + e_t^p. \quad (\text{TP})$$

$$e_t^p \equiv p_t - E_{t-1}(p_t),$$

$$\Rightarrow E_t(\pi_{t+1}) = \pi_t - 2k(y_{t-1} + y_t) + e_t^p, \quad (\text{TP}')$$

2. (e) Price persistence 1: Taylor (1979, 1980)

- Taylor's model implies:

$$E_t(p_{t+1}) = 2p_t - p_{t-1} - 2k(y_{t-1} + y_t) + e_t^p, \quad (\text{TP})$$

$$E_t(\pi_{t+1}) = \pi_t - 2k(y_{t-1} + y_t) + e_t^p. \quad (\text{TP}')$$

- Equations (TP) and (TP') imply that price growth is persistent.
- Equation (TP') shows that inflation growth is **not** persistent.
- Equation (TP') shows that permanent changes in inflation have no effect on output: money can be superneutral but not neutral.

2. The Simplified and Linearized Model

(f) Price persistence 2: Other Models

- Calvo's (1983) model
 - Each period, a random fraction q of firms renegotiate wages.
 - Dynamics are similar to Taylor's.
- Imperfect competition and menu costs
 - If firms have monopoly power, small deviations from the optimal monopoly price have small effects on profits
 - If there are **menu costs** to adjusting prices, prices will be sticky.
 - These assumptions are often used to justify Taylor- or Calvo-style adjustment processes

2. The Simplified and Linearized Model

(f) Price persistence 2: Other Models

- Criticism 1: arbitrary price adjustments
 - Neither the Taylor nor the Calvo adjustment process is derived from an optimizing model.
- Caplin and Spulber (1987)
 - The firms most likely to adjust prices are the ones with the most suboptimal prices.
 - The distribution of prices (relative to optimal) is stationary.
 - A monetary shock changes the rate of adjustment, but not the distribution of prices: aggregate prices are **not** sticky.

2. The Simplified and Linearized Model

(f) Price persistence 2: Other Models

- Criticism 1: arbitrary price adjustments
- Golosov and Lucas (2007): In a calibrated, less-stylized version of Caplin and Spulber's model, money shocks have few real effects.
- Criticism 2: Chari, Kehoe and McGrattan (2000)
 - A reasonably-calibrated sticky-price model does not produce persistent responses to monetary shocks.