Lecture 5
Exogenous Economic Growth

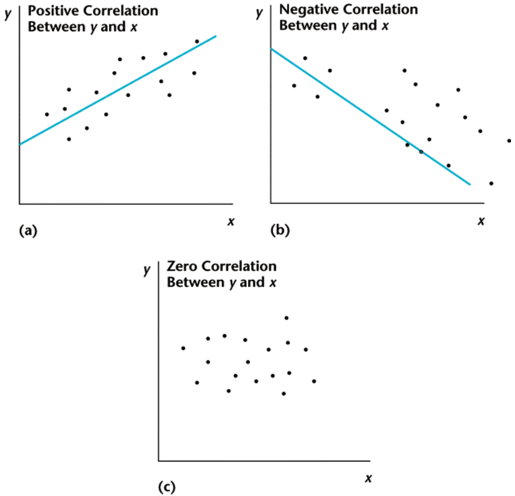
October 2009

Background: Correlation

- Suppose we have multiple observations of the variables x and y. For example, x could be average education and y could be per capita GDP.
- If we plot the observations as points in (x, y) space, we have a **scatter plot**.
- The **correlation** between x and y measures the strength and direction of the co-movement between these two variables.
- Correlations take on values between -1 and $+1$.
- A large **positive** correlation implies:
  - When an observation of x is above its average value, the corresponding value of y is usually above average as well.
  - The points in the scatter plot cluster around an upward sloping line.
- A large **negative** correlation implies the opposite.
- A zero **correlation** implies the variables are **linearly** unrelated.

Correlation Illustrated

Figure 3.4 Correlations between Variables *y* and *x*



Background: Growth Rates and Natural Logarithms

- For any variable y_t, the **gross** growth rate, G_t, is related to the **net** growth rate, g_t,

$$G_t = \frac{y_t}{y_{t-1}} = 1 + g_t = 1 + \frac{y_t - y_{t-1}}{y_{t-1}}.$$

- If x is close to zero, $\ln(1+x) \approx x$.
- For small values of g_t, $\ln(G_t) = \ln(1+g_t) \approx g_t$.
- This result implies that

$$\begin{aligned} g_t &\approx \ln(G_t) \\ &= \ln\left(\frac{y_t}{y_{t-1}}\right) \\ &= \ln(y_t) - \ln(y_{t-1}). \end{aligned}$$

- Punchline: If we graph the natural logarithm of an economic time series, the graph's slope gives the series' growth rate.

Background: The Rule of 70

- Suppose the variable y_t has a constant net growth rate of g. How many years will it take y_t to double in size?
- What we want is the value of T that solves

$$y_t\left(1+g\right)^T = 2y_t \Rightarrow \left(1+g\right)^T = 2.$$

- Log both sides

$$\begin{aligned} T \times \ln\left(1+g\right) &= \ln\left(2\right) \\ \Rightarrow T &= \frac{\ln\left(2\right)}{\ln\left(1+g\right)} \\ &\approx \frac{0.7}{g} = \frac{70}{g \text{ in percentages}}. \end{aligned}$$

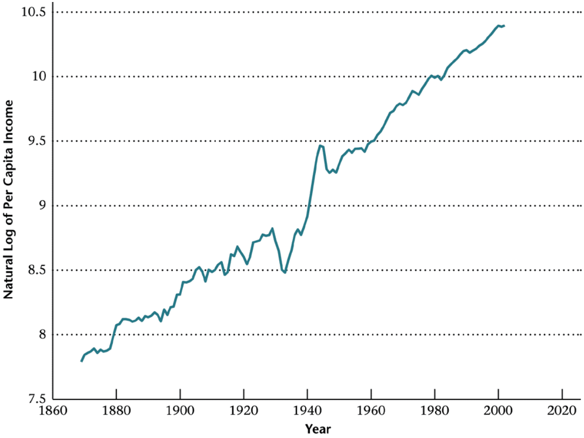
- This is the **rule of 70**.
- Example: Suppose GDP grows at an annual rate of 2 percent. Then GDP will double in $70/2 = 35$ years.

Economic Growth Facts

- Before the Industrial Revolution (around 1800), standards of living differed little over time and across countries.
- Since the Industrial Revolution, per capita income has steadily grown in the richest countries.
    - Ex. In the U.S. per capita income growth has been about 1.7% per year since 1869.
- Differences in per capita income between the "western" and "non-western" countries increased dramatically between 1800 and 1950.

Sustained Economic Growth in the United States

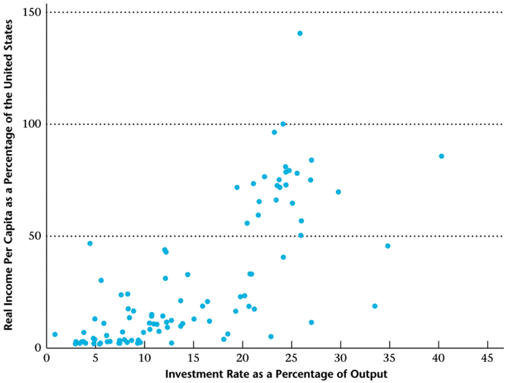
Figure 6.1 Natural Log of Real per Capita Income in the United States, 1869-2002



Economic Growth Facts (Continued)

- There is a positive correlation between the rate of investment and output per worker across countries.

Figure 6.2 Real Income Per Capita vs. the Investment Rate

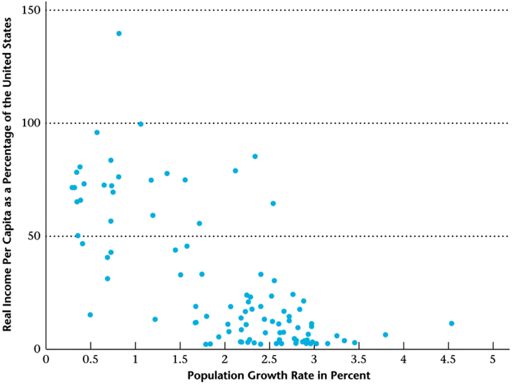


Source: A. Heston, R. Summers, and B. Aten, *Penn World Table Version 6.1*, Center for International Comparisons at the University of Pennsylvania (CICUP), October 18, 2002, available at pwt.econ.upenn.edu.

Economic Growth Facts (Continued)

- There is a negative correlation between the population growth rate and output per worker across countries.

Figure 6.3 Real Income Per Capita vs. the Population Growth Rate



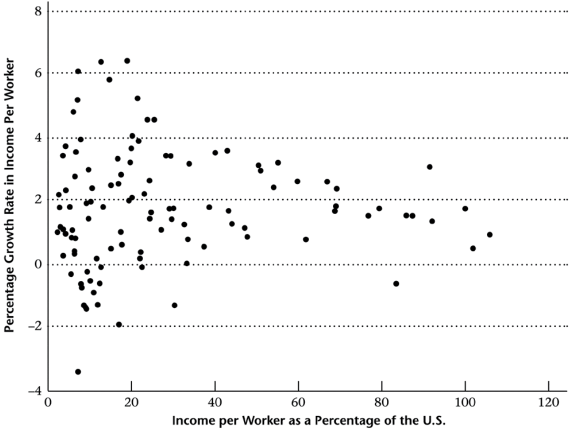
Source: A. Heston, R. Summers, and B. Aten, *Penn World Table Version 6.1*, Center for International Comparisons at the University of Pennsylvania (CICUP), October 18, 2002, available at pwt.econ.upenn.edu.

Economic Growth Facts (Continued)

- Overall, there is essentially no correlation between per capita income in 1960 and the per capita income growth rate for the years 1960-1995.
- Among the richest countries, there is a negative correlation between per capita income in 1960 and the per capita income growth rate for the years 1960-1995.
    - In the subset of rich countries, low-income countries catch up with high-income countries.
    - This is called convergence.
- Among the poorest countries, there is essentially no correlation between per capita income in 1960 and the per capita income growth rate for the years 1960-1995.

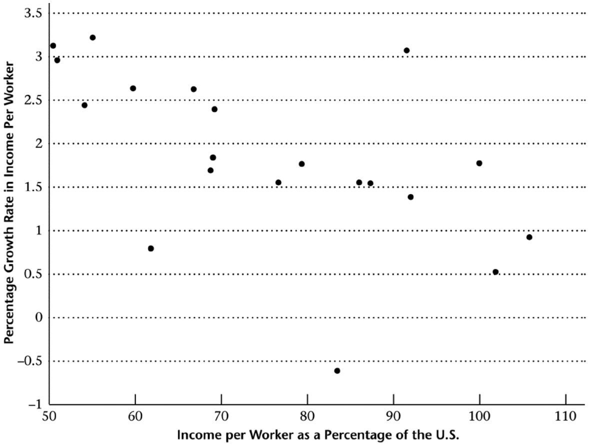
Economic Growth Facts (Continued)

Figure 6.4 No Convergence Among All Countries



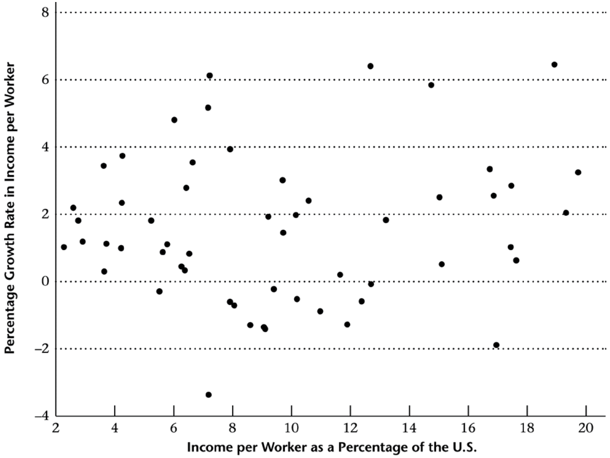
Economic Growth Facts (Continued)

Figure 6.5 Convergence Among the Richest Countries



Economic Growth Facts (Continued)

Figure 6.6 No Convergence Among the Poorest Countries



The Malthusian Model

- As the population, N, increases, per capita consumption, $c = C/N$, decreases.
  - Total consumption, C, will grow more slowly than N.
  - Reflects limited supply of land.
- A large population grows more slowly.
  - Large populations have lower per capita consumption, c.
  - Lower values of c imply higher mortality (death) rates.
- There is a subsistence level of consumption, c^*, where the population is constant at N^*.
- Increases in technology or land increase N^*, not c^*.
  - c initially increases.
  - But higher c implies population growth: N grows.
  - As the population grows, c returns to the subsistence level c^*.

Critique of the Malthusian Model

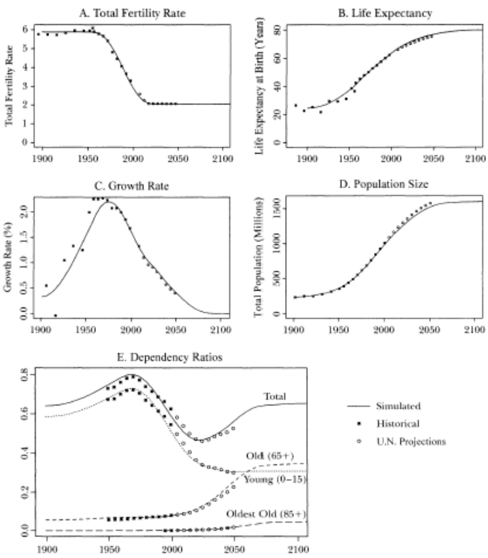
- The Malthusian model implies that per capita income (consumption) will not grow unless:
  - Population growth is controlled.
  - Productivity growth exceeds population growth.
- The model is a good description of pre-industrial economies.
- The model is inconsistent with the post-1800 experience in developed countries:
  - Increases in $y = Y/N$ and c have led to decreases in fertility.
- Why has fertility dropped? Kids have become more expensive.
  - Child-rearing technology has not improved as much as other technology $\Rightarrow$ parents forgo more labor income.
  - Human capital accumulation (schooling) takes longer.
- Also, governments and financial markets can now provide retirement income and risk sharing: less need for kids.

The “Classic” Demographic Transition

- Mortality rates drop.
  - Medical improvements increase life expectancy.
  - Drop in mortality most pronounced for the very young.
  - Fertility rates do **not** drop.
  - Population growth increases.
  - Population gets younger.
- Fertility rates drop.
  - Population growth decreases.
  - In some countries, fertility is below the replacement rate.
  - Population gets older. This is a major fiscal issue: e.g., Social Security and Medicare.
- “Western” countries are much further along in their demographic transitions $\Rightarrow$ most population growth will be occuring in “non-western” countries.

Figure 6

A Classic Demographic Transition: Actual and Projected for India and Simulated, 1900–2100



Source: Lee (*The Journal of Economic Perspectives*, 2003), page 181.

Solow Model: Consumers

- N = labor = population.
- Let N' denote next period's population: if $N = N_t$, $N' = N_{t+1}$.
- Population grows exogenously at the net rate n: $N' = (1+n)N$, with $n > -1$.
- Own firms, pay no taxes $\Rightarrow$ consumers' disposable income $= Y$.
- Allocate income between consumption (C) and saving (S).
- Assume saving is the constant fraction s of income: $S = sY \Rightarrow C = (1-s)Y$.

Solow Model: Firms

- The representative firm:
  - Production function: $Y = zF(K, N^d) = zF(K, N)$.
  - K = capital.
  - N^d = labor = population.
  - z = total factor productivity.
  - $F()$ has standard properties, including constant returns to scale (CRTS) in capital and labor.
  - $K' = (1-d)K + I$, where I is investment and d is the depreciation rate.
- (Capital Market) Equilibrium:

$$\begin{aligned} I &= S \\ &= sY \\ &= s \cdot zF(K, N) \end{aligned}$$

which implies that

$$K' = (1-d)K + s \cdot zF(K, N).$$

Solow Model: Per Worker Equilibrium

- CRTS $\Rightarrow$ output per worker is

$$
\begin{aligned}
y &= \frac{Y}{N} = \frac{1}{N} zF(K, N) \\
&= zF\left(\frac{K}{N}, \frac{N}{N}\right) = zF(k, 1) \\
&= zf(k),
\end{aligned}
$$

  where $k = K/N$.
- $zf(k)$ is the per worker production function.
- $zf'(k) =$ slope of per worker production function $= MP_K$.
  - Intuition: with CRTS, giving one worker an additional unit of capital is equivalent to giving N workers $1/N$ units of capital.

Solow Model: Per Worker Equilibrium (Continued)

- CRTS $\Rightarrow$ output per worker is

$$y = \frac{Y}{N} = \frac{1}{N} zF(K, N) = zf(k),$$

where $k = K/N$.

- Write the equilibrium expression for K' as:

$$\begin{aligned} K' \cdot \frac{N'}{N'} &= (1-d)K + s \cdot zF(K, N) \\ \Rightarrow \frac{K'}{N'} \cdot \frac{N'}{N} &= (1-d)\frac{K}{N} + s \cdot \frac{1}{N} zF(K, N) \\ \Rightarrow k'(1+n) &= (1-d)k + s \cdot zf(k) \\ \Rightarrow k' &= \frac{1-d}{1+n} k + \frac{s}{1+n} \cdot zf(k). \end{aligned}$$

Solow Model: Steady State

- Steady state capital stock:

$$\begin{aligned} k' &= k = k^* \\ \Rightarrow k^*(1+n) &= (1-d)k^* + s \cdot zf(k^*), \\ \Rightarrow i^* &= s \cdot zf(k^*) \\ &= (n+d)k^*. \end{aligned}$$

  The value of k that solves $(n+d)k = s \cdot zf(k)$ is the steady-state capital stock.

- Output and consumption are:

$$\begin{aligned} y^* &= zf(k^*), \\ c^* &= y^* - i^* = zf(k^*) - (n+d)k^*. \end{aligned}$$

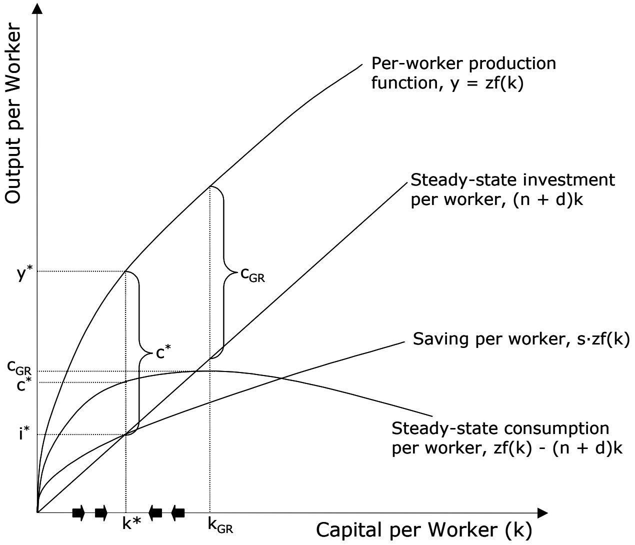
Solow Model: Golden Rule

- **Golden rule** capital stock (k_{GR}): Highest sustainable level of per worker consumption.
- k_{GR} = value of k^* that maximizes $c^* = zf(k^*) - (n+d)k^*$.
  - First order condition:

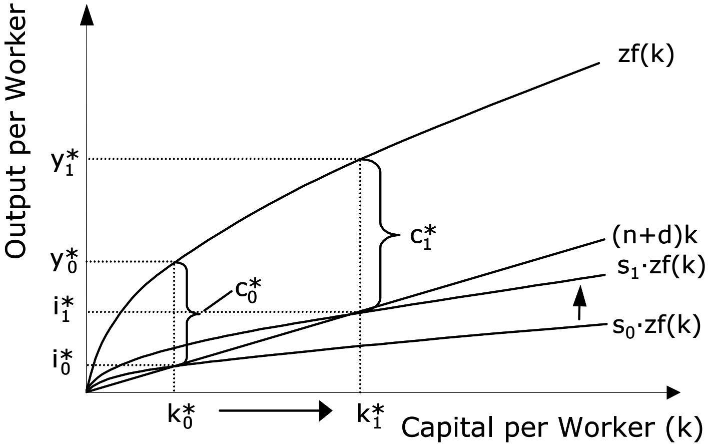
$$MP_K = zf'(k_{GR}) = n + d.$$

  - Intuition: marginal output of last unit of capital just covers marginal cost of "maintaining" it.
  - It can be the case that $k^* > k_{GR}$ but this is generally thought to be unlikely.

Steady State in the Solow Model

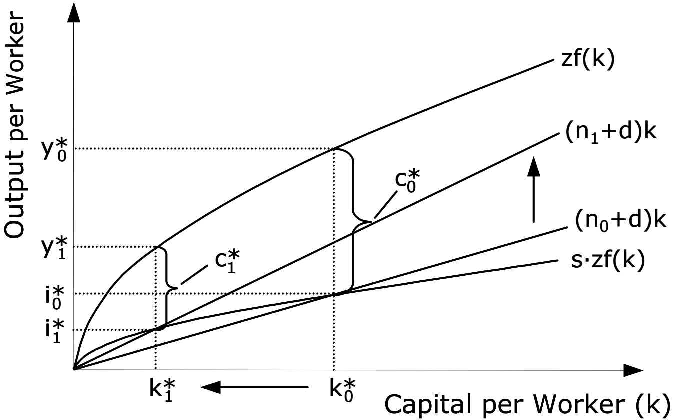


- $s \uparrow \Rightarrow k^* \uparrow$; if $k^* < k_{GR}$, $c^* \uparrow$ as well.
- Higher s does not necessarily mean a country is better off.



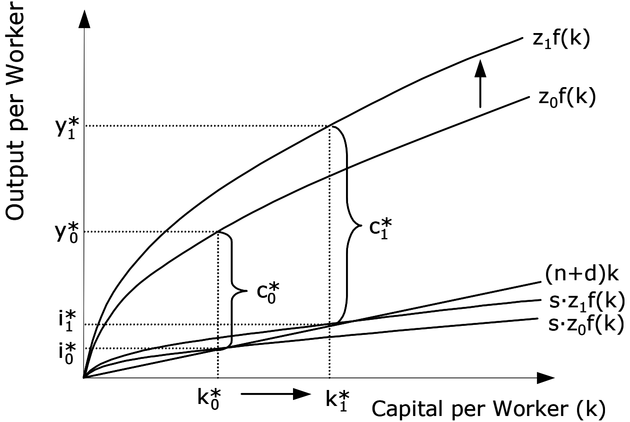
Solow Model: Effects of Changing the Population Growth Rate

- $n \uparrow \Rightarrow k^* \downarrow$, $c^* \downarrow$.
- More resources dedicated to equipping new workers.



Solow Model: Effects of Changing Total Factor Productivity

- $z \uparrow \Rightarrow k^* \uparrow, c^* \uparrow$.
- Capital is more productive **and** there is more of it.



The Role of Productivity

- Sustained per-worker output **growth** requires sustained productivity growth.
- Saving rates, etc. affect the **level** of output permanently, but affect the **growth rate** of output only temporarily.
  - Increasing s or decreasing n increases y^* by increasing k^*.
  - But diminishing $MP_K \Rightarrow$ diminishing increases in y^*.
  - And we can't increase s or decrease n indefinitely.
- But productivity improvement seems limitless.
- What causes productivity to improve?
  - In the Solow model, productivity is exogenous.
  - Endogenous growth theory tries to explain productivity.

Growth Accounting

- **Growth Accounting**: How much of the growth in Y is due to growth in z? Growth in K? Growth in N?
- If $Y = zK^{\alpha}N^{1-\alpha}$, then $z = Y/[K^{\alpha}N^{1-\alpha}]$.
- Moreover,

$$\begin{aligned} G_Y &= \frac{Y'}{Y} = \frac{z'(K')^{\alpha}(N')^{1-\alpha}}{zK^{\alpha}N^{1-\alpha}} = \frac{z'}{z}\left(\frac{K'}{K}\right)^{\alpha}\left(\frac{N'}{N}\right)^{1-\alpha} \\ &= G_z G_K^{\alpha} G_N^{1-\alpha} \\ \Rightarrow g_Y &\approx \ln(G_Y) = \ln\left(G_z G_K^{\alpha} G_N^{1-\alpha}\right) \\ &= \ln(G_z) + \alpha \ln(G_K) + (1-\alpha)\ln(G_N) \\ &\approx g_z + \alpha g_K + (1-\alpha) g_N. \end{aligned}$$

- We do not observe z, but we can calculate its growth as

$$g_z = g_Y - \alpha g_K - (1-\alpha) g_N.$$

Growth Accounting (Continued)

- U.S. data show a productivity slowdown over 1970-1990, a speed-up in the 1990s.

Average Annual Growth Rates: U.S.

Years	Y	K	N	z
1950-1960	3.48	3.68	1.11	1.42
1960-1970	4.19	3.86	1.80	1.61
1970-1980	3.19	3.24	2.36	0.50
1980-1990	3.26	2.85	1.81	1.05
1990-2000	3.28	2.72	1.43	1.36
2000-2005	2.39	2.71	0.69	0.96

Source: Williamson, Macroeconomics (2008), Table 6.3

Growth Accounting (Continued)

- Why do we care? Low productivity growth rates $\Rightarrow$ low output growth rates $\Rightarrow$ major differences in affluence.
  - Country A: GDP grows at 2% per year ($g_y = 2\%$). 1900: $GDP = 100$. 2000: $GDP = 724$.
  - Country B: GDP grows at 1.5% per year. 1900: $GDP = 100$. 2000: $GDP = 443$.

Principal Sources

1. Andrew B. Abel and Ben S. Bernanke, Macroeconomics, fourth edition update, (Addison-Wesley, 2003), chapter 6.
2. Ronald Lee, "The Demographic Transition: Three Centuries of Fundamental Change," Journal of Economic Perspectives, 17(4), Fall 2003, pp. 167-190.
3. Wikipedia: "Rule of 72," (http://en.wikipedia.org/wiki/Rule_of_72).
4. Stephen D. Williamson, Macroeconomics, second edition, (Addison-Wesley, 2005), chapter 6. (Figures downloaded from http://wps.aw.com/aw_williamson_macroecon_2/0,9327,1432147-content,00.html.)
5. Stephen D. Williamson, Macroeconomics, third edition, (Addison-Wesley, 2008), chapters 1, 3 and 6. (Figures downloaded from http://wps.aw.com/aw_williamson_macroecon_3/69/17800/4557009.cw/index.html.)