Economics 701: Macroeconomics II

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Lecture 2: The Keynesian View, Rational Expectations and Policy (In)effectiveness

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9. Learning

(a) Overview

- Rational expectations makes strong assumptions:
  - People know the model and how to solve it.
  - People know the model's parameters.
- An alternative model is learning:
  - People assume economy follows simple model.
  - Each period, re-estimate model's parameters.
- Questions
  - Estimated model parameters affect economic outcomes. Economic outcomes affect estimated parameters. Will this process converge to an "equilibrium"?
  - If there is convergence, then to what?

9. Learning

(b) Consider an “Ad Hoc” Model

- The model:

$$y_t = -\chi\left(w_t - p_t\right), \quad \chi > 0,$$

$$y_t = m_t - p_t + \theta_t,$$

$$w_t = \widetilde{E}_{t-1}\left(p_t\right),$$

$$\theta_t = \phi\theta_{t-1} + \eta_t, \quad |\phi| < 1,$$

$$\eta_t = \text{zero-mean, i.i.d. process with } V\left(\eta_t\right) = \sigma_\eta^2,$$

$$m_t = \overline{m}.$$

9. (b) “Ad Hoc” Model

- Reduced form

$$p_t = (1 - \alpha)\left(\overline{m} + \theta_t\right) + \alpha \widetilde{E}_{t-1}\left(p_t\right), \qquad \text{(EQP)}$$

$$y_t = \alpha\left(\overline{m} + \theta_t\right) - \alpha \widetilde{E}_{t-1}\left(p_t\right), \qquad \text{(EQY)}$$

$$\alpha = \frac{\chi}{1+\chi}; \quad (1-\alpha) = \frac{1}{1+\chi} = \frac{\alpha}{\chi}.$$

- Under rational expectations, this simplifies (do it!) to

$$p_t = \overline{m} + \phi\theta_{t-1} + (1-\alpha)\eta_t, \qquad \text{(REP)}$$

$$y_t = \alpha\eta_t. \qquad \text{(REY)}$$

9. (b) “Ad Hoc” Model

- Forecasting rule under learning

$$\widetilde{E}_t\left(p_{t+1}\right) = a_t + b_t\phi\theta_t, \qquad \text{(FR)}$$

$$c_t = \begin{pmatrix} a_t \\ b_t \end{pmatrix}$$

$$= \left[\frac{1}{t}\sum_{j=1}^{t} x_j x_j'\right]^{-1} \frac{1}{t}\sum_{j=1}^{t} x_j p_j, \qquad \text{(OLS)}$$

$$x_j = \begin{pmatrix} 1 \\ \phi\theta_{j-1} \end{pmatrix}.$$

9. (c) Recursive Forecasting

- Moment matrix 1

$$
\begin{aligned}
R_t &= \frac{1}{t}\sum_{j=1}^{t} x_j x_j' \\
&= \frac{1}{t}\left[\sum_{j=1}^{t-1} x_j x_j' + x_t x_t'\right], \\
&= \left(\frac{t-1}{t}\right)\left(\frac{1}{t-1}\right)\sum_{j=1}^{t-1} x_j x_j' + \frac{1}{t} x_t x_t', \\
&= \frac{t-1}{t} R_{t-1} + \frac{1}{t} x_t x_t', \qquad \text{(RMM1)} \\
&= R_{t-1} + \frac{1}{t}\left(x_t x_t' - R_{t-1}\right).
\end{aligned}
$$

9. (c) Recursive Forecasting

- Moment matrix 2

$$\begin{aligned} Q_t &= \frac{1}{t}\sum\nolimits_{j=1}^{t} x_j p_j \\ &= \left(\frac{t-1}{t}\right)\left(\frac{1}{t-1}\right)\sum\nolimits_{j=1}^{t-1} x_j p_j + \frac{1}{t} x_t p_t, \\ &= \frac{t-1}{t} Q_{t-1} + \frac{1}{t} x_t p_t. \qquad \text{(RMM2)} \end{aligned}$$

9. (c) Recursive Forecasting

- Use (RMM1) and (RMM2) to get recursive OLS:

$$
\begin{aligned}
c_t &= R_t^{-1} Q_t, \\
\Rightarrow Q_t &= R_t c_t, \\
Q_t &= \frac{t-1}{t} Q_{t-1} + \frac{1}{t} x_t p_t \\
&= \frac{t-1}{t} R_{t-1} c_{t-1} + \frac{1}{t} x_t p_t \\
&= \frac{t-1}{t} \left[\frac{t}{t-1} \left(R_t - \frac{1}{t} x_t x_t' \right) \right] c_{t-1} + \frac{1}{t} x_t p_t, \\
\Rightarrow c_t &= R_t^{-1} Q_t \\
&= c_{t-1} + \frac{1}{t} R_t^{-1} x_t \left(p_t - x_t' c_{t-1} \right). \qquad \text{(ROLS)}
\end{aligned}
$$

9. (d) Laws of motion

- The Perceived Law of Motion (PLM) for prices is given by the forecasting rule:

$$\widetilde{E}_{t-1}(p_t) = a_{t-1} + b_{t-1}\phi\theta_{t-1}. \qquad \text{(FR)}$$

- The Actual Law of Motion for prices is found by combining (EQP) and (FR):

$$\begin{aligned} p_t &= (1-\alpha)\left[\overline{m} + \theta_t\right] + \alpha\left[a_{t-1} + b_{t-1}\phi\theta_{t-1}\right], \\ &= a^*_{t-1} + b^*_{t-1}\phi\theta_{t-1} + (1-\alpha)\eta_t, \qquad \text{(ALM)} \\ a^*_{t-1} &= \overline{m} + \alpha(a_{t-1} - \overline{m}), \\ b^*_{t-1} &= 1 + \alpha(b_{t-1} - 1). \end{aligned}$$

9. (d) Laws of motion

- Rational expectations
  - Using equation (REP), suppose $a_t = \overline{a} = \overline{m}$, and $b_t = \overline{b} = 1$.
  - Then $a_t^* = \overline{a}$ and $b_t^* = \overline{b}$.
  - As $t \to \infty$, regression estimates of the coefficients in equation (REP) would converge to $\overline{a}$ and $\overline{b}$.
- But this isn't what happens! The economy instead follows the following recursive system:
  - a_t and b_t are updated with equations (RMM1) and (ROLS).
  - This updating uses p_t and θ_t.
  - But equation (ALM) shows that p_t depends on a_{t-1} and b_{t-1}.
  - Will this process converge?

9. (e) Convergence

- Define the bivariate function $T : \Re^2 \to \Re^2$ that converts the PLM to the ALM:

$$c_t^* = \begin{pmatrix} a_t^* \\ b_t^* \end{pmatrix}$$

$$= T(c_t) = \begin{pmatrix} \overline{m} + \alpha(a_t - \overline{m}) \\ 1 + \alpha(b_t - 1) \end{pmatrix}. \qquad \text{(TDEF)}$$

- Equation (ALM) becomes:

$$p_t = x_t' T(c_{t-1}) + (1 - \alpha)\eta_t.$$

9. (e) Convergence

- The recursive forecasting system becomes equation (RMM1) and

$$c_t = c_{t-1} + \frac{1}{t} R_t^{-1} x_t \left(x_t' \left[T(c_{t-1}) - c_{t-1} \right] + (1-\alpha)\eta_t \right) . \tag{ROLS$'$}$$

- c_t will converge to $\bar{c}$ iff $T(\cdot)$ exhibits <u>E-Stability</u>, that is, the solution to the differential equation

$$\dot{d}_t = T(d_t) - d_t, \tag{ES}$$

converges to a steady state from all nearby d_0.

9. (e) Convergence

- For the Ad Hoc model, condition (ES) involves

$$\dot{a}_t = (1-\alpha)\left(\overline{m} - a_t\right),$$
$$\dot{b}_t = (1-\alpha)\left(1 - b_t\right).$$

- The solution (check it!) to this differential equation is

$$a_t = \overline{m} + e^{(\alpha-1)t}\left(a_0 - \overline{m}\right),$$
$$b_t = 1 + e^{(\alpha-1)t}\left(b_0 - 1\right).$$

- $\chi > 0 \Rightarrow \alpha < 1 \Rightarrow$ E-Stability.
- E-Stability $\Rightarrow$ the recursive forecasting system given by (RMM1) and (ROLS′) converges to the rational expectations equilibrium.