Economics 701: Macroeconomics II

Spring 2009

Lecture 2: The Keynesian View, Rational Expectations and Policy (In)effectiveness

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January, 2009

2. Conditional distributions

(a) Work with random vector $\underline{x} = (X, Y) \sim F(x, y)$.

(b) Conditional probability: when $\Pr(\underline{x} \in B) > 0$,

$$\Pr(\underline{x} \in A | \, \underline{x} \in B) = \frac{\Pr(A \cap B)}{\Pr(B)}.$$

(c) Conditional distribution $F(\, y | \, x)$ (handles $\Pr(B) = 0$).

- Marginal distribution: $F_x\,(x) = \Pr\,(X \leq x)$.
- $F\,(\, y | \, x)$ is a function s.t. for any set A,

$$\int_{x \in A} F\,(\, y | \, x)\, dF_x\,(x) = \Pr\,((Y \leq y) \cap (X \in A))\,.$$

- c.f. $\Pr(B)\Pr(\, A | \, B) = \Pr(A \cap B)$.

2. (d) Independence:

$$F(x,y) = F_x(x)\,F_y(y) \Leftrightarrow$$
$$F(y|\,x) = F_y(y) \Leftrightarrow$$
$$F(x|\,y) = F_x(x)\,.$$

- i.i.d.: X and Y are independent and $F_x(\cdot) = F_y(\cdot)$.

(e) Conditional (mathematical, rational) expectation:

$$E(Y|\,x) = \int_{-\infty}^{\infty} y dF(y|\,x)\,.$$

3. Introduction to ARMA processes

(a) White noise: $\{\varepsilon_t\}_{t=-\infty}^{\infty}$ where

- $E(\varepsilon_t) = 0, \ \forall t$
- $V(\varepsilon_t) = \sigma_\varepsilon^2, \ \forall t$
- $C(\varepsilon_t, \varepsilon_{t-j}) = 0, \ \forall t, \ j \neq 0.$

(b) First-order autoregressive (AR(1)) process

- $x_t = \alpha + \phi x_{t-1} + \varepsilon_t$, where
  - ε_t is white noise
  - $|\phi| < 1$: stationarity restriction.

3. (b) First-order autoregressive (AR(1)) process

- $x_t = \alpha + \phi x_{t-1} + \varepsilon_t$.
- By recursive substitution:

$$
\begin{aligned}
x_t &= \alpha + \varepsilon_t + \phi \left[\alpha + \phi x_{t-2} + \varepsilon_{t-1}\right] \\
&= \frac{\alpha}{1-\phi} + \sum_{j=0}^{\infty} \phi^j \varepsilon_{t-j}.
\end{aligned}
$$

- $E\left(x_t\right) = \alpha / \left(1-\phi\right)$.

3. (b) First-order autoregressive (AR(1)) process

- Recall that

$$V(aX + bY) = a^2 V(X) + b^2 V(Y) + 2abC(X, Y),$$

$$C(aX + bY, cX + dY) = acV(X) + bdV(Y) + (ad + bc)C(X, Y).$$

- Then

$$x_t = \frac{\alpha}{1-\phi} + \sum_{j=0}^{\infty} \phi^j \varepsilon_{t-j} \Rightarrow$$

$$V(x_t) = \sum_{j=0}^{\infty} \phi^{2j} \sigma_\varepsilon^2 = \frac{\sigma_\varepsilon^2}{1-\phi^2}.$$

3. (b) First-order autoregressive (AR(1)) process

$$
\begin{aligned}
C(x_t, x_{t-1}) &= C(\alpha + \phi x_{t-1} + \varepsilon_t, x_{t-1}) \\
&= 0 + \phi V(X) + 0 = \phi \frac{\sigma_\varepsilon^2}{1-\phi^2}, \\
C(x_t, x_{t-k}) &= C\left(\phi^k x_{t-k} + \sum_{j=0}^{k-1} \phi^j \varepsilon_{t-j}, x_{t-k}\right) \\
&= \phi^k \frac{\sigma_\varepsilon^2}{1-\phi^2} = \phi^k V(x_t).
\end{aligned}
$$

- If $\{\varepsilon_t\}$ is i.i.d. and $\alpha = 0$, $E(x_t | x_{t-k}) = \phi^k x_{t-k}$.

3. (c) First-order moving average (MA(1)) process

- $x_t = \alpha + \varepsilon_t + \theta\varepsilon_{t-1}$.
- ε_t is white noise.
- $E(x_t) = \alpha$.
- $V(x_t) = \left(1 + \theta^2\right)\sigma_\varepsilon^2$.
- $C(x_t, x_{t-1}) = C(\varepsilon_t + \theta\varepsilon_{t-1}, \varepsilon_{t-1} + \theta\varepsilon_{t-2}) = \theta\sigma_\varepsilon^2$.
- $C(x_t, x_{t-k}) = C(\varepsilon_t + \theta\varepsilon_{t-1}, \varepsilon_{t-k} + \theta\varepsilon_{t-k-1})$
  $= 0,\ k > 1$.
- $E(x_t | x_{t-k})$ discussed later.

3. (c) First-order moving average (MA(1)) process

- $x_t = \alpha + \varepsilon_t + \theta\varepsilon_{t-1}$.
- When $|\theta| < 1$,

$$\begin{aligned}\varepsilon_t &= x_t - \left(\alpha + \theta\varepsilon_{t-1}\right) \\ &= x_t - \alpha - \theta\left(x_{t-1} - \left(\alpha + \theta\varepsilon_{t-2}\right)\right) \\ &= \frac{-\alpha}{1+\theta} + \sum_{j=0}^{\infty} (-\theta)^j \, x_{t-j},\end{aligned}$$

and x_t is said to be <u>invertible</u>.

3. (d) ARMA(p,q) Process

$$x_t = \alpha + \phi_1 x_{t-1} + \phi_2 x_{t-2} + ... + \phi_p x_{t-p}$$
$$+ \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + ... + \theta_q \varepsilon_{t-q}.$$

- Moving Average Representation
  - Stationarity condition: The roots of

$$1 - \phi_1 z - \phi_2 z^2 - ... - \phi_p z^p = 0,$$

    lie outside unit circle ($|\cdot| > 1$ when real).
  - For an AR(1), $z = \phi^{-1}$, so that $|\phi| < 1 \Leftrightarrow |z| > 1$.
  - If this condition holds, we can rewrite x_t as

$$x_t = \mu + \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j}.$$

3. (d) ARMA(p,q) Process

$$x_t = \alpha + \phi_1 x_{t-1} + \phi_2 x_{t-2} + ... + \phi_p x_{t-p}$$
$$+ \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + ... + \theta_q \varepsilon_{t-q}.$$

- Invertibility:
  - Roots of

    $$1 + \theta_1 z + \theta_2 z^2 + ... + \theta_q z^q = 0,$$

    lie on or outside the unit circle ($|| \geq 1$ when real).
  - For an MA(1), $z = (-\theta)^{-1}$. We include $|\theta| = 1$ as a limit.

3. (d) ARMA(p,q) Process

- Stationarity and invertibility imply:
  - ε_t is the <u>innovation</u> to x_t:

$$\varepsilon_t = x_t - \widehat{E}\left(x_t \middle| x_{t-1}, x_{t-2}, \ldots.\right).$$

  - If $\{\varepsilon_t\}$ is i.i.d.,

$$\varepsilon_t = x_t - E\left(x_t \middle| x_{t-1}, x_{t-2}, \ldots.\right).$$

  - Knowledge of the <u>entire</u> sequences $\{\varepsilon_{t-j}\}_{j=0}^{\infty}$ and $\{x_{t-j}\}_{j=0}^{\infty}$ is equivalent.

4. AD-AS model with expectations

(a) Basic “Ad Hoc” Model

$$y_t = -(w_t - p_t), \qquad \text{(AS)}$$

$$y_t = m_t - p_t + \theta_t, \qquad \text{(AD)}$$

$$w_t = \widetilde{E}_{t-1}(p_t), \qquad \text{(WS)}$$

$y_t = \ln$ (output),

$w_t = \ln$ (nominal wage,)

$m_t = \ln$ (nominal money supply), set by central bank,

$p_t = \ln$ (price level),

$\theta_t =$ exogenous AD shock,

$\widetilde{E}_{t-1}(\cdot) =$ subjective expectations.

4. (b) Full employment output: prices are perfectly anticipated: $y_n = 0$.

   (c) Solve

$$y_t = p_t - \widetilde{E}_{t-1}(p_t) = m_t - p_t + \theta_t,$$

$$p_t = \frac{1}{2}\left[m_t + \theta_t + \widetilde{E}_{t-1}(p_t)\right], \qquad \text{(EQP)}$$

$$y_t = \frac{1}{2}\left[m_t + \theta_t - \widetilde{E}_{t-1}(p_t)\right]. \qquad \text{(EQY)}$$

4. (d) Optimal monetary policy under non-rational expectations

- Common type is adaptive expectations:

$$\widetilde{E}_{t-1}(p_t) = \sum_{j=1}^{\infty} \psi_j p_{t-j}, \quad \{\psi_j\} \text{ fixed.}$$

- Assume θ_t is an AR(1):

$$\theta_t = \phi\theta_{t-1} + \eta_t,$$

where η_t is an i.i.d. zero-mean process with

$$V(\eta_t) = \sigma_\eta^2.$$

4. (d) Optimal monetary policy under non-rational expectations

- When setting m_t, the central bank's information set is

$$I_{t-1} = \left\{ \widetilde{E}_{t-j}\left(p_{t-j+1}\right), \theta_{t-j}, p_{t-j}, m_{t-j}, y_{t-j}, w_{t-j} \right\}_{j=1}^{\infty}.$$

- C-bank uses the policy rule:

$$m_t = \lambda_0 + \lambda_1 \theta_{t-1} + \lambda_2 \widetilde{E}_{t-1}\left(p_t\right) + \varepsilon_t.$$

  - ε_t is an i.i.d. zero-mean process with variance σ_ε^2, and η_t and ε_{t-j} are independent, $\forall t, j$.
  - ε reflects uncertainties in policy-setting or implementation.

4. (d) Optimal m. policy under non-rational expectations

- C-bank picks $\left(\lambda_0, \lambda_1, \lambda_2, \sigma_\varepsilon^2\right)$ to:
  - Minimize $V\left(y_t\right)$,
  - Set $E\left(y_t\right) = y_n = 0$.
- Insert monetary rule and process for θ_t into (EQY):

$$y_t = \frac{1}{2}\left[\begin{array}{l} \lambda_0 + \left(\lambda_1 + \phi\right)\theta_{t-1} + \left(\lambda_2 - 1\right)\widetilde{E}_{t-1}\left(p_t\right) \\ +\eta_t + \varepsilon_t \end{array}\right],$$

$$V\left(y_t\right) = \frac{1}{4}\left[\begin{array}{l} \left(\lambda_1 + \phi\right)^2 V\left(\theta_{t-1}\right) \\ +\left(\lambda_2 - 1\right)^2 V\left(\widetilde{E}_{t-1}\left(p_t\right)\right) + \\ 2\left(\lambda_1 + \phi\right)\left(\lambda_2 - 1\right) C\left(\theta_{t-1}, \widetilde{E}_{t-1}\left(p_t\right)\right) \\ +\sigma_\varepsilon^2 + \sigma_\eta^2 \end{array}\right].$$

4. (d) Optimal m. policy under non-rational expectations

- Minimize with $\lambda_1 = -\phi$, $\lambda_2 = 1$, $\sigma_\varepsilon^2 = 0$, so that

$$\begin{aligned} y_t &= \frac{1}{2}\left[\lambda_0 + \eta_t\right], \\ E\left(y_t\right) &= \frac{1}{2}\lambda_0, \\ V\left(y_t\right) &= \frac{1}{4}\sigma_\eta^2, \end{aligned}$$

- Set $\lambda_0 = 0$, so that

$$\begin{aligned} m_t &= -\phi\theta_{t-1} + \widetilde{E}_{t-1}\left(p_t\right), \\ y_t &= \frac{1}{2}\eta_t, \\ y_t &= p_t - \widetilde{E}_{t-1}\left(p_t\right) \Rightarrow p_t = \frac{1}{2}\eta_t + \widetilde{E}_{t-1}\left(p_t\right) \end{aligned}$$

4. (e) Optimal monetary policy under rational expectations

- Rational expectations

$$\widetilde{E}_{t-1}(p_t) = E_{t-1}(p_t) = E(p_t | I_{t-1}).$$

  - Agents know the model and economic environment.
  - Fixed point: behavior consistent with beliefs about future outcomes—beliefs about future outcomes consistent with behavior.
- Law of iterated expectations: if $I^b \subseteq I^a$,

$$\begin{aligned} E\left(E(X|I^a)|I^b\right) &= E\left(X|I^b\right) \\ \Rightarrow E(Y) &= \int_{-\infty}^{\infty} E(Y|x)\, dF(x). \end{aligned}$$

4. (e) Optimal monetary policy under rational expectations

- Apply L.I.E. to equation (EQP):

$$
\begin{aligned}
E_{t-1}\left(p_t\right) &= \frac{1}{2} E_{t-1}\left(m_t + \theta_t + E_{t-1}\left(p_t\right)\right), \\
E_{t-1}\left(p_t\right) &= E_{t-1}\left(m_t + \theta_t\right), \\
y_t &= \frac{1}{2}\left[m_t + \theta_t - E_{t-1}\left(m_t + \theta_t\right)\right].
\end{aligned}
$$

4. (e) Optimal monetary policy under rational expectations

- Bank uses the policy rule:

$$m_t = \mu_t + \sum_{j=1}^{\infty} \lambda_j \eta_{t-j} + \varepsilon_t.$$

- μ_t is known at time $t-1$; ε_t and η_t are the same as before. Then

$$\begin{aligned} E_{t-1}(p_t) &= E_{t-1}(m_t + \theta_t) \\ &= m_t - \varepsilon_t + \phi\theta_{t-1}, \\ y_t &= \frac{1}{2}\left[m_t + \theta_t - (m_t - \varepsilon_t + \phi\theta_{t-1})\right] \\ &= \frac{1}{2}\left[\varepsilon_t + \eta_t\right]. \end{aligned}$$

4. (e) Optimal monetary policy under rational expectations

- It follows that

$$E(y_t) = 0,$$
$$V(y_t) = \frac{1}{4}\left[\sigma_\eta^2 + \sigma_\varepsilon^2\right].$$

- The only thing the C-bank can affect is σ_ε^2. All other elements of the policy rule are anticipated and neutralized, and are thus irrelevant for output.

4. (f) Intuition

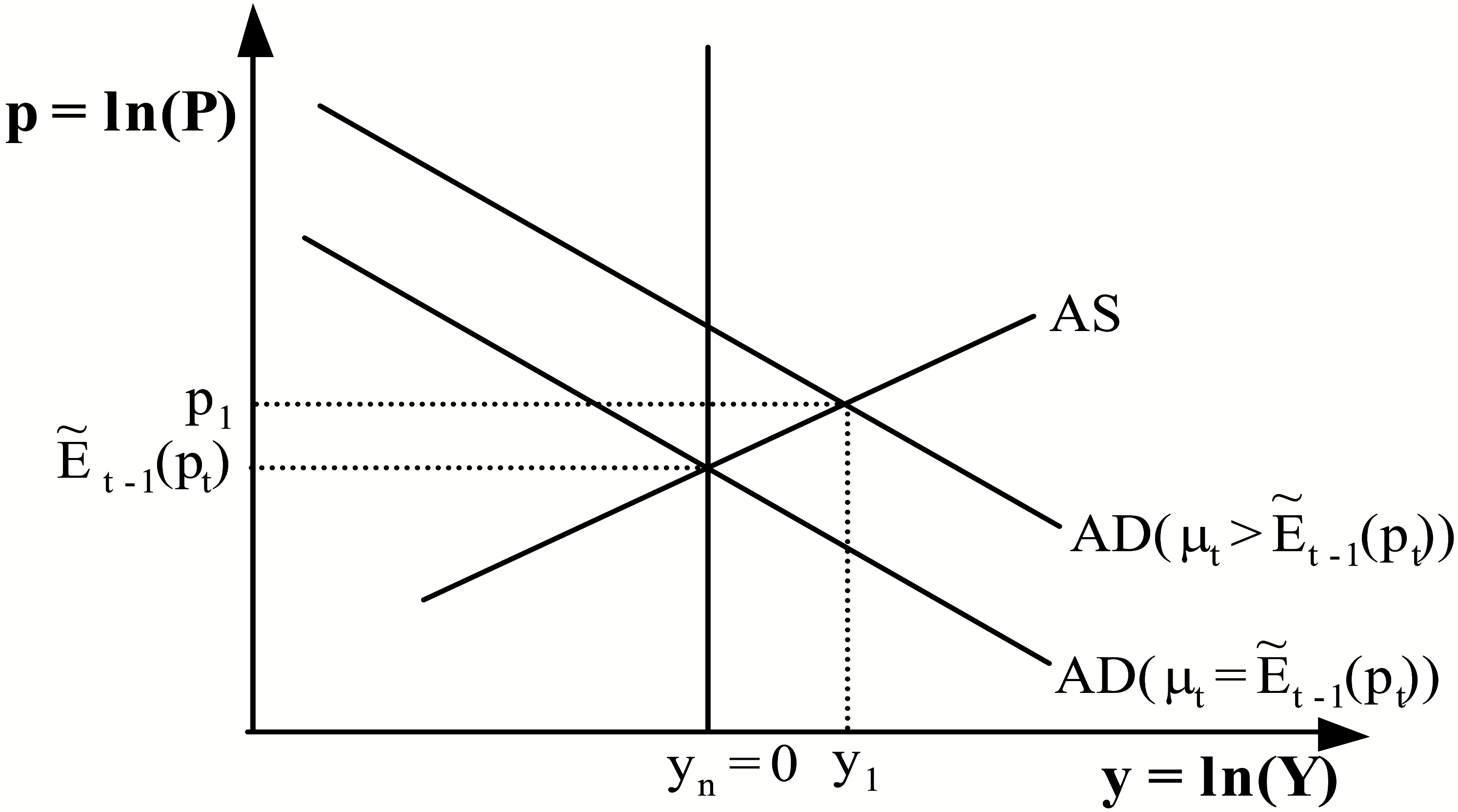
- Simplify: $\theta_t = \varepsilon_t = 0$; $m_t = \mu_t$, known at time $t - 1$.
- Non-rational expectations
  - AD and AS curves become

$$y_t = \mu_t - p_t, \tag{AD}$$

$$y_t = p_t - \widetilde{E}_{t-1}(p_t). \tag{AS}$$

  - Anticipated changes in money affect output because nominal wages do not incorporate these changes.
  - Under adaptive expectations ($\widetilde{E}_{t-1}(p_t)$ *reflects* μ_{t-1}), the effect on output of a change in money might not be permanent.

4. (f) • Non-rational expectations



4. (f) Intuition

- Rational expectations
  - AD and AS curves become

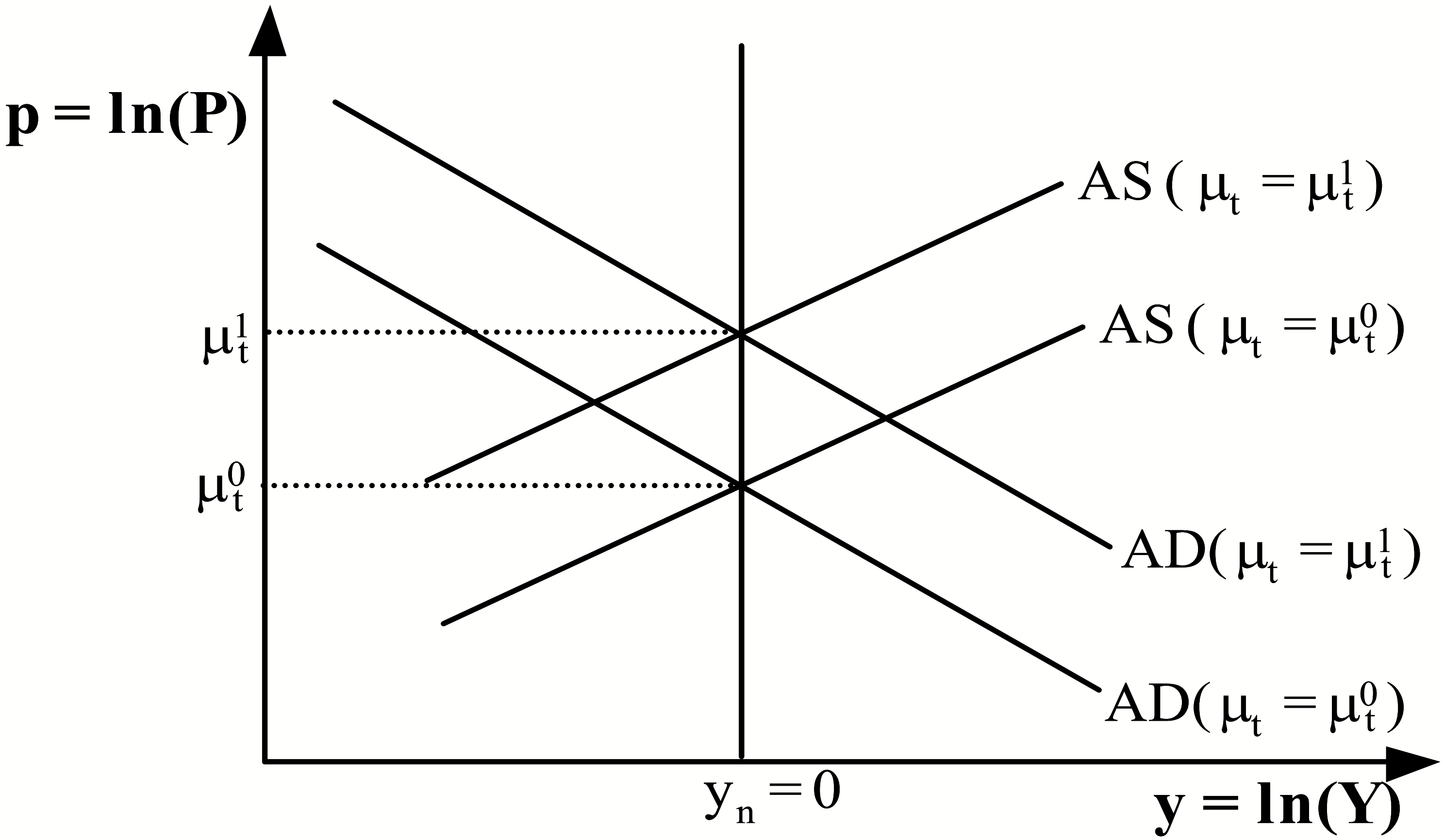
$$y_t = \mu_t - p_t, \qquad \text{(AD)}$$

$$y_t = p_t - E\left(p_t \middle| I_{t-1}\right) = p_t - \mu_t, \qquad \text{(AS)}$$

$$\Rightarrow y_t = 0 = y_n.$$

  - Anticipated changes in money do not affect output because nominal wages incorporate the changes.
- Do people have rational expectations?
  - One testable implication: under RE, only unexpected changes in money affect output.
  - Econometric problem: monetary policy is endogenous—is money driving output or output driving money?

4. (f) Rational expectations



4. (g) Rational expectations with an informational advantage

- C-bank now knows η_t:

$$I_{t-1}^{govt} = \left\{ I_{t-1}^{priv}, \eta_t \right\}.$$

so that

$$\begin{aligned} m_t &= \mu_t + \sum_{j=0}^{\infty} \lambda_j \eta_{t-j} + \varepsilon_t, \\ E\left(m_t + \theta_t | \, I_{t-1}^{priv} \right) &= \mu_t + \sum_{j=1}^{\infty} \lambda_j \eta_{t-j} + \sum_{j=1}^{\infty} \phi^j \eta_{t-j} \\ &= m_t - \lambda_0 \eta_t - \varepsilon_t + \phi \theta_{t-1}, \\ y_t &= \frac{1}{2} \left[\eta_t + \lambda_0 \eta_t + \varepsilon_t \right]. \end{aligned}$$

4. (g) Rational expectations with an informational advantage

- C-bank now knows η_t:

$$I_{t-1}^{govt} = \left\{ I_{t-1}^{priv}, \eta_t \right\}.$$

so that

$$y_t = \frac{1}{2}\left[\eta_t + \lambda_0 \eta_t + \varepsilon_t\right].$$

- Set $\lambda_0 = -1$, $\sigma_\varepsilon^2 = 0 \Rightarrow y_t = 0 \Rightarrow V(y_t) = 0$.
- C-bank can exploit informational advantage to stabilize economy.
- Same results if government is more flexible: e.g., wages must be set two periods in advance, so that $w_t = E(p_t | I_{t-2})$.