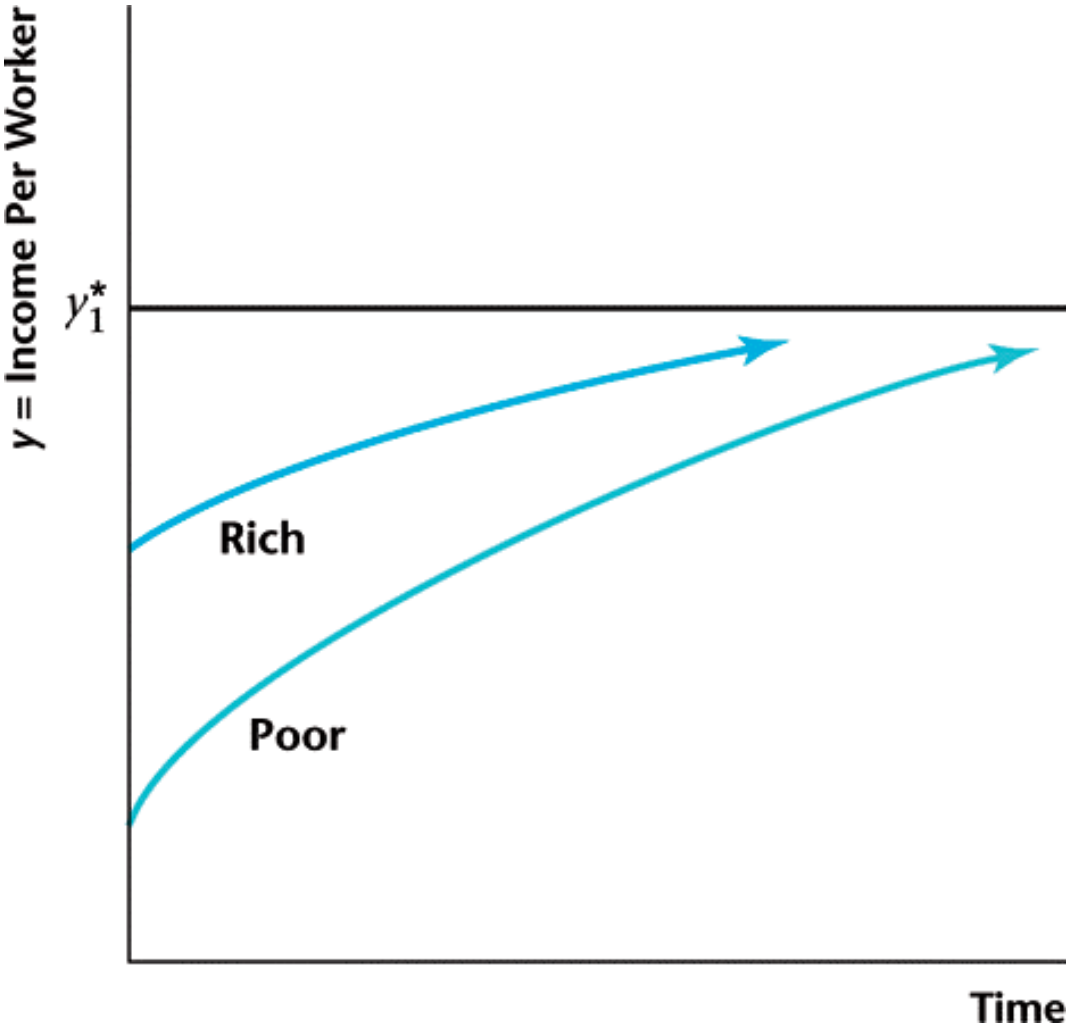
Lecture 6
Endogenous Growth

October 2009

Convergence and the Solow Model

- **Convergence**: Poor countries grow more quickly than rich countries, i.e., standards of living converge.
- Solow Model generates convergence.
  - Recall that in the Solow model, $k' = \frac{1-d}{1+n}k + \frac{s}{1+n} \cdot zf(k)$.
  - This implies that the gross growth rate of capital per worker, G_k, is
$$G_k = \frac{k'}{k} = \frac{1-d}{1+n} + \frac{s}{1+n} \cdot z\frac{1}{k}f(k).$$
  - $zf(k)/k$ is the average product of capital, or AP_K.
  - Diminishing $MP_K \Rightarrow$ diminishing AP_K.
  - Example: Cobb-Douglas $\Rightarrow AP_K = z = zk^{\alpha}/k = zk^{\alpha-1}$, where $\alpha - 1$ is negative.
  - Diminishing $AP_K \Rightarrow G_k$ decreasing in $k \Rightarrow$ richer countries grow more slowly.

Figure 7.2 Convergence in Income per Worker Across Countries in the Solow Growth Model



Sustained Growth in the Solow Model

- Sustained per-worker output growth requires sustained productivity growth.
- Countries with identical long-run growth rates in productivity will have identical long-run growth rates in per-worker output.
- Cobb-Douglas example:
  - $Y = zK^{\alpha}N^{1-\alpha} \Rightarrow y = Y/N = zk^{\alpha}$, where $k = K/N$.
  - Note that

$$\begin{aligned} G_y &= \frac{y'}{y} = \frac{z'\left(k'\right)^{\alpha}}{zk^{\alpha}} \\ &= \left(\frac{z'}{z}\right)\left(\frac{k'}{k}\right)^{\alpha} = G_z G_k^{\alpha}. \end{aligned}$$

- Along a **balanced growth path**,

$$\begin{aligned} G_y &= G_k \\ \Rightarrow G_k &= G_z G_k^{\alpha} \\ \Rightarrow G_k &= G_z^{1/(1-\alpha)} = G_y. \end{aligned}$$

Solow Model vs. the Data

- Data show:
  - No convergence in general.
  - Convergence among the richest countries.
  - No convergence among the poorest countries.
- How can the Solow Model explain this?
- Explanation 1: Different saving and population growth rates
  - Lead to differences in k^* and y^*.
  - Generally viewed as insufficient.
- Explanation 2: Different levels of productivity
  - Isn't technology global?
  - No, if governments raise barriers to competition. (Parente and Prescott)
  - No, if productivity is expanded to include performance of government and legal institutions (beer truck story).

Endogenous Growth Theory

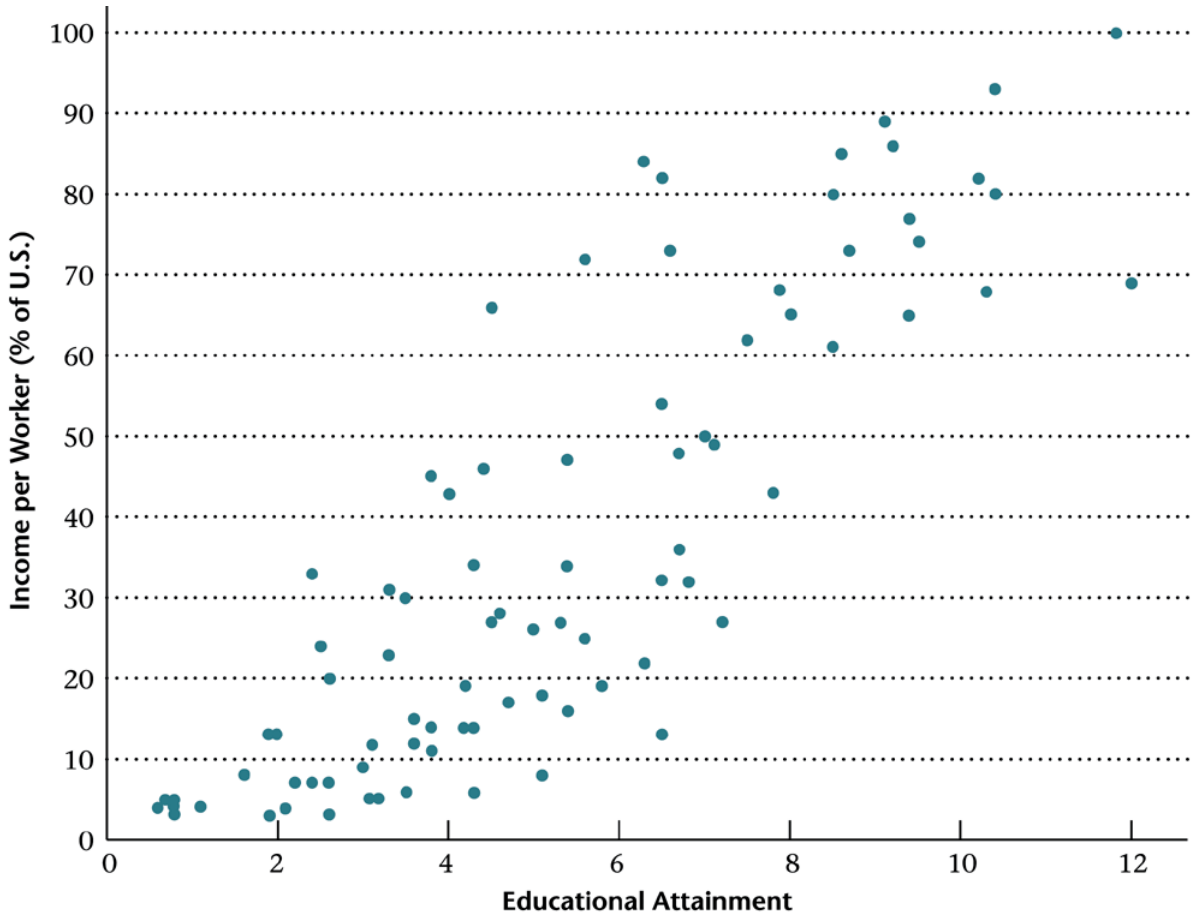
- Tries to explain productivity growth.
  - **Human capital**: Skills and knowledge embodied in workers.
  - Ideas (blueprints)
- With endogenous productivity growth, MP_K might not be decreasing.
  - Reason: Increasing K now increases productivity as well.
  - Without diminishing returns to capital, convergence might never occur.
- Savings rates, etc. can now affect growth rates as well as levels, because they affect productivity growth.

Human Capital

- Human capital:
  - Skills and knowledge embodied in workers.
  - One source: education.
  - Another source: learning-by-doing (shipbuilding picture).

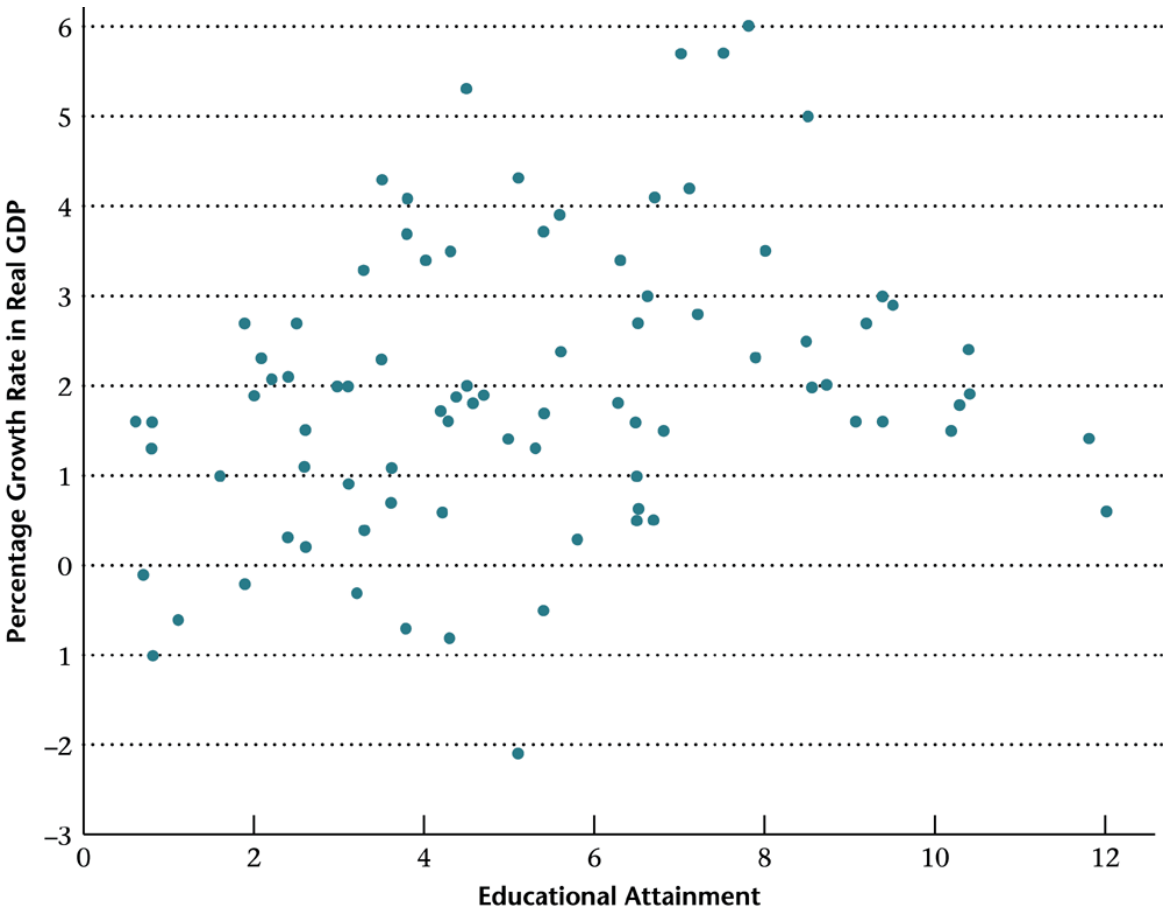
Human Capital Accumulation through Education

Figure 7.10 Income per Worker and Education



Human Capital and Education (Continued)

Figure 7.9 Growth and Education



Human Capital Accumulation through Learning by Doing

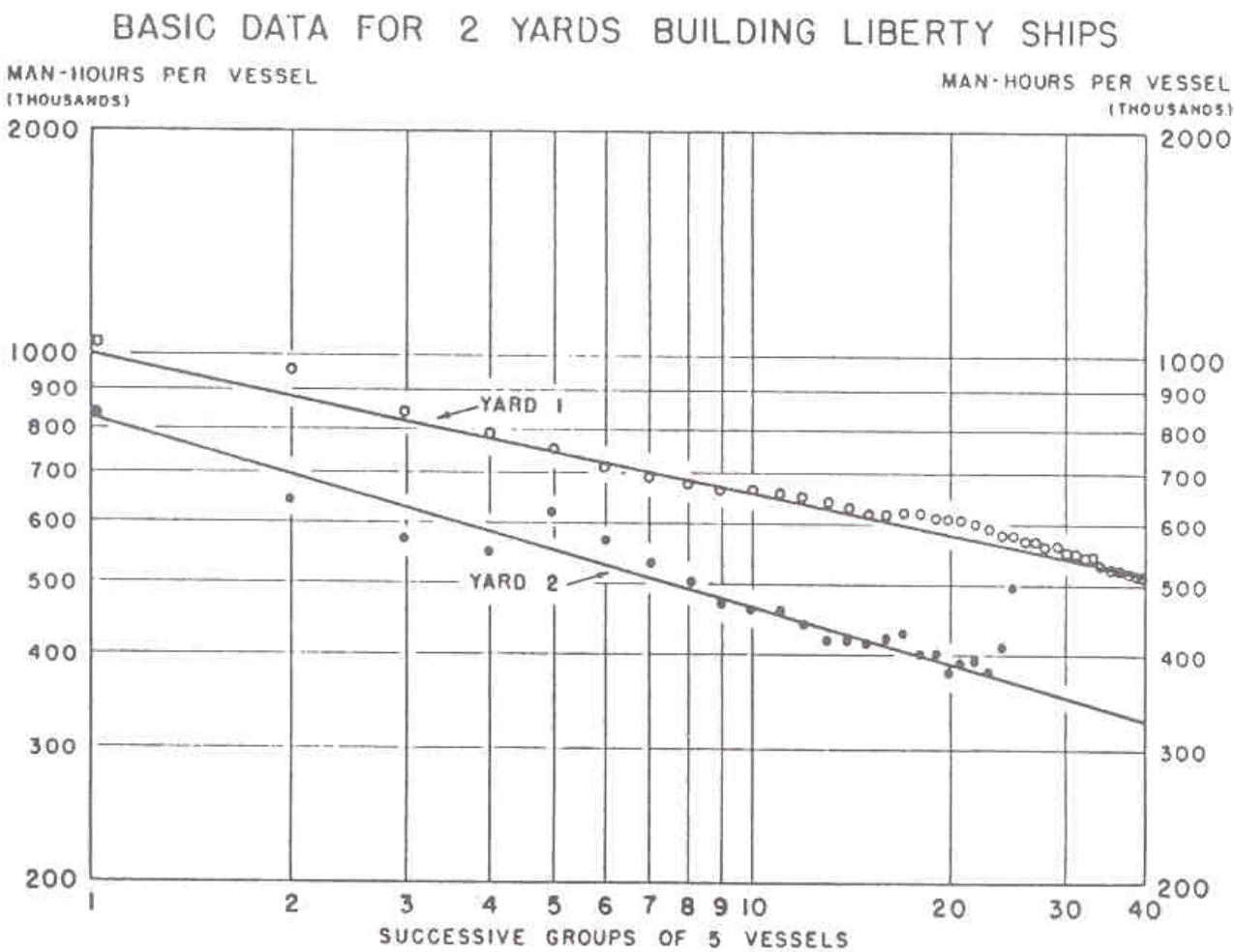


FIGURE 1.—Reductions in man-hours per vessel with increasing production. Merchant shipyards.

Source: Lucas (*Econometrica*, 1993), page 260.

Ideas

- Ideas (blueprints)
  - Innovation fueled by R&D expenditures.
  - Key feature: Ideas are **non-rival** goods that are costless to spread. (Paul Romer)

A Growth Model with Human Capital

- Consumers
  - N = labor = population.
  - Population grows exogenously at the net rate n: $N' = (1+n)N$, with $n > -1$.
  - Own firms, pay no taxes $\Rightarrow$ consumers' disposable income = Y.
  - Allocate income between consumption (C) and saving (S).
  - Assume $S = sY \Rightarrow C = (1-s)Y$.
- The representative firm:
  - Production function: $Y = aF(K, hN^d) = aF(K, hN)$.
  - a is a positive constant.
  - h = human capital per worker.
  - $F(.)$ has standard properties, including CRTS.
  - $K' = (1-d)K + I$.
- (Capital Market) Equilibrium

$$K' = (1-d)K + s \cdot aF(K, hN).$$

Growth with Human Capital: Per Worker Equilibrium

- CRTS $\Rightarrow$ output per worker is

$$\begin{aligned} y &= \frac{Y}{N} = \frac{1}{N} aF(K, hN) \\ &= aF\left(\frac{K}{N}, h\frac{N}{N}\right) \\ &= aF(k, h). \end{aligned}$$

- $aF(k, h)$ is the per worker production function.
- Assume that $h = k$:
  - Human capital improves with physical capital
  - Learning by doing.
  - Educational expenses proportional to saving.
- CRTS $\Rightarrow y = aF(k, h) = aF(k, k) = akF(1, 1) = Ak$.
  - Intuition: Doubling $k \Rightarrow$ doubling $h \Rightarrow$ doubling y.
  - Cobb-Douglas example: $y = zk^{\alpha}h^{1-\alpha} = zk^{\alpha}k^{1-\alpha} = zk \Rightarrow A = z$.
- $MP_K = A$ is constant.

Growth with Human Capital: Balanced Growth Path

- Write the equilibrium expression for K' as:

$$\begin{aligned}
\frac{K'}{N} \cdot \frac{N'}{N'} &= (1-d)\frac{K}{N} + s \cdot \frac{1}{N} aF(K, hN) \\
\Rightarrow k'(1+n) &= (1-d)k + s \cdot aF(k, h) \\
&= (1-d)k + s \cdot Ak \\
\Rightarrow k' &= \left(\frac{1-d+sA}{1+n}\right) k.
\end{aligned}$$

- This implies that the gross growth rate of capital per worker, G_k, is given by

$$G_k = \frac{k'}{k} = \frac{1-d+sA}{1+n}.$$

Growth with Human Capital: Balanced Growth Path

- The gross growth rate of capital per worker, G_k, is given by

$$G_k = \frac{k'}{k} = \frac{1 - d + sA}{1 + n}.$$

- The growth rate of output per worker is

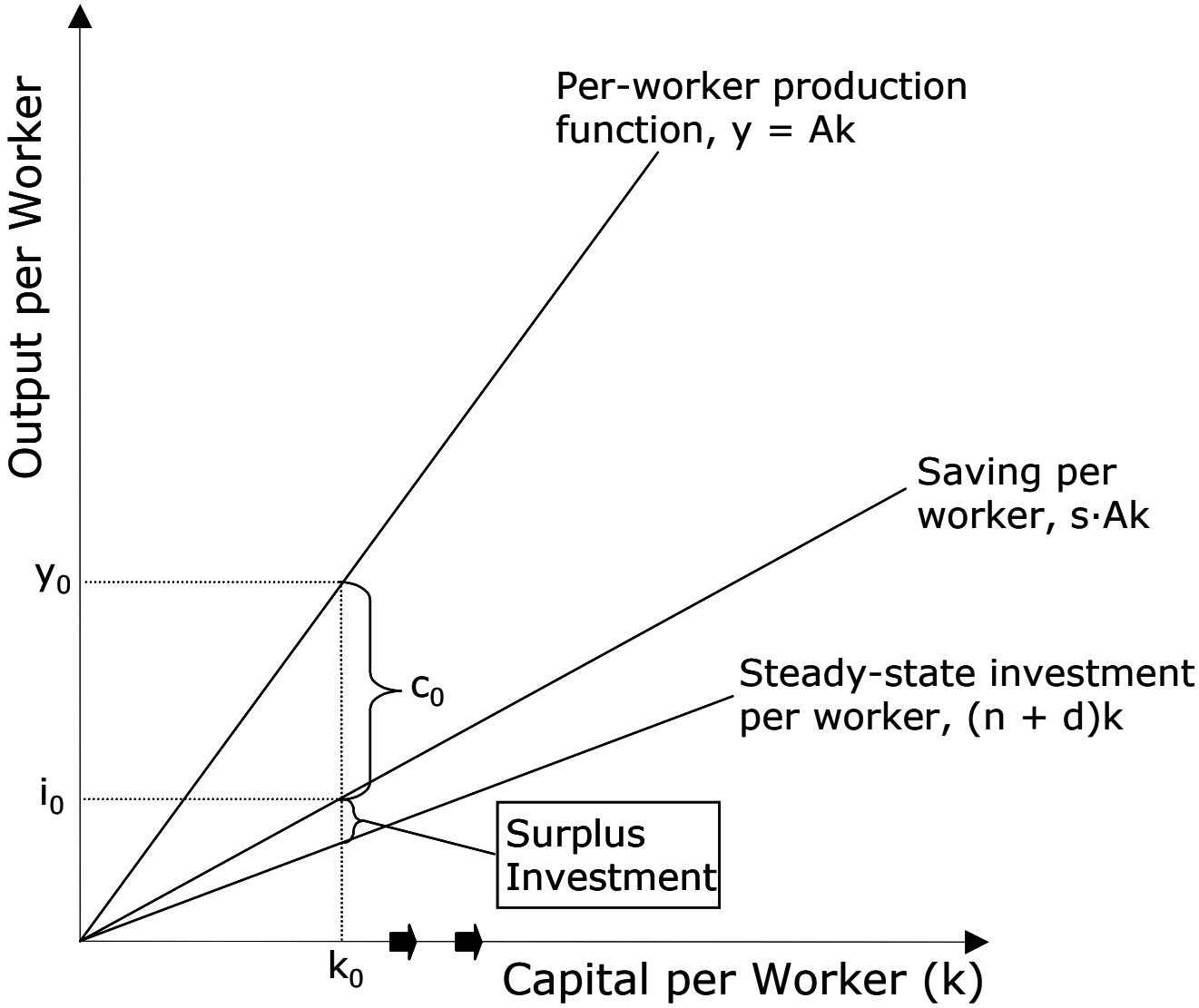
$$G_y = \frac{y'}{y} = \frac{Ak'}{Ak} = \frac{k'}{k} = G_k.$$

- The growth rate of consumption per worker is

$$G_c = \frac{c'}{c} = \frac{(1-s)y'}{(1-s)y} = \frac{y'}{y} = G_y = G_k.$$

- Note that **growth rates**—as opposed to steady-state **levels**—depend on A, s, d and n.

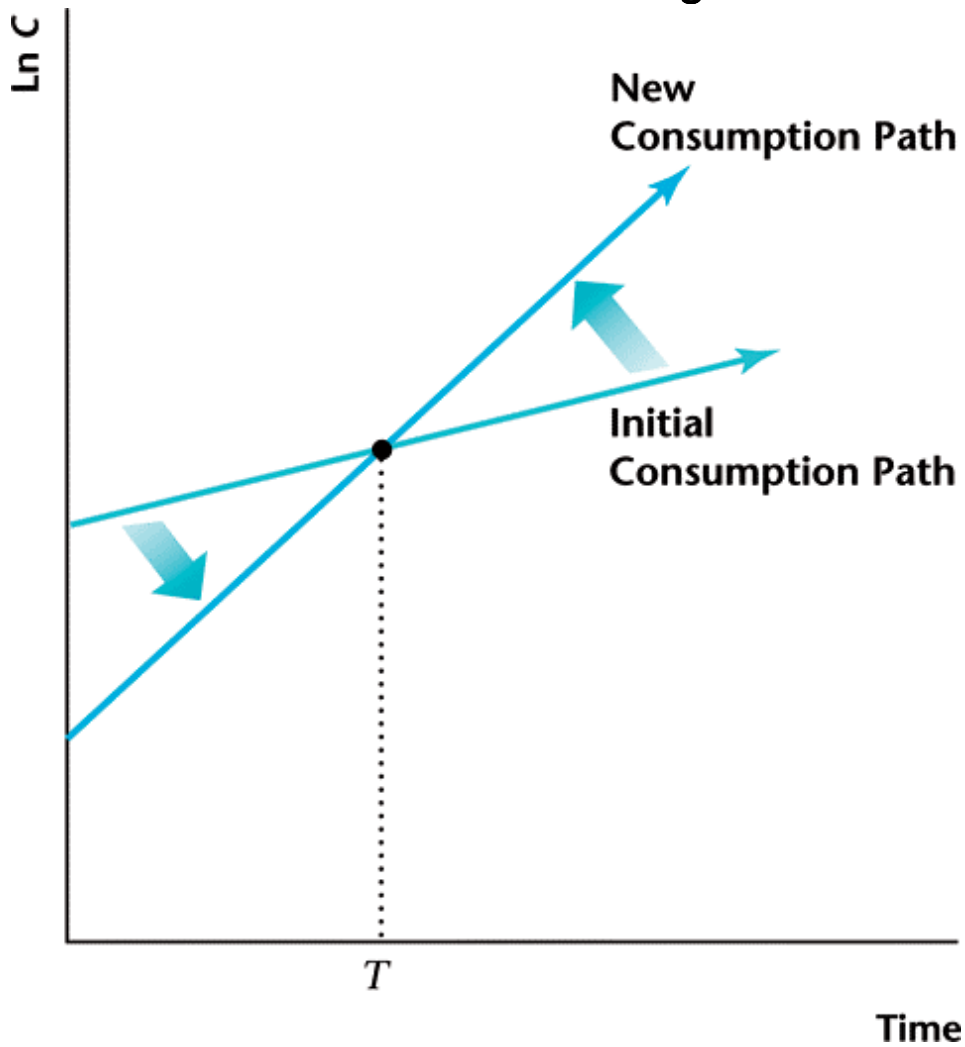
Balanced Growth in the y=Ak Model



A Growth Model with Human Capital: Implications

- No convergence:
  - Poor economies might grow more or less quickly than rich ones.
  - Even if all economies grow at similar rates, poor countries never catch up.
- Savings rates, etc. can now affect growth rates as well as levels, because they affect productivity growth.
- Government policies that affect s, n, d and A now affect growth rates as well as levels.
- Increasing s reduces current consumption in favor of future consumption.
  - For a given value of k, increasing s reduces current consumption: $c = (1-s)y = (1-s)Ak$.
  - Increasing s increases consumption growth: $G_c = \dfrac{1-d+sA}{1+n}$.

Figure 7.7 Effect of an Increase in s on the Consumption Path in the Endogenous Growth Model



Productivity Booms

- Four historical episodes of productivity growth:
  - 1873-1890: Railroads and telegraphs.
  - 1917-1927: Electrification of factories (also internal combustion engines).
  - 1948-1973: Commercial applications of military innovations, e.g., plastics and transistors.
  - 1995-2003: High tech computing and communications.
- Productivity booms require four things:
  - Technological improvements.
  - Organizational change to handle increased economies of scale and scope and/or greater complexity.
  - Financial market improvements to fund bigger, more complex, or more fluid organizations.
  - Changes in the educational system.

U.S. Average Productivity Growth,1873-2003

(Average Annual Growth Rates, in percent)

Period	Labor Productivity	Total Factor Productivity	Capital Deepening + Labor Composition
1873-2003	2.2	1.3	0.9
Episode 1			
1873-1890	2.6	1.5	1.1
1890-1917	1.5	0.8	0.7
Episode 2			
1917-1927	3.8	2.8	1.6
1927-1948	1.8	1.7	0.1
Episode 3			
1948-1973	2.9	1.9	1.0
1973-1995	1.4	0.4	1.0
Episode 4			
1995-2003	3.0	1.0	2.0

Source: Ferguson and Wascher (*The Journal of Economic Perspectives*, Spring 2004), page 6.

Principal Sources I

1. Andrew B. Abel and Ben S. Bernanke, Macroeconomics, fourth edition update, (Addison-Wesley, 2003), chapter 6.
2. The Economist, "The Road to Hell is Unpaved," December 19, 2002. (Downloaded from http://www.economist.com/displayStory.cfm?Story_ID=1487583.)
3. Roger W. Ferguson Jr. and William L. Wascher, "Distinguished Lecture on Economics in Government: Lessons from Past Productivity Booms," Journal of Economic Perspectives, 18(2), Spring 2004, pp. 3-28.
4. Robert E. Lucas, Jr., "Making a Miracle," Econometrica, 61(2), March 1993, pp. 251-272.
5. Stephen D. Williamson, Macroeconomics, second edition, (Addison-Wesley, 2005), chapter 7. (Figures downloaded from http://wps.aw.com/aw_williamson_macroecon_2/0,9327,1432147-content,00.html.)

Principal Sources II

6. Stephen D. Williamson, Macroeconomics, third edition, (Addison-Wesley, 2008), chapters 6 and 7. (Figures downloaded from http://wps.aw.com/aw_williamson_macroecon_3/69/17800/4557009.cw/index.html.)