AMAT 326 Quiz #9 Name: BG

① Show that $[6]$ is a zero divisor in $\mathbb{Z}/18\mathbb{Z}$

$(6,18)=6$ and $1<6<18$ so $[6]$ is classified as a zero divisor by the Theorem in class. Another Solution could be simply writing down the equation $[6][3]=[18]=[0]$.

② Find all zero divisors in $\mathbb{Z}/18\mathbb{Z}$ which are complementary to $[6]$.

$[6]X=[0]$ gives $6x+18y=0$, so $x=0+\frac{18}{3}k=6k$. This gives the list of all solutions as $[0],[6],[12],[18]=[0]$, stop!

out of these $[6]$ and $[12]$ are the complementary zero divisors.

③ Prove that a unit cannot be a zero divisor, in any ring.

Suppose a is a unit with the inverse a^{-1}, and suppose it is a zero divisor. Then there is $b\neq 0$ such that $ab=0$. Multiply both sides by a^{-1}: $a^{-1}(ab)=a^{-1}\cdot 0$ so $(a^{-1}a)b=0 \Rightarrow 1\cdot b=0 \Rightarrow b=0$. This contradicts $b\neq 0$.

So a can't be a zero divisor.