

**Midterm Examination**  
**Answer Key**

1. (32 points) Consider a version of the Solow model where total output is given by

$$Y = zK^{0.35}N^{0.65},$$

Assume that  $z = 10$ ,  $N$  equals 3,000,  $s = 0.20$ ,  $d = 0.08$ , and  $n = 0.2$ .

- (a) Output per worker is given by

$$y = \frac{Y}{N} = \frac{1}{N}zK^{0.35}N^{0.65} = zK^{0.35}\left(\frac{N^{0.65}}{N}\right) = zK^{0.35}N^{-0.35} = z\left(\frac{K}{N}\right)^{0.35} = zk^{0.35}.$$

- (b) Steady state capital per worker can be derived as follows

$$\begin{aligned} s \cdot zf(k^*) &= (n + d)k^*, \\ 0.2 \cdot 10(k^*)^{0.35} &= 0.1k^*, \\ 20 &= (k^*)^{1-0.35} = (k^*)^{0.65}, \\ k^* &= 20^{1/0.65} = 100.366. \end{aligned}$$

The remaining steady state quantities can be calculated as follows

$$\begin{aligned} y^* &= z(k^*)^{0.35} = 10 \cdot 100.366^{0.35} = 50.183 \\ i^* &= s \cdot y^* = 0.2 \cdot 50.183 = 10.037 \\ &= (n + d)k^* = 0.1 \cdot 100.366 = 10.037, \\ c^* &= y^* - i^* = 50.183 - 10.037 = 40.146. \end{aligned}$$

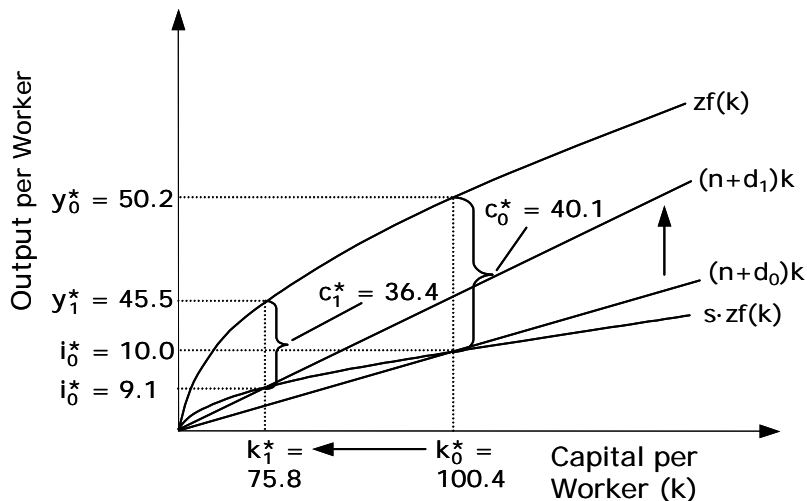
- (c) Now suppose that the depreciation rate productivity permanently increases from 8 to 10 percent. Steady state capital per worker can be derived as follows

$$\begin{aligned} 50/3 &= (k^*)^{0.65}, \\ k^* &= (50/3)^{1/0.65} = 75.817. \end{aligned}$$

The remaining steady state quantities can be calculated as follows

$$\begin{aligned} y^* &= z(k^*)^{0.35} = 10 \cdot 75.817^{0.35} = 45.490, \\ i^* &= s \cdot y^* = 0.2 \cdot 45.490 = 9.098 \\ &= (n + d)k^* = 0.12 \cdot 75.817 = 9.098, \\ c^* &= y^* - i^* = 45.490 - 9.098 = 36.392. \end{aligned}$$

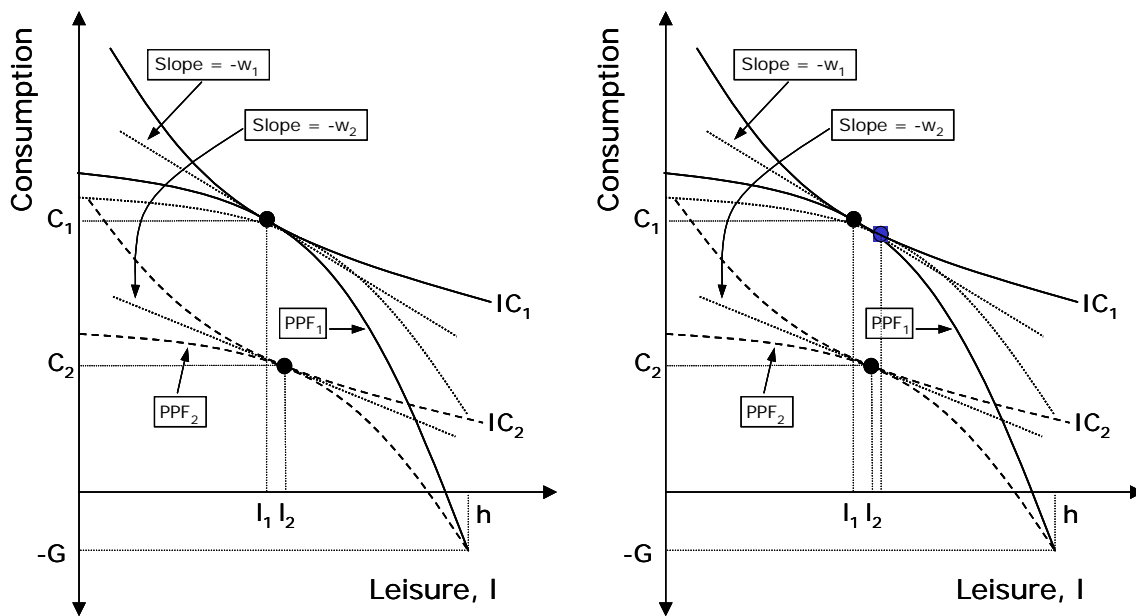
- (d) Recalling our answer to part (b), we see that increasing the depreciation rate reduces steady state capital, output, investment and consumption. Graphically, we see that a higher depreciation rate rotates the SS investment curve counter-clockwise, leading to lower values of  $k^*$ ,  $y^*$ ,  $i^*$  and  $c^*$ . (In the interest of clarity, the graph has not been drawn to scale.)



2. (16 points)

	2007	2057
Quantity Cell Phones (billions)	6	5
Quantity Haircuts (billions)	15	30
Price Cell Phones (\$/phone)	50	250
Price Haircuts (\$/haircut)	25	75
Nominal GDP (\$billions)	$6 \times 50 + 15 \times 25 = 675$	$5 \times 250 + 30 \times 75 = 3,500$
Real GDP Using Year-2007 Prices (billions of year-2007 \$)	$6 \times 50 + 15 \times 25 = 675$	$5 \times 50 + 30 \times 25 = 1,000$
Gross Growth Rate	NA	$1000/675 = 1.481$
Real GDP Using Year-2057 Prices (billions of year-2057 \$)	$6 \times 250 + 15 \times 75 = 2,625$	$5 \times 250 + 30 \times 75 = 3,500$
Gross Growth Rate	NA	$3500/2625 = 1.3$
2-year Geometric Averages	NA	$\sqrt{1.481 \times 1.3} = 1.40546$
Chain-Weighted Real GDP (billions of year-2007 \$)	Base Year: 675	$675 \times 1.40546 = 948.7$

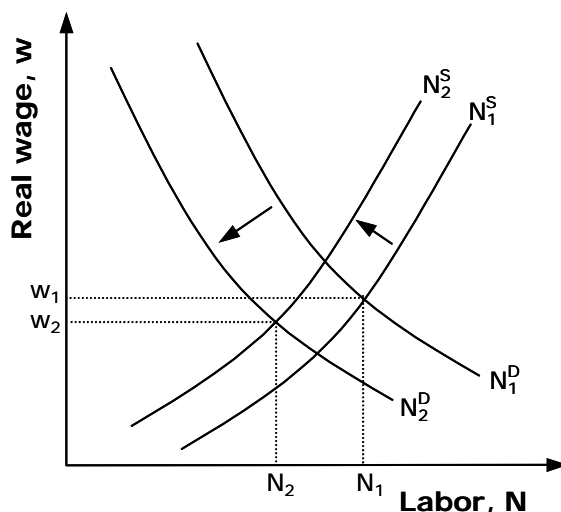
3. (8 points) In the Malthusian framework, climate change can be modelled as a decrease in productivity or a decrease in the stock of arable land (due to coastal flooding, etc.). In either case, climate change will decrease total output and thus total consumption. Initially, before the population adjusts, this decrease in total consumption will reduce per capita consumption as well. But a decrease in per capita consumption will increase mortality rates, and the population will decrease. In the long run, per capita consumption will return to its sustainable level ( $c^*$ ) and the population will stabilize at a new, lower value.
4. (14 points) A reduction in total factor productivity rotates the production possibilities frontier (PPF) counterclockwise. Consumption, being a normal good, decreases. The decrease in consumption implies that output declines as well (this follows from  $Y = C + G$ , with  $C$  decreasing and  $G$  unchanged.) The overall movement in labor hours is ambiguous. The substitution effect of the TFP decrease is to increase the relative price of consumption—it now takes more hours to produce the same amount of goods—and discourage work. The income effect, on the other hand, encourages work; the consumer is poorer, and thus purchases less leisure. Without more information, we cannot ascertain which effect is bigger. Following the arguments set forth in your textbook, you can show that the real wage decreases. Below are two figures. On the left is a simpler figure that would be an acceptable answer. On the right is the same figure, with “ghost” PPFs and indifference curves that clarify why the real wage decreases.



5. (10 points) Let's look at the national accounts.
- (a) Ignoring any responses, the **effects** of climate change—higher temperatures, higher sea levels, different weather patterns—would be captured in the National Accounts if they led to changes in market output. For example, if climate change reduced TFP (as in question 4), leading to less production, the reduction in output would be captured as reduction in GDP. If, however, effects such as environmental degradation do not generate market responses, they will not be accounted for.

- (b) Any response to climate change will almost surely involve the installation of new technologies. Expenditures by firms would appear as investment,  $I$ , or be intermediate expenditures not captured in GDP. Expenditures by consumers would be consumption,  $C$  (primarily consumer durables) or housing investment. Expenditures by the government would appear as government consumption,  $G$ . (Expenditures financed by government transfers to private citizens—e.g., FEMA benefits—would be assigned to  $I$  or  $C$ , as income transfers are not part of  $G$ .)
6. (20 points) Suppose that governments respond to climate change by imposing an output tax on firms, and then refunding the proceeds to consumers as lump-sum transfers.

- (a) From the firm's perspective, the output tax acts like a decrease in TFP—revenues shrink from  $F(K, N)$  to  $(1 - t)F(k, N)$ —that shifts the labor demand curve down and to the left. Because individual consumers take the transfer as given, the transfer will have an income effect that discourages work, shifting the labor supply curve to the left as well. Given that the transfer will offset the income effect of any change in wages, labor hours will decrease even if the labor supply curve is backward bending. With TFP and capital unchanged, a drop in labor hours must decrease output, and with government spending unchanged, consumption will fall as well. The effect on the real wage is uncertain: in the picture below wages fall.



- (b) The First Welfare Theorem states that if markets are fully competitive, the economy has no distorting taxes, and there are no externalities, a competitive equilibrium will be Pareto Optimal, and the tax/transfer scheme cannot improve efficiency. It is likely, however, that climate change is at least in part a negative externality, so that the welfare theorems do not hold. In this case the output tax, by diminishing production, might actually improve the economy's allocation: if producers are not considering the full environmental consequences of their actions, they may be producing too much output.