Economics 701: Macroeconomics II

Spring 2009

Lecture 6: Multiple Equilibria and Sunspots

University at Albany

State University of New York

John Bailey Jones

April, 2009

1. Overview

   (a) Two Underlying Assumptions

   - Spillovers (externalities) among agents
     - Technology spillovers (Paul Romer, 1986):

$$y_i = f\left(k_i, \ell_i, \sum_j k_j\right).$$

     - Demand externalities: Blanchard and Kiyotaki (1987), Murphy et al. (1989).
     - Search/communication externalities: Diamond (1982), Howitt and McAfee (1992).
     - Sociological spillovers: neighborhood effects.
   - Lack of markets to "deal" with these effects = lack of explicit coordination mechanism.

1. Overview

   (b) Features (Results)

   - Multiple rational expectations equilibria: Self-fulfilling prophecies.
   - Inefficiency: Equilibria are Pareto-rankable.
   - Sunspots: Expectations-driven fluctuations.

2. Spillovers and Strategic Complementarities

   (a) Bryant (QJE, 1983)

   - Setup
     - N identical agents.
     - Endowment: E units of "effort" per agent.
     - Preferences for agent i

$$u_i = U\left(c_i\right) + V\left(E - e_i\right),$$

       with $U' > 0$, $V' > 0$, $U'' \leq 0$, $V'' \leq 0$, and $U'\left(0\right) - V'\left(E\right) > 0$.

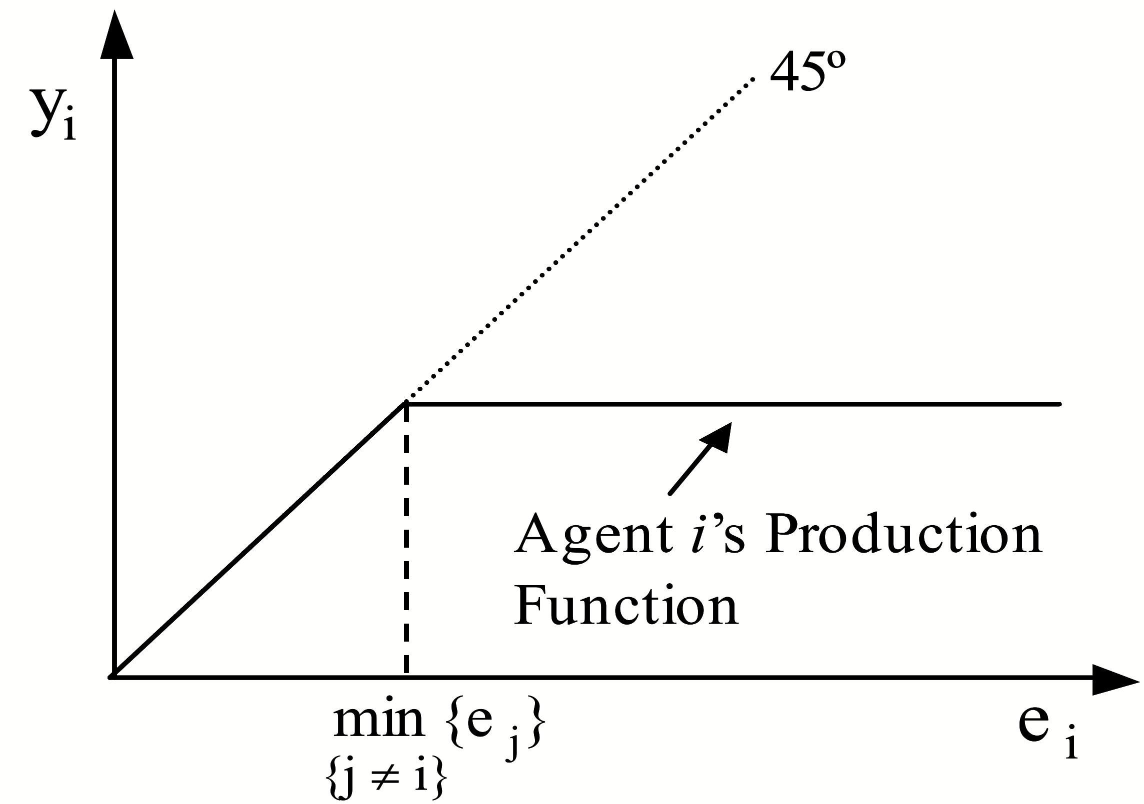
   - Production technology: Leontief

$$\begin{aligned} Y &= N \times \min\left\{e_1, e_2, ..., e_N\right\}, \\ y_i &= N^{-1} Y \\ &= \min\left\{e_1, e_2, ..., e_N\right\}. \end{aligned}$$

2. Spillovers and Strategic Complementarities

   (a) Bryant (QJE, 1983)

      - Production technology in the Bryant Model



2. Spillovers and Strategic Complementarities

   (a) Bryant (QJE, 1983)

- Setup
  - Storage technology: none $\Rightarrow c_i = y_i$.
  - Define $e^i_{min} = \min\limits_{j \neq i} \{e_j\}$. Then

$$\frac{\partial U\left(c_i\left(e_i\right)\right)}{\partial e_i} = \begin{cases} U'\left(e_i\right), & e_i < e^i_{min}, \\ 0, & e_i > e^i_{min}, \end{cases} .$$

2. Spillovers and Strategic Complementarities

   (a) Bryant (QJE, 1983)

- Cooperative equilibrium:
  - Symmetry (as excess effort is wasted) implies

$$c_i = y_i = y = e = e_i.$$

  - Social planner solves

$$\max_{0 \le e \le E} U(e) + V(E - e),$$

  - First-order condition:

$$U'(e) - V'(E - e) \ge 0.$$

  - Let e^* be the largest solution.

2. Spillovers and Strategic Complementarities

   (a) Bryant (QJE, 1983)

- Nash (uncooperative) equilibrium
  - Any $e \leq e^*$ constitutes a symmetric Nash equilibrium.
  - Proof. $e^i_{min} = e$ implies that

$$\frac{\partial}{\partial e_i}\Big[U\left(c_i\left(e_i\right)\right) - V\left(E - e_i\right)\Big] =$$

$$\begin{cases} U'\left(e_i\right) - V'\left(E - e_i\right) \geq 0, & e_i < e \leq e^*, \\ U'\left(e_i\right) - V'\left(E - e_i\right) < 0, & e^* < e_i < e, \\ -V'\left(E - e_i\right) < 0, & e_i > e, \end{cases}.$$

2. Spillovers and Strategic Complementarities

(b) Cooper and John (1988)

- General conditions for multiple equilibria
- N identical agents: each chooses effort $e_i \in [0, E]$.
- Payoff function for agent i
  - Deal with symmetric case:

$$\begin{aligned} u_i &= \phi(e_1, e_2, ..., e_i, ..., e_N) \\ &= \phi(e_i, \overline{e}_{-i}), \end{aligned}$$

where $e_j = \overline{e}_{-i}, \forall j \neq i$.

2. (b) Cooper and John (1988)

- Payoff function for agent i: $u_i = \phi(e_i, \overline{e}_{-i})$
  - Technicalities

$$\lim_{e \to 0} \frac{\partial \phi(e,e)}{\partial e_i} > 0, \quad \lim_{e \to E} \frac{\partial \phi(e,e)}{\partial e_i} < 0,$$

$$\frac{\partial^2 \phi(e_i, \overline{e}_{-i})}{\partial e_i^2} < 0,$$

  - Interiority assumptions.
  - Positive spillovers

$$\frac{\partial \phi(e_i, \overline{e}_{-i})}{\partial \overline{e}_{-i}} > 0.$$

2. (b) Cooper and John (1988)

- Payoff function for agent i: $u_i = \phi(e_i, \overline{e}_{-i})$
  - Strategic Complementarities

$$\frac{\partial^2 \phi(e_i, \overline{e}_{-i})}{\partial e_i \partial \overline{e}_{-i}} > 0.$$

  - Strategic complementaries and positive spillovers are not the same. Spillovers involve total returns; str. comps involve marginal returns.
  - For example, consider

$$\max_{0 \leq e \leq E} U(e - \overline{e}_{-i}) + V(E - e)$$

If U is increasing and concave, U is decreasing in $\overline{e}_{-i}$, but U' is increasing.

2. (b) Cooper and John (1988)

- Symmetric Nash Equilibria (SNE)
  - Agent i has a reaction function $e_i^*\left(\overline{e}_{-i}\right)$ where

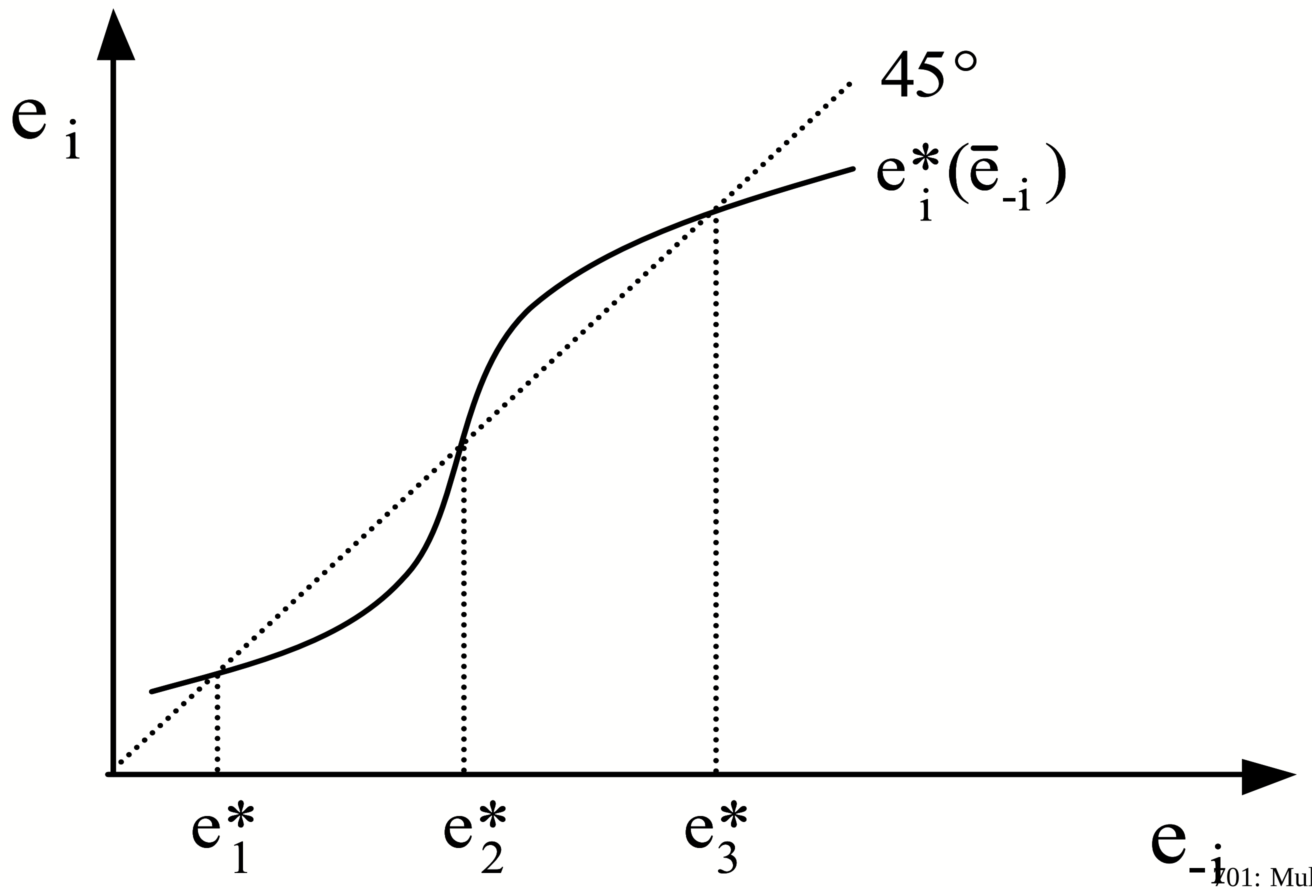
$$\left.\frac{\partial \phi\left(e_i, \overline{e}_{-i}\right)}{\partial e_i}\right|_{e_i = e_i^*\left(\overline{e}_{-i}\right)} = 0.$$

  - An SNE has $e_i^*\left(e^*\right) = e^*$, so that

$$\left.\frac{\partial \phi\left(e_i, e^*\right)}{\partial e_i}\right|_{e_i = e^*} = 0. \qquad \text{(SNE)}$$

2. (b) • Strategic Complementarity is necessary for multiple SNE.

- **Proof**: Multiple SNE require that over a range $e_i^*\left(\overline{e}_{-i}\right)$ has a positive slope.



2. (b) Cooper and John (1988)

- Strategic Complementarity is necessary for multiple SNE.
  - **Proof** (continued): Find the slope of $e_i^*\left(\overline{e}_{-i}\right)$ by implicitly differentiating equation (SNE):

$$\frac{\partial \phi\left(e^*, e^*\right)}{\partial e_i} = 0$$

$$\Rightarrow \frac{\partial^2 \phi\left(e^*, e^*\right)}{\partial^2 e_i} de_i + \frac{\partial^2 \phi\left(e^*, e^*\right)}{\partial e_i \partial \overline{e}_{-i}} d\overline{e}_{-i} = 0,$$

$$\Rightarrow \frac{de_i^*\left(\overline{e}_{-i}\right)}{d\overline{e}_{-i}} = -\frac{\partial^2 \phi\left(e^*, e^*\right)}{\partial e_i \partial \overline{e}_{-i}} \Big/ \frac{\partial^2 \phi\left(e^*, e^*\right)}{\partial^2 e_i} > 0$$

$$\Leftrightarrow \frac{\partial^2 \phi\left(e^*, e^*\right)}{\partial e_i \partial \overline{e}_{-i}} > 0.$$

2. (b) Cooper and John (1988)

- Symmetric Cooperative Equilibria
  - In an SCE, $e_i = e^{**}$, with

$$\left.\frac{\partial \phi\left(e, e\right)}{\partial e}\right|_{e=e^{**}}$$

$$= \left.\frac{\partial \phi\left(e_i, e^{**}\right)}{\partial e_i}\right|_{e_i=e^{**}} + \left.\frac{\partial \phi\left(e^{**}, \overline{e}_{-i}\right)}{\partial \overline{e}_{-i}}\right|_{\overline{e}_{-i}=e^{**}}$$

$$= 0.$$

  - Note that $e^{**} > e^{*}$ because of positive spillovers. Thus, any value of e^{*} where there are positive spillovers is inefficient.

2. (b) Cooper and John (1988)

- SNE with higher levels of e^* are Pareto-superior.
  - **Proof**:

$$\frac{d\phi\left(e_i^*\left(\overline{e}_{-i}\right),\overline{e}_{-i}\right)}{d\overline{e}_{-i}} = \frac{\partial\phi\left(e_i^*\left(\overline{e}_{-i}\right),e^*\right)}{\partial e_i}\frac{de_i^*\left(\overline{e}_{-i}\right)}{d\overline{e}_{-i}} + \frac{\partial\phi\left(e_i^*\left(\overline{e}_{-i}\right),\overline{e}_{-i}\right)}{\partial\overline{e}_{-i}} = \frac{\partial\phi\left(e_i^*\left(\overline{e}_{-i}\right),\overline{e}_{-i}\right)}{\partial\overline{e}_{-i}} > 0,$$

    because of equation (SNE) (envelope theorem) and positive spillovers.
  - When the economy resides in a less-efficient SNE, it exhibits <u>coordination failure</u>.