Economics 701: Macroeconomics II

Spring 2009

Lecture 2: The Keynesian View, Rational Expectations and Policy (In)effectiveness

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January, 2009

5. Lucas Critique (1976)

   (a) Old view

$$
\begin{aligned}
\mathbf{z}_t &= F(\mathbf{x}_t, \mathbf{g}_t, \mathbf{e}_t), \\
\mathbf{z}_t &= \text{(private sector) economic variables}, \\
\mathbf{x}_t &= \text{exogenous and predetermined/lagged variables}, \\
\mathbf{g}_t &= \text{government policy variables} = G(\mathbf{z}_t, \mathbf{x}_t, \mathbf{e}_t), \\
\mathbf{e}_t &= \text{subjective expectations of future variables} \\
&= H(\mathbf{z}_t, \mathbf{x}_t, \mathbf{g}_t).
\end{aligned}
$$

Taking $H(\cdot)$ as given, estimate $F(\mathbf{x}_t, \mathbf{g}_t, \mathbf{e}_t)$ from historical data, then set $G(\cdot)$ to achieve goals.

5. (b) Lucas' critique

- $H(\cdot)$ and thus $F(\cdot)$ will change with $G(\cdot)$ (*as well as changes in other aspects of* $F(\cdot)$).
- Estimates of (the parameters of) $F(\cdot)$ based on historical $G(\cdot)$ are <u>reduced-form</u> estimates that are not invariant to changes in other parameters.
- Need estimates of the <u>structural</u> parameters of the underlying model (e.g., preference, technology parameters).

(c) Example: AD-AS model

- Old approach: non-rational expectations.
- Lucas's approach: rational expectations.

5. (d) Summary

- The response of private agents to changes in policy variables depends on the government's policy rules.
- Why? Agents are forward-looking, and their predictions of the future depend on the policy rules in effect.
- Estimating the way in which agents respond to policy variables under the rules in effect for the data does not tell us how agents will respond if the policy rules are changed.

6. Phillips Curve

(a) Phillips (1958): data show inverse relationship between nominal wage growth and unemployment:

$$\frac{\Delta W_t}{W_{t-1}} = g(u_t), \qquad g'(\cdot) < 0$$

$$u = \text{unemployment rate}.$$

(b) If inflation, $\pi_t = \Delta P_t / P_{t-1} \approx \Delta \ln(P_t)$, increases in $\Delta W_t / W_{t-1}$, we get an inflation-unemployment trade-off.

6. Phillips Curve

(c) Then if Y (or ΔY) increases in employment, we get an inflation-output trade-off.

- Close link to earlier AS curve:

$$y_t = p_t - \widetilde{E}_{t-1}(p_t),$$

and if $\widetilde{E}_{t-1}(p_t) = p_{t-1}$, $y_t = \pi_t$.

- N.B. even though price expectations are updated, there is a permanent inflation-output trade-off.
- Money can be neutral, but not superneutral: the growth rate of money can affect real quantities even if the level doesn't.

6. (d) Friedman (1968) and Phelps (1970): no permanent π/u trade-off.

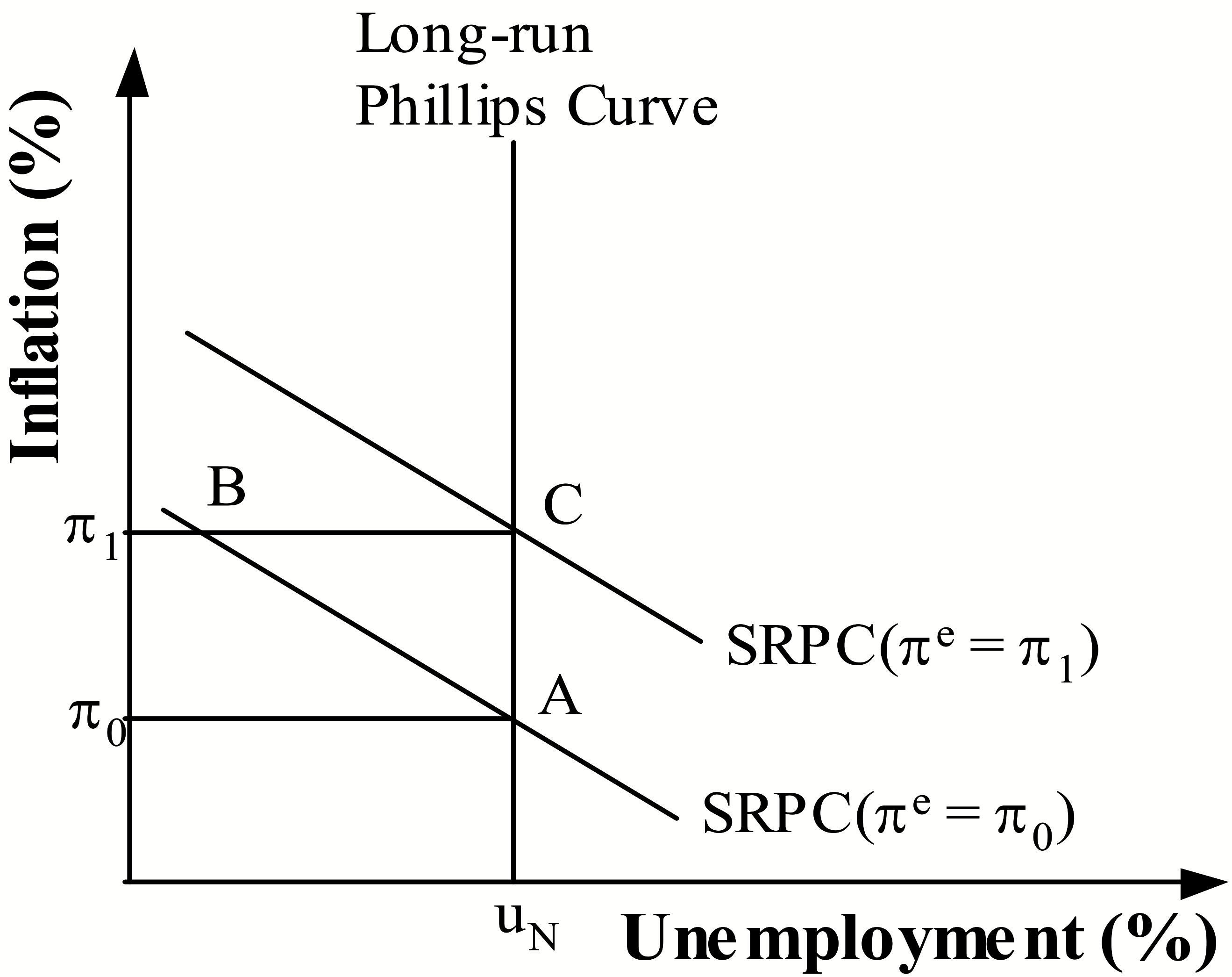
- Expectations-augmented Phillips curve:

$$\begin{aligned} \pi_t &= \widetilde{E}_{t-1}(\pi_t) + h(u_t - u_n), \\ h(0) &= 0, \quad h'(\cdot) < 0, \\ u_n &= \text{natural rate of unemployment}. \end{aligned}$$

- The AS curve is essentially an expectations-augmented Phillips curve:

$$\begin{aligned} y_t &= p_t - \widetilde{E}_{t-1}(p_t) \\ &= (p_t - p_{t-1}) - \left(\widetilde{E}_{t-1}(p_t) - p_{t-1}\right) \\ &= \pi_t - \widetilde{E}_{t-1}(\pi_t). \end{aligned}$$

6. (d) Long-run Phillips curve is vertical at u_n, as $\widetilde{E}_{t-1}(\pi_t)$ adjusts.



6. (d)
- Friedman & Phelps: $\widetilde{E}_{t-1}(\pi_t) = \pi_{t-1}$.
  - No permanent π/u trade-off. Money is superneutral.
  - One-time increase in π worsens trade-off forever.
  - Long-term employment increase requires accelerating inflation.
- Lucas: applied Muth's (1960) concept of rational expectations.
- Evidence: Phillips curve shifted up in 1970s.
  - Supply shocks: π and u <u>both</u> increased (unlike AD shocks)
  - Expansionary monetary policy
  - $\widetilde{E}_{t-1}(\pi_t)$ adapted to the inflation caused by the previous two events.

7. Conditional expectations and linear projections

   (a) Conditional expectation: function that minimizes the mean squared forecast error.

$$E\left(Y|\,\underline{X}\right) = \underset{\{f(\cdot)\}}{\text{argmin}}\; E\left(\left[Y - f\left(\underline{X}\right)\right]^2\right).$$

   N.B. $\underline{X}$ is a vector.

   (b) Best linear predictor or linear projection: <u>linear</u> function that minimizes the mean squared forecast error.

$$\widehat{E}\left(Y|\,\underline{X}\right) = \underset{\{linear\;\, f(\cdot)\}}{\text{argmin}}\; E\left(\left[Y - f\left(\underline{X}\right)\right]^2\right).$$

7. Conditional expectations and linear projections

(c) $\widehat{E}\left(Y|\,\underline{X}\right) = E\left(Y|\,\underline{X}\right)$ when $E\left(Y|\,\underline{X}\right)$ is linear

$$\begin{aligned} Y &= \alpha + \sum\nolimits_{j=1}^{J} X_j \beta_j + \varepsilon, \\ E\left(\varepsilon|\,\underline{x}\right) &= 0, \quad \text{(WLOG)}. \end{aligned}$$

- $E\left(\varepsilon|\,\underline{x}\right) = 0$ is an example of mean-independence, weaker than independence.
- If $(Y, \underline{X})$ are distributed multivariate normal:
  - $E\left(Y|\,\underline{X}\right)$ is linear.
  - If $C\left(Y, \underline{X}\right) = \underline{0}$, $\underline{X}$ and Y are independent. (Usually works only the other way.)

7. (d) Least squares normal equations

- Objective:

$$\min_{\{\alpha,\{\beta_j\}_{j=1}^{J}\}} E\left(\left[Y-\left(\alpha+\sum_{j=1}^{J} X_j\beta_j\right)\right]^2\right)$$

- First order conditions (*Note: problem is convex*)

$$\begin{aligned} -2E\left(\left[Y-\left(\alpha+\sum_{j=1}^{J} X_j\beta_j\right)\right]\right) &= 0, \\ -2E\left(\left[Y-\left(\alpha+\sum_{j=1}^{J} X_j\beta_j\right)\right]X_k\right) &= 0, \\ for\ k = 1,2,...,J. \end{aligned}$$

7. (d) Least squares normal equations

- Forecast error $\varepsilon = Y - \left(\alpha + \sum_{j=1}^{J} X_j \beta_j\right)$ is:
  - Zero-mean.
  - Uncorrelated with $\underline{X}$.
- In matrix form,

$$E\begin{pmatrix} Y \\ X_1 Y \\ \vdots \\ X_J Y \end{pmatrix} = E\begin{pmatrix} 1 & X_1 & \cdots & X_J \\ X_1 & X_1 X_1 & \vdots & X_1 X_J \\ \vdots & \vdots & \vdots & \vdots \\ X_J & X_J X_1 & \vdots & X_J X_J \end{pmatrix}\begin{pmatrix} \alpha \\ \beta_1 \\ \vdots \\ \beta_J \end{pmatrix}.$$

7. (d) Least squares normal equations

- Let $\mathbf{X}$ be a row vector and rewrite in matrix form:

$$E\begin{pmatrix} Y \\ \mathbf{X}'Y \end{pmatrix} = E\begin{bmatrix} 1 & \mathbf{X} \\ \mathbf{X}' & \mathbf{X}'\mathbf{X} \end{bmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix}.$$

  - First line: $\alpha = E(Y) - E(\mathbf{X})\beta$.
  - Second line:

$$\begin{aligned} E\left(\mathbf{X}'Y\right) &= E\left(\mathbf{X}'\right)\alpha + E\left(\mathbf{X}'\mathbf{X}\right)\beta \\ &= E\left(\mathbf{X}'\right)E(Y) - E\left(\mathbf{X}'\right)E(\mathbf{X})\beta + E\left(\mathbf{X}'\mathbf{X}\right)\beta \\ &= \left[E\left(\mathbf{X}'\mathbf{X}\right) - E(\mathbf{X})'E(\mathbf{X})\right]\beta + E\left(\mathbf{X}'\right)E(Y) \\ &= E\left(\mathbf{X}'\right)E(Y) + V(\mathbf{X})\beta. \end{aligned}$$

7. (d) Matrix form

- First line:

$$\alpha = E(Y) - E(\mathbf{X})\,\beta.$$

- Second line:

$$E\left(\mathbf{X}'Y\right) - E\left(\mathbf{X}'\right)E(Y) = V(\mathbf{X})\,\beta$$

- Final result:

$$\begin{aligned} \beta &= \left[V\left(\mathbf{X}'\right)\right]^{-1} C\left(\mathbf{X}', Y\right), \\ \widehat{E}\left(Y|\,\mathbf{X}\right) &= \alpha + \mathbf{X}\beta \\ &= E(Y) + \left[\mathbf{X} - E(\mathbf{X})\right]\beta. \end{aligned}$$

7. (d)

- Matrix form
  - Final result:

$$\beta = \left[V\left(\mathbf{X}'\right)\right]^{-1} C\left(\mathbf{X}', Y\right),$$
$$\widehat{E}\left(Y|\,\mathbf{X}\right) = E\left(Y\right) + \left[\mathbf{X} - E\left(\mathbf{X}\right)\right]\beta.$$

  - Analytical shortcut: First find β, using variances and covariances; then find α, using β, $E\left(Y\right)$ and $E\left(\mathbf{X}\right)$.
- Scalar case:

$$\widehat{E}\left(Y|\,X\right) = \alpha + \beta X = E\left(Y\right) + \frac{\sigma_{xy}}{\sigma_x^2}\left[X - E\left(X\right)\right].$$

8. Lucas' (1972) Signal Extraction Model

(a) Outline

- One good
- Many isolated markets
- Random disturbances
    - Distribution of aggregate demand: market-specific real shocks
    - Aggregate monetary shocks
- Information set
    - No direct info about current monetary policy. Infer from prices in own market.
    - Perfect info about all previous shocks in all markets.

8. (b) The model

$$y_t(i) = y_t + z_t(i) - z_t - r_t(i), \tag{1}$$

$$r_t(i) \equiv p_t(i) - p_t, \tag{2}$$

$$y_t(i) = \alpha E\left(r_t(i) \mid I_t(i)\right), \quad \alpha > 0, \tag{3}$$

$$y_t = \frac{1}{N}\sum_{i=1}^{N} y_t(i), \tag{4}$$

$$p_t = \frac{1}{N}\sum_{i=1}^{N} p_t(i), \tag{5}$$

$$z_t = \frac{1}{N}\sum_{i=1}^{N} z_t(i), \tag{6}$$

$$y_t = m_t - p_t, \tag{7}$$

8. (b) The model

$$
\begin{aligned}
p_t(i) &= \ln(\text{price of good } i), \\
z_t(i) &= \text{real shock, i.i.d. across mkts. and time,} \\
&\quad \text{with } E(z_t(i)) = 0, V(z_t(i)) = \sigma_z^2, \\
y_t(i) &= \ln(\text{output in mkt. } i), \\
y_t &= \ln(\text{aggr. output}), \\
p_t &= \ln(\text{aggr. price level}), \\
m_t &= \ln(\text{money supply}) \\
&= \text{i.i.d. process with } E(m_t) = \overline{m}, V(m_t) = \sigma_m^2, \\
&\quad m_t \text{ and } z_s(i) \text{ independent, } \forall t, s, i,
\end{aligned}
$$

8. (b) The model

$$I_t(i) = \begin{cases} z_s(k), p_s(k), y_s(k), & \forall k,\ s < t, \\ p_s, z_s, m_s, y_s, & \forall s < t, \\ p_t(i) \end{cases} . \quad (8)$$

$I_t(i) =$ information set for mkt. i at time t.

8. (c) Signal extraction problem: Extracting $r_t(i)$ from $p_t(i)$.

- Assume m_t and $z_t(i)$ are each $\sim \mathcal{N}$.
- Guess and verify
  - Minimal guess: $r_t(i)$ and p_t are mutally and serially independent and $\sim \mathcal{N}$.
  - Time saver: Assume that

$$y_t(i) = b\left(p_t(i) - E(p_t)\right).$$

which implies (show this!)

$$p_t = \frac{1}{1+b} m_t + \frac{b}{1+b} \overline{m}, \tag{9}$$

$$r_t(i) = \frac{1}{1+b}\left[z_t(i) - z_t\right]. \tag{10}$$

8. (c)
- Guess and verify
  - Then $\{\{r_t(i)\}_i, \{p_t(i)\}_i\} \sim MV\mathcal{N}$.
  - Multivariate normality stronger than individual normality.
  - Collections of independent normal variables are $MV\mathcal{N}$.
  - Linear combinations of $MV\mathcal{N}$ variables are $MV\mathcal{N}$.
- The guess implies

$$\begin{aligned} E\left(r_t(i)\middle| I_t(i)\right) &= \widehat{E}\left(r_t(i)\middle| p_t(i)\right) \\ &= E\left(r_t(i)\right) + \\ &\quad \frac{C\left(r_t(i), p_t(i)\right)}{V\left(p_t(i)\right)}\left[p_t(i) - E\left(p_t(i)\right)\right]. \end{aligned}$$

8. (c) Extracting $r_t(i)$ from $p_t(i)$
Use equations (2), (6), (9) and (10):

$$
\begin{aligned}
E(p_t(i)) &= E(p_t + r_t(i)) \\
&= E\left(p_t + \frac{1}{1+b}\left[z_t(i) - z_t\right]\right) = E(p_t), \\
V(p_t(i)) &= V(p_t + r_t(i)) \\
&= V\left(\frac{m_t + b\overline{m}}{1+b}\right. \\
&\qquad \left. + \frac{1}{1+b}\left[z_t(i) - \frac{1}{N}\sum_{j=1}^{N} z_t(j)\right]\right) \\
&= \left(\frac{1}{1+b}\right)^2 \left[\sigma_m^2 + \eta\sigma_z^2\right], \quad \eta \equiv 1 - \frac{1}{N}.
\end{aligned}
$$

8. (c) Extracting $r_t(i)$ from $p_t(i)$
Continue with equations (2), (6), (9) and (10):

$$\begin{aligned} C\left(r_t(i), p_t(i)\right) &= C\left(r_t(i), p_t + r_t(i)\right) \\ &= V\left(r_t(i)\right) \\ &= \left(\frac{1}{1+b}\right)^2 \eta\sigma_z^2. \end{aligned}$$

8. (c) We thus have

$$
\begin{aligned}
E\left(\left. r_t(i)\right| I_t(i)\right) &= E\left(r_t(i)\right) + \\
&\quad \frac{C\left(r_t(i), p_t(i)\right)}{V\left(p_t(i)\right)}\left[p_t(i) - E\left(p_t(i)\right)\right] \\
&= 0 + \frac{\eta\sigma_z^2}{\sigma_m^2 + \eta\sigma_z^2}\left[p_t(i) - E\left(p_t\right)\right].
\end{aligned}
$$

which with (3) yields

$$
y_t(i) = b\left(p_t(i) - E\left(p_t\right)\right), \tag{3'}
$$

$$
b = \alpha\frac{\eta\sigma_z^2}{\sigma_m^2 + \eta\sigma_z^2},
$$

confirming our guess!

8. (c) Extracting $r_t(i)$ from $p_t(i)$

- We found that

$$y_t(i) = \alpha \frac{\eta \sigma_z^2}{\sigma_m^2 + \eta \sigma_z^2} \left[p_t(i) - E(p_t) \right].$$

- Intepretation
  - $\sigma_z^2 = 0 \Rightarrow y_t(i) = 0.$ All shocks are aggregate price (money) shocks, and money is neutral.
  - $\sigma_m^2 = 0 \Rightarrow y_t(i) = \frac{\alpha}{1+b} \left[z_t(i) - z_t \right]$. All shocks are local and real, and responded to fully.

8. (d) The Lucas AS curve

- Use (3′), (4) and (5) to aggregate across firms and get:

$$
\begin{aligned}
y_t &= \frac{1}{N}\sum\nolimits_{i=1}^{N} y_t(i) \\
&= \frac{1}{N}\sum\nolimits_{i} b\left(p_t(i) - E(p_t)\right) \\
&= b\left(p_t - E(p_t)\right).
\end{aligned}
$$

- Insert (9) to get the Lucas AS curve:

$$
y_t = \frac{b}{1+b}\left(m_t - \overline{m}\right). \qquad (11)
$$

- Standard result: only unexpected money matters.

8. (e) Illustrates the Lucas critique.

- What is the effect of increasing σ_m^2?
- Old approach: take b as given $\Rightarrow V(y_t)$ increases in σ_m^2.
- Lucas critique $\Rightarrow b$ is not invariant to σ_m^2. Rather,

$$
\begin{aligned}
V(y_t) &= \left(\frac{b}{1+b}\right)^2 \sigma_m^2 \\
&= \alpha^2 \left(\frac{\eta\sigma_z^2}{\sigma_m^2 + (1+\alpha)\,\eta\sigma_z^2}\right)^2 \sigma_m^2.
\end{aligned}
$$

and $V(y_t)$ decreases in σ_m^2.