This is a solution of the problem I didn't finish in class.
The problem was to show that $n^{101} - n$ is always divisible by 33.
What I could do was to show this when $n$ is a unit modulo 33:

\[ \phi(33) = 20 \]

Then $n^{20} = 1 \mod 33$, so

\[ n^{100} = (n^{20})^5 = 1 \mod 33, \]

and $n^{101} = n \mod 33$.

This is not enough to show $33 \mid n^{101} - n$ for all $n$ because $\mathbb{Z}/33\mathbb{Z}$ has zero divisors. Instead, I will use the fact that $33 = 3 \cdot 11 = [3, 11]$ and a familiar strategy using Fermat's
Theorem for the primes 3 and 11.
It is enough to check that $3|n^{101}-n$ and $11|n^{101}-n$ for all $n$, because these facts are equivalent to
$n^{101} \equiv n \mod 3$, $n^{101} \equiv n \mod 11$
So then $n^{101} \equiv n \mod [3, 11] = 33$.  
So here we go:

- When $n$ is a unit mod 3 then
  $n^2 \equiv 1 \mod 3 \Rightarrow n^{100} = (n^2)^{50} \equiv 1 \mod 3$
  So $n^{101} \equiv n \mod 3$

- When $n$ is not a unit then $3|n$ \Rightarrow
  Still $n^{101} \equiv n \equiv 0 \mod 3$.

- If $n$ is a unit mod 11 then
  $n^{10} \equiv 1 \mod 11 \Rightarrow n^{100} = (n^{10})^{10} \equiv 1 \mod 11$
So \( n^{101} \equiv n \mod 11 \)

- if \( 11 \mid n \) then still \( n^{101} \equiv n \equiv 0 \mod 11 \).

This verifies all cases.