The total number of points is 15.

**Part 1.** Here you need to give complete proofs.

**Question 1** (5 points). Prove that $f : X \to Y$ is continuous if and only if for all $A \subset Y$

$$f^{-1}(A) \subset f^{-1}(\overline{A}).$$

**Question 2** (6 points). Let $A$ be a subset of $X$ given the subspace topology.
(a) If $X$ is Hausdorff, does $A$ have to be Hausdorff? Prove you are correct.
(b) If $X$ is not Hausdorff, can $A$ be Hausdorff? Prove you are correct.

**Part 2** (4 points, each question is worth half point). **True-False.** The questions in this section can be answered either “true” or “false”. You do not need to give reasons for your answers, though a wrong answer with a largely correct explanation will receive partial credit.

1. Let $X = \{0, 1\}$ with the topology in which the open sets are $\emptyset$, $\{0\}$, and $X$.
   - Is $X$ Hausdorff?
   - Is $X$ metrizable?

2. Let $X = \mathbb{R}^2 / \{y - \text{axis}\}$, i.e., the plane with the $y$-axis collapsed to a point, with the quotient topology.
   - Is $X$ Hausdorff?
   - Is $X$ metrizable?

3. Let $X$ be the “line with two origins”. As a set, this is the real line, except there are two points $0'$ and $0''$ in place of single “zero”. There is a set map $\pi$ to the usual real line mapping all nonzero numbers to themselves, and both $0'$ and $0''$ to 0. The topology on $X$ is the coarsest topology which makes $\pi$ continuous.
   - Is $X$ Hausdorff?
   - Is $X$ metrizable?

4. Let $X = \prod_{n=1}^{\infty} [0, n]$, an infinite product of the indicated closed intervals with the product topology.
   - Is $X$ Hausdorff?
   - Is $X$ metrizable?