MAT 326  Quiz#1  Name: BG

The following are two relations on \( \mathbb{Z} \). Check which properties of the equivalence relations are satisfied by each of them.

(1) \( a \sim b \) if \( a-b<0 \)

(2) \( a \sim b \) if \( ab \) is divisible by 5.

(i) \( 5 \sim 5 \) \( \iff 5-5=0 \leq 0 \) \( \Rightarrow \) FALSE

(ii) \( 3 \sim 5 \) \( \iff 3-5<0 \) but \( 5 \times 3 \) \( \not\mid 5-3 \leq 0 \).

(iii) \( a \sim b, b \sim c \Rightarrow a-b<0, b-c<0 \Rightarrow \) after adding \( (a-b) + (b-c) = a-c < 0 \), TRUE

(iv) \( 3 + 3 \) \( \not\mid 3.3 \) is not divisible by 5

(v) \( a \sim b \Rightarrow ab \) is div. by 5 \( \Rightarrow \) \( b = ab \) is div. by 5 \( \Rightarrow \) \( b \mid a \). TRUE

(vi) \( 3 \sim 5 \) \( \iff 3.5=15 \) is div. by 5 \( \Rightarrow \)

\( 5 \sim 9 \) \( \iff 5-9=45 \) is div. by 5 \( \Rightarrow \)

but \( 3 \times 9 \) \( \not\mid 3.9 \) is not div. by 5, FALSE