Economics 701: Macroeconomics II

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Lecture 6: Multiple Equilibria and Sunspots

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3. Sunspots in the Stochastic Growth Model
   (a) Expectation-driven fluctuations
      - People coordinate beliefs with some (possibly meaningless) stochastic process, so that economy fluctuates with the belief-coordinating process.
      - Reappearance of Keynes' "animal spirits."
   (b) Two main types:
      - Forecast errors, especially for vector LEDEs.
      - Serially correlated sunspots.
   (c) Issues
      - Are the parameter values that support sunspots plausible?
      - Can these models generate realistic fluctuations?
      - Where are the sunspot variables?

3. Sunspots in the Stochastic Growth Model

   (d) Farmer and Guo (1994)

      - RBC model with spillovers.
         - Many firms, with production given by

$$Y_{it} = K_{it}^{\alpha} L_{it}^{1-\alpha} Y_t^{1-\theta}, \quad 0 < \theta \leq 1.$$

where Y_t is aggregate output, taken as given by individual firms.

3. Sunspots in the Stochastic Growth Model

(d) Farmer and Guo (1994)

- RBC model with spillovers.
  - In a symmetric equilibrium, $Y_{it} = Y_t$ (*up to scale*) so that

$$
\begin{aligned}
Y_t &= K_t^{\alpha/\theta} L_t^{(1-\alpha)/\theta}, \\
r_t &= \alpha \frac{Y_t}{K_t}, \\
w_t &= (1-\alpha)\frac{Y_t}{L_t}.
\end{aligned}
$$

  - Otherwise the basic RBC Model with no technology shocks.

3. (d) Farmer and Guo (1994)

- Log-linearize to get

$$\begin{pmatrix} \widehat{k}_{t+1} \\ \widehat{c}_{t+1} \end{pmatrix} = \mathbf{A} \begin{pmatrix} \widehat{k}_t \\ \widehat{c}_t \end{pmatrix} + \begin{pmatrix} 0 \\ \varepsilon^c_{t+1} \end{pmatrix}.$$

- Constant returns ($\theta = 1$). System is saddle path stable, so that $\widehat{c}_t = \lambda \widehat{k}_t$. In the absence of technology shocks, $\varepsilon^c_{t+1} = 0$.
- High increasing returns ($\theta \ll 1$). Both roots of the system have modulus less than 1. ε^c_{t+1} can be non-zero. Sunspots!
- Criticism: required returns to scale too high.
- Multi-sector models can generate sunspots with far lower returns to scale.

3. (e) Harrison and Weder (2006)

- Consumer confidence fell during the Great Depression; was this a sunspot shock?
- Fit the Farmer-Guo model to the Great Depression.
- Sunspots are measured as changes in the interest rate spread (high- vs. low quality bonds) that remain after controlling for output, money, inflation and short-term interest rates.
- Using these sunspots, the model fits the Great Depression events very well.

3. Sunspots in the Stochastic Growth Model

(f) Schmitt-Grohe (AER, 2000)

- Analyzes multi-sector sunspot models.
- Model Structure:
  - Flow utility

$$u\left(C_t, \ell_t\right) = \ln\left(C_t\right) - \chi \ell_t.$$

  - Note the indivisible labor assumption: The more elastic labor supply is, the lower the returns to scale that are needed for indeterminacy.

3. (f) Schmitt-Grohe (AER, 2000)

- Model Structure
  - Intermediate goods production. For sector $i \in \{1, 2, \ldots, J\}$,

$$Y_t^i = F\left(K_t^i, Z_t \ell_t^i\right),$$

    where Z_t is an aggregate technology shock.
  - Final goods production, consumption:

$$C_t = J^{1/(1-\rho)} \left(\sum_{i=1}^{J} \left[y_{Ct}^i \right]^{(\rho-1)/\rho} \right)^{\rho/(\rho-1)}, \quad \rho > 0,$$

    where y_{Ct}^i is the amount of intermediate good i devoted to consumption.

3. (f) Schmitt-Grohe (AER, 2000)

- Model Structure
  - Final goods production, investment:

$$I_t = A_t J^{1/(1-\eta)} \left(\sum_{i=1}^{J} \left[y_{It}^i \right]^{(\eta-1)/\eta} \right)^{\eta/(\eta-1)} , \quad \eta > 0,$$

where y_{It}^i is the amount of intermediate good i devoted to investment.

3. (f) Schmitt-Grohe (AER, 2000)

- Model Structure
  - A_t is an "external factor" given by

$$A_t = \left(\frac{\overline{I}_t}{Z_t}\right)^{\theta/(1+\theta)}, \quad \theta \geq 0,$$

    where $\overline{I}_t$ is the average output of each investment-sector firm, taken as given by individual firms.
  - In equilibrium, $\overline{I}_t = I_t$ and the returns to scale in investment are $1 + \theta$.
  - θ calibrated to 0.1.

3. (f) Schmitt-Grohe (AER, 2000)

- Types of shocks
  - Technology

$$z_t \equiv \ln(Z_t) = g + z_{t-1} + \sigma_z \varepsilon_t^z,$$

where the i.i.d. variable ε_t^z has a variance of 1.
  - Sunspots: the i.i.d. variable $\sigma_b \varepsilon_t^b$.
  - Joint process: $(\sigma_z \varepsilon_t^z, \sigma_b \varepsilon_t^b)$ with

$$\begin{aligned} \sigma_b &= 0.228\sigma_z, \\ corr\left(\varepsilon_t^z, \varepsilon_t^b\right) &= -0.963, \end{aligned}$$

calibrated to make the model best fit the data.

3. (f) Schmitt-Grohe (AER, 2000)

- Results
  - Sunspot model with both shocks outperforms RBC model in matching variance and serial correlation of Δc, Δi and $\Delta \ell$.
  - Two-shock model also does better at matching $corr\,(\ell, y/\ell)$.
  - Sunspot model does better at replicating the autocorrelation function of Δy, but is still rejected by data.
  - Sunspot model does not match observed responses to transitory and permanent shocks.

3. Sunspots in the Stochastic Growth Model

(g) Wang and Wen (2006)

- Motivation: In existing RBC Models, sunspots are:
  - Local—steady state deviations—instead of global.
  - Sensitive to parameters such as IES_L.
  - Not robust to capital or labor adjustment costs.
- Wang and Wen construct a model where:
  - The economy has constant returns to scale.
  - Sunspots are global.
  - Sunspots are robust.
- Key mechanisms
  - Imperfect competition and demand spillovers alá Blanchard and Kiyotaki (1987).
  - Prices/quantities are set under uncertainty.

3. Sunspots in the Stochastic Growth Model

(g) Wang and Wen (2006)

- Final goods
  - Let $i \in [0, 1]$ index intermediate goods.
  - Final output follows

$$Y = \left(\int_0^1 Y(i)^{[\sigma-1]/\sigma} \, di \right)^{\sigma/[\sigma-1]}, \quad \sigma > 1.$$

  - Producer solves

$$\max_{[Y(i)]_0^1} P \left(\int_0^1 Y(i)^{[\sigma-1]/\sigma} \, di \right)^{\sigma/[\sigma-1]} - \int_0^1 P(i) \, Y(i) \, di.$$

3. (g) Wang and Wen (2006)

- Final goods
  - FOC is

$$PY(i)^{-1/\sigma} Y^{1/\sigma} = P(i)$$
$$\Rightarrow Y(i) = P^{\sigma} P(i)^{-\sigma} Y. \qquad \text{(iD)}$$

  - Assume perfect competition, and insert equation (iD) to get

$$\begin{aligned} P &= \text{Average Cost} = \frac{1}{Y}\int_0^1 P(i)\,Y(i)\,di \\ &= \left(\int_0^1 P(i)^{1-\sigma}\,di\right)^{1/(1-\sigma)}. \end{aligned}$$

3. (g) Wang and Wen (2006)

- Intermediate goods
  - Cost minimization

$$\mathcal{L} = RK(i) + W\ell(i) - \phi(i)\left[K(i)^{\alpha}\,\ell(i)^{1-\alpha} - Y(i)\right].$$

  - FOC imply

$$R = \phi(i)\alpha\frac{Y(i)}{K(i)}, \qquad \text{(MRPK)}$$

$$W = \phi(i)(1-\alpha)\frac{Y(i)}{\ell(i)}, \qquad \text{(MRPL)}$$

3. (g) Wang and Wen (2006)

- Intermediate goods
  - The FOC for cost minimization imply that

$$R = \phi(i)\alpha\frac{Y(i)}{K(i)}, \qquad \text{(MRPK)}$$

$$W = \phi(i)\left(1-\alpha\right)\frac{Y(i)}{\ell(i)}, \qquad \text{(MRPL)}$$

while marginal cost, $\phi(i)$, is given by

$$\phi(i) = \left(\phi(i)\frac{Y(i)}{K(i)}\right)^{\alpha}\left(\phi(i)\frac{Y(i)}{\ell(i)}\right)^{1-\alpha}$$

$$= \left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{W}{1-\alpha}\right)^{1-\alpha}.$$

3. (g) Wang and Wen (2006)

- Intermediate goods
  - Producer i is a monopolist, solves

$$\max_{P_t(i)} E_{t-1}\left(\left[P_t(i) - \phi_t\right] Y_t(i)\right),$$

subject to equation (iD).

  - Producer doesn't know the aggregate quantities (P_t, Y_t, ϕ_t) that affect his profits; he only knows his own price, $P_t(i)$.
  - FOC is

$$P_t(i) = \left(\frac{\sigma}{\sigma - 1}\right) \frac{E_{t-1}\left(\phi_t P_t^{\sigma} Y_t\right)}{E_{t-1}\left(P_t^{\sigma} Y_t\right)}.$$

3. (g) Wang and Wen (2006)

- Intermediate goods
  - Imposing symmetry $\Rightarrow P_t(i) = P_t \equiv 1$, $Y_t(i) = Y_t$, $K_t(i) = K_t$, $\ell_t(i) = \ell_t$, and the producer's FOC becomes

$$\frac{E_{t-1}(\phi_t Y_t)}{E_{t-1}(Y_t)} = \frac{\sigma - 1}{\sigma}. \qquad \text{(iFOC)}$$

  - Producer's uncertainty is $extrinsic$, not based on any fundamental shock.
  - Note: High values of $\phi_t \Rightarrow$ high marginal costs $\Rightarrow$ higher input prices $\Rightarrow$ more capital and labor $\Rightarrow$ more output.

3. (g) Wang and Wen (2006)

- Household's FOC

$$W_t \cdot U_C\left(C_t, 1-\ell_t\right) = U_{1-\ell}\left(C_t, 1-\ell_t\right), \tag{LL}$$

$$U_C\left(C_t, 1-\ell_t\right) = \tag{EE}$$

$$\beta E_t\Big(U_C(C_{t+1}, 1-\ell_{t+1})\left[R_{t+1}+1-\delta\right]\Big).$$

- Deterministic steady state. Find

$$\phi_{SS} = \frac{\sigma - 1}{\sigma},$$

and proceed in the usual fashion.

3. (g) Wang and Wen (2006)

- Sunspot example 1
  - Set $\delta = 1$, $U(C, 1-\ell) = \ln(C) - \ell$.
  - Guess that

$$C_t = Y_t - K_{t+1} = (1 - \beta\alpha\theta)\, Y_t.$$

  - Using this guess, (MRPK), and the specializations, equation (EE) becomes

$$\begin{aligned} \frac{1}{C_t} &= \beta E_t \left(\frac{1}{C_{t+1}} \phi_{t+1} \alpha \frac{Y_{t+1}}{K_{t+1}} \right) \\ \Rightarrow E_t\left(\phi_{t+1}\right) &= \theta. \end{aligned}$$

3. (g) Sunspot example 1

- Similarly, (LL) and (MRPL) become

$$W_t = C_t \Rightarrow \ell_t(i) = \phi_t \frac{1-\alpha}{1-\beta\alpha\theta}.$$

- Plug this into the i-goods prod. function and impose symmetry:

$$Y_t = \Lambda K_t^{\alpha} \phi_t^{1-\alpha}, \quad \Lambda \equiv \left(\frac{1-\alpha}{1-\beta\alpha\theta}\right)^{1-\alpha}.$$

- (iFOC) becomes

$$\frac{E_{t-1}\left(\phi_t^{2-\alpha}\right)}{E_{t-1}\left(\phi_t^{1-\alpha}\right)} = \frac{\sigma-1}{\sigma}. \qquad \text{(iFOC}'\text{)}$$

3. (g) • Sunspot example 1

- Suppose that

$$
\begin{aligned}
\phi_t &= \frac{\sigma - 1}{\sigma}\varepsilon_t, \\
\varepsilon_t &= \begin{cases} 0 \text{ with probability } p \\ 1 \text{ with probability } 1-p \end{cases},
\end{aligned}
$$

- It can be shown that this satisfies (iFOC′). Sunspot equilibrium!

3. (g) Wang and Wen (2006)

- Sunspot example 2
  - Log-linearize to find

$$\begin{pmatrix} \widehat{k}_{t+1} \\ \widehat{c}_{t+1} \end{pmatrix} = \mathbf{A} \begin{pmatrix} \widehat{k}_{t} \\ \widehat{c}_{t} \end{pmatrix} + \mathbf{\Gamma} \hat{\phi}_t,$$

  - Even though **A** is saddle-path configured, we can still allow $\hat{\phi}_t$ to vary.

3. (g) Wang and Wen (2006)

- Problem
  - Model will generate persistent fluctuations only if $\hat{\phi}_t$ is persistent, and it is not clear that (iFOC) allows persistence.
  - Standard sunspot models (Farmer and Guo, 1994) alter configuration of $\mathbf{A}$, which allows i.i.d. sunspots to generate persistent fluctuations.

3. (g) Wang and Wen (2006)

- Wang and Wen (2006b)
  - Two classes of intermediate goods

$$Y = \left(\int_0^1 Y(i)^{[\sigma_h - 1]/\sigma_h} \, di\right)^{\sigma_h/[\sigma_h - 1]} + \left(\int_0^1 Y(i)^{[\sigma_l - 1]/\sigma_l} \, di\right)^{\sigma_l/[\sigma_l - 1]},$$

$\sigma_h > \sigma_l > 1.$

3. (g) • Wang and Wen (2006b)

  - Equation (iD) becomes

$$Y_t\left(i\right)=\left[\theta_t P_t(i)^{-\sigma_h}+\left(1-\theta_t\right)P_t(i)^{-\sigma_l}\right]Y_t,$$

    where θ_t is the fraction of output produced with high-elasticity goods.

  - It can be shown that under symmetry, equation (iFOC) becomes

$$\begin{aligned}\phi_t &= \frac{\sigma_t-1}{\sigma_t},\\ \sigma_t &\equiv \theta_t\sigma_h+\left(1-\theta_t\right)\sigma_l.\end{aligned}$$

  - Model is otherwise identical to earlier model.

3. (g)
- Wang and Wen (2006b)
  - θ_t, and thus ϕ_t, can follow a large number of ARMA processes.
  - Economic booms driven by increases in the demand elasticity σ_t: $\sigma_t \uparrow \Rightarrow$ markups $(\text{Price}/\text{Cost} - 1 = 1/\phi_t - 1)\downarrow \Rightarrow$ intermediate goods cheaper.
  - Is this a plausible spillover mechanism?
  - However, markups are countercyclical in the data.
  - Smets and Wouters (2003) find that "cost-push" shocks—shocks to σ—are important in their DSGE forecasting model.

3. (g) • Wang and Wen (2006b)

- Suppose production and depreciation both depend on capital utilization:

$$\begin{aligned} Y_t &= (u_t K_t)^{\alpha}\, \ell_t^{1-\alpha}, \\ K_{t+1} &= (1 - \delta_0 \frac{1}{\nu} u_t^{\nu}) K_t + Y_t - C_t, \end{aligned}$$

  where u_t is the rate of capital utilization.
- Then pro-cyclical capital utilization will lead to pro-cyclical measured TFP.

3. (g) Wang and Wen (2006b)

- Suppose that sectoral composition evolves dynamically:

$$\begin{aligned} \theta_t &= \frac{M_{h,t}}{M_{h,t} + M_{l,t}}, \\ M_{h,t} &= (1-\omega)M_{h,t-1} + s_t m_t, \\ M_{l,t} &= (1-\omega)M_{l,t-1} + (1-s_t)m_t. \end{aligned}$$

  where: M_h and M_l are the number of high- and low- elasticity firms; ω is the firm exit rate; m_t is the # of new firms; and s_t is an extrinsic shock.
- The number of new firms, m_t is determined by comparing expected profits to a fixed entry cost.
- If $\omega < 1$, this model generates persistence.