Analysis of Social Network Data

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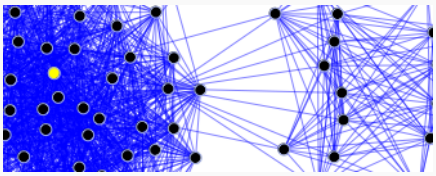

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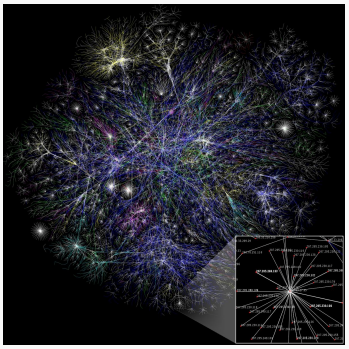
Network science



Theory · History

Graph · Complex network · Contagion
Small-world · Scale-free ·
Community structure · Percolation · Evolution · Controllability · Topology · Graph drawing · Social capital · Link analysis · Optimization
Reciprocity · Closure · Homophily
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Balance · Network effect · Influence

Types of Networks

Information · Telecommunication
Social · Biological · Neural · Semantic
Random · Dependency · Flow

Graphs

Vertex · Edge · Component Directed · Multigraph · Bipartite Weighted · Hypergraph · Random Cycle · Loop · Path Neighborhood · Clique · Complete · Cut Data structure · Adjacency list & matrix Incidence list & matrix
Metrics and Algorithms
Centrality · Degree · Betweenness Closeness · PageRank · Motif Clustering · Degree distribution · Assortativity · Distance · Modularity
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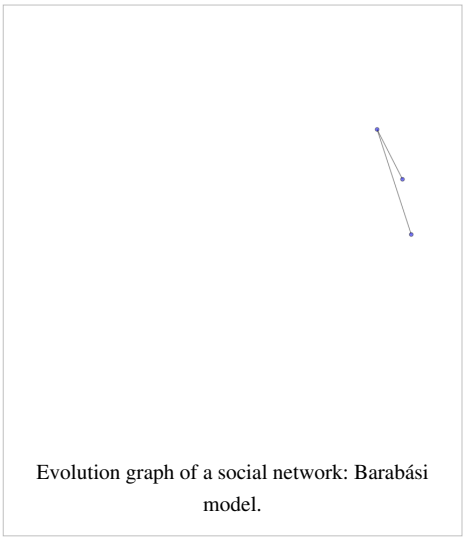
A **social network** is a social structure made up of a set of actors (such as individuals or organizations) and the dyadic ties between these actors. The social network perspective provides a clear way of analyzing the structure of whole social entities.[] The study of these structures uses social network analysis to identify local and global patterns, locate influential entities, and examine network dynamics.

Social networks and the analysis of them is an inherently interdisciplinary academic field which emerged from social psychology, sociology, statistics, and graph theory. Georg Simmel authored early structural theories in sociology emphasizing the dynamics of triads and "web of group affiliations."[1] Jacob Moreno is credited with developing the first sociograms in the 1930s to study interpersonal relationships. These approaches were mathematically formalized in the 1950s and theories and methods of social networks became pervasive in the social and behavioral sciences by the 1980s.[][] Social network analysis is now one of the major paradigms in contemporary sociology, and is also employed in a number of other social and formal sciences. Together with other complex networks, it forms part of the nascent field of network science.[2][]

Overview



Evolution graph of a social network: Barabási model.

A social network is a theoretical construct useful in the social sciences to study relationships between individuals, groups, organizations, or even entire societies (social units, see differentiation). The term is used to describe a social structure determined by such interactions. The ties through which any given social unit connects represent the convergence of the various social contacts of that unit. This theoretical approach is, necessarily, relational. An axiom of the social network approach to understanding social interaction is that social phenomena should be primarily conceived and investigated through the properties of relations between and within units, instead of the properties of these units themselves. Thus, one common criticism of social network theory is that individual agency is often ignored,[3] although this may not be

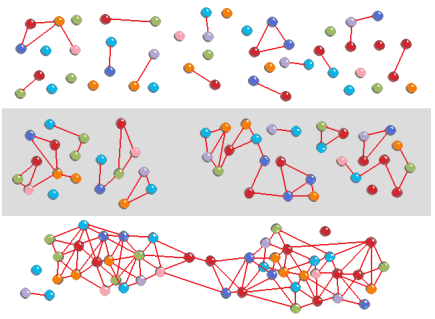
the case in practice (see agent-based modeling). Precisely because many different types of relations, singular or in combination, form these network configurations, network analytics are useful to a broad range of research enterprises. In social science, these fields of study include, but are not limited to anthropology, biology, communication studies, economics, geography, information science, organizational studies, social psychology, sociology, and sociolinguistics.

History

Some of the ideas of the social network approach are found in writings going back to the ancient Greeks [*citation needed*]. In the late 1800s, both Émile Durkheim and Ferdinand Tönnies foreshadow the idea of social networks in their theories and research of social groups. Tönnies argued that social groups can exist as personal and direct social ties that either link individuals who share values and belief (*Gemeinschaft*, German, commonly translated as "community") or impersonal, formal, and instrumental social links (*Gesellschaft*, German, commonly translated as "society").[4] Durkheim gave a non-individualistic explanation of social facts arguing that social phenomena arise when interacting individuals constitute a reality that can no longer be accounted for in terms of the properties of individual actors.[5] Georg Simmel, writing at the turn of the twentieth century, pointed to the nature of networks and the effect of network size on interaction and examined the likelihood of interaction in loosely-knit networks rather than groups.[6]

Major developments in the field can be seen in the 1930s by several groups in psychology, anthropology, and mathematics working independently.[3] .[7][8] In psychology, in the 1930s, Jacob L. Moreno began systematic recording and analysis of social interaction in small groups, especially classrooms and work groups (see sociometry). In anthropology, the foundation for social network theory is the theoretical and ethnographic work of Bronislaw Malinowski,[9] Alfred Radcliffe-Brown,[10][11] and Claude Lévi-Strauss.[12] A group of social anthropologists associated with Max Gluckman and the Manchester School, including John A. Barnes,[13] J. Clyde Mitchell and Elizabeth Bott Spillius,[14][15] often are credited with performing some of the first fieldwork from which network analyses were performed, investigating community networks in southern Africa, India and the United Kingdom.[3] Concomitantly, British anthropologist S.F. Nadel codified a theory of social structure that was influential in later network analysis.[16] In sociology, the early (1930s) work of Talcott Parsons set the stage for taking a relational approach to understanding social structure.[17][18] Later, drawing upon Parsons' theory, the work of sociologist Peter Blau provides a strong impetus for analyzing the relational ties of social units with his work on social exchange theory.[19][20][21] By the 1970s, a growing number of scholars worked to combine the different tracks and traditions. One group consisted of sociologist Harrison White and his students at the Harvard University Department of Social Relations. Also independently active in the Harvard Social Relations department at the time were Charles Tilly, who focused on networks in political and community sociology and social movements, and Stanley Milgram, who developed the "six degrees of separation" thesis.[22]Mark Granovetter[23] and Barry Wellman[24] are among the former students of White who elaborated and championed the analysis of social networks.[25][26][27][28]

Levels of analysis

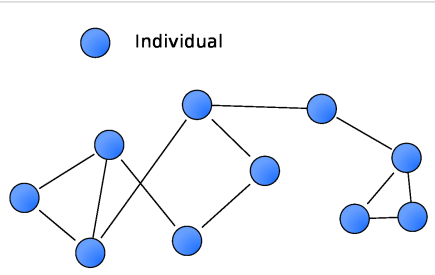


Self-organization of a network, based on Nagler, Levina, & Timme, (2011)[29]

In general, social networks are self-organizing, emergent, and complex, such that a globally coherent pattern appears from the local interaction of the elements that make up the system.[30][31] These patterns become more apparent as network size increases. However, a global network analysis of, for example, all interpersonal relationships in the world is not feasible and is likely to contain so much information as to be uninformative. Practical limitations of computing power, ethics and participant recruitment and payment also limit the scope of a social network analysis.[32][33] The nuances of a local system may be lost in a large network analysis, hence the quality of information may be more important than its scale for understanding network properties. Thus, social networks are analyzed at the scale relevant to the researcher's theoretical question. Although levels of analysis are not necessarily mutually exclusive, there are three general levels into which networks may fall: micro-level, meso-level, and macro-level.

Micro level

At the micro-level, social network research typically begins with an individual, snowballing as social relationships are traced, or may begin with a small group of individuals in a particular social context.



Social network diagram, micro-level.

Dyadic level: A dyad is a social relationship between two individuals. Network research on dyads may concentrate on structure of the relationship (e.g. multiplexity, strength), social equality, and tendencies toward reciprocity/mutuality.

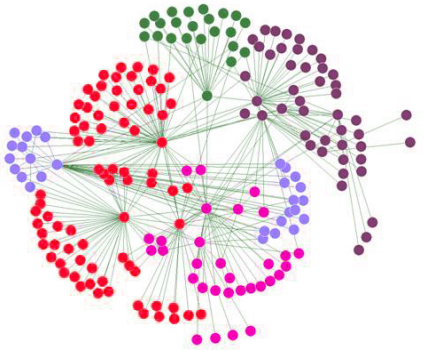
Triadic level: Add one individual to a dyad, and you have a triad. Research at this level may concentrate on factors such as balance and transitivity, as well as social equality and tendencies toward reciprocity/mutuality.[32]

Actor level: The smallest unit of analysis in a social network is an individual in their social setting, i.e., an "actor" or "ego". Egonetwork analysis focuses on network characteristics such as size, relationship strength, density, centrality, prestige and roles such as isolates, liaisons, and bridges.[34] Such analyses, are most commonly used in the fields of psychology or social psychology, ethnographic kinship analysis or other genealogical studies of relationships between individuals.

Subset level: Subset levels of network research problems begin at the micro-level, but may crossover into the meso-level of analysis. Subset level research may focus on distance and reachability, cliques, cohesive subgroups, or other group action, group actions or behavior[*citation needed*].

Meso level

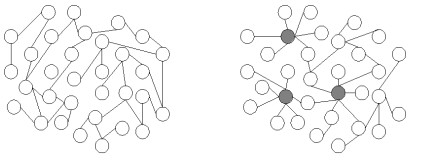
In general, meso-level theories begin with a population size that falls between the micro- and macro-levels. However, meso-level may also refer to analyses that are specifically designed to reveal connections between micro- and macro-levels. Meso-level networks are low density and may exhibit causal processes distinct from interpersonal micro-level networks.[35]



Social network diagram, meso-level

Organizations: Formal organizations are social groups that distribute tasks for a collective goal.[36] Network research on organizations may focus on either intra-organizational or inter-organizational ties in terms of formal or informal relationships. Intra-organizational networks themselves often contain multiple levels of analysis, especially in larger organizations with multiple branches, franchises or semi-autonomous departments. In these cases, research is often conducted at a workgroup level and organization level, focusing on the interplay between the two structures.[37]

Randomly-distributed networks: Exponential random graph models of social networks became state-of-the-art methods of social network analysis in the 1980s. This framework has the capacity to represent social-structural effects commonly observed in many human social networks, including general degree-based structural effects commonly observed in many human social networks as well as reciprocity and transitivity, and at the node-level, homophily and attribute-based activity and popularity effects, as derived from explicit hypotheses about dependencies among network ties. Parameters are given in terms of the prevalence of small subgraph configurations in the network and can be interpreted as describing the combinations of local social processes from which a given network emerges. These probability models for networks on a given set of actors allow generalization beyond the restrictive dyadic independence assumption of micro-networks, allowing models to be built from theoretical structural foundations of social behavior.[38]



(a) Random network (b) Scale-free network

Examples of a random network and a scale-free network. Each graph has 32 nodes and 32 links. Note the "hubs" in the scale-free diagram (on the right).

Scale-free networks: A scale-free network is a network whose degree distribution follows a power law, at least asymptotically. In network theory a scale-free ideal network is a random network with a degree distribution that unravels the size distribution of social groups.[39] Specific characteristics of scale-free networks vary with the theories and analytical tools used to create them, however, in general, scale-free networks have some common characteristics. One notable characteristic in a scale-free network is the relative commonness of vertices with a degree that greatly exceeds the average. The highest-degree nodes are often called "hubs", and may serve specific purposes in their networks, although this depends greatly on the social context. Another general characteristic of scale-free networks is the clustering coefficient distribution, which decreases as the node degree increases. This distribution also follows a power law.[40] The Barabási model of network evolution shown above is an example of a scale-free network.

Macro level

Rather than tracing interpersonal interactions, macro-level analyses generally trace the outcomes of interactions, such as economic or other resource transfer interactions over a large population.

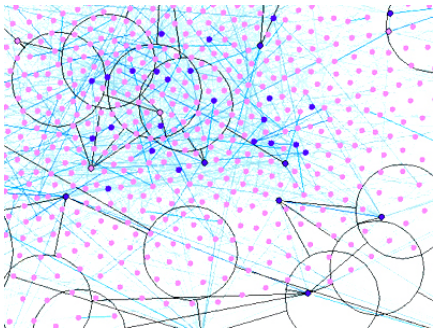


Diagram: section of a large-scale social network

Large-scale networks: Large-scale network is a term somewhat synonymous with "macro-level" as used, primarily, in social and behavioral sciences, in economics. Originally, the term was used extensively in the computer sciences (see large-scale network mapping).

Complex networks: Most larger social networks display features of social complexity, which involves substantial non-trivial features of network topology, with patterns of complex connections between elements that are neither purely regular nor purely random (see, complexity science, dynamical system and chaos theory), as do biological, and technological networks. Such complex network features include a heavy tail in the degree distribution, a high clustering coefficient, assortativity or disassortativity among vertices, community structure, and hierarchical structure. In the case of agency-directed networks these features also include reciprocity, triad significance profile (TSP, see network motif), and other features. In contrast, many of the mathematical models of networks that have been studied in the past, such as lattices and random graphs, do not show these features.[41]

Theoretical links

Imported theories

Various theoretical frameworks have been imported for the use of social network analysis. The most prominent of these are Graph Theory, Balance Theory, Social Comparison Theory, and more recently, the Social identity approach.[42]

Indigenous theories

Few complete theories have been produced from social network analysis. Two that have are Structural Role Theory and Heterophily Theory.

The basis of Heterophily Theory was the finding in one study that more numerous weak ties can be important in seeking information and innovation, as cliques have a tendency to have more homogeneous opinions as well as share many common traits. This homophilic tendency was the reason for the members of the cliques to be attracted together in the first place. However, being similar, each member of the clique would also know more or less what the other members knew. To find new information or insights, members of the clique will have to look beyond the clique to its other friends and acquaintances. This is what Granovetter called "the strength of weak ties".[43]

Structural holes

In the context of networks, social capital exists where people have an advantage because of their location in a network. Contacts in a network provide information, opportunities and perspectives that can be beneficial to the central player in the network. Most social structures tend to be characterized by dense clusters of strong connections[44] . Information within these clusters tends to be rather homogeneous and redundant. Non-redundant information is most often obtained through contacts in different clusters.[45] When two separate clusters possess non-redundant information, there is said to be a structural hole between them.[46] Thus, a network that bridges structural holes will provide network benefits that are in some degree additive, rather than overlapping. An ideal network structure has a vine and cluster structure, providing access to many different clusters and structural holes.[47]

Information benefits

Networks rich in structural holes are a form of social capital in that they offer information benefits. The main player in a network that bridges structural holes is able to access information from diverse sources and clusters.[48] This is beneficial to an individual's career because he is more likely to hear of job openings and opportunities if his network spans a wide range of contacts in different industries/sectors. This concept is similar to Mark Granovetter's theory of weak ties, which rests on the basis that having a broad range of contacts is most effective for job attainment.

Social capital mobility benefits

In many organizations, members tend to focus their activities inside their own groups, which stifles creativity and restricts opportunities. A player whose network bridges structural holes has an advantage in detecting and developing rewarding opportunities.[49] Such a player can mobilize social capital by acting as a "broker" of information between two clusters that otherwise would not have been in contact, thus providing access to new ideas, opinions and opportunities. British philosopher and political economist John Stuart Mill, writes, "it is hardly possible to overrate the value...of placing human beings in contact with persons dissimilar to themselves…Such communication [is] one of the primary sources of progress."[50] Thus, a player with a network rich in structural holes can add value to an organization through new ideas and opportunities. This in turn, helps an individual's career development and advancement.

A social capital broker also reaps control benefits of being the facilitator of information flow between contacts. In the case of consulting firm Eden McCallum, the founders were able to advance their careers by bridging their connections with former big 3 consulting firm consultants and mid-size industry firms.[51] By bridging structural holes and mobilizing social capital, players can advance their careers by executing new opportunities between contacts.

There has been research that both substantiates and refutes the benefits of information brokerage. A study of high tech Chinese firms by Zhixing Xiao found that the control benefits of structural holes are "dissonant to the dominant firm-wide spirit of cooperation and the information benefits cannot materialize due to the communal sharing values" of such organizations.[52] However, this study only analyzed Chinese firms, which tend to have strong communal sharing values. Information and control benefits of structural holes are still valuable in firms that are not quite as inclusive and cooperative on the firm-wide level. In 2004, Ronald Burt studied 673 managers who ran the supply chain for one of America's largest electronics companies. He found that managers who often discussed issues with other groups were better paid, received more positive job evaluations and were more likely to be promoted.[53] Thus, bridging structural holes can be beneficial to an organization, and in turn, to an individual's career.

Research clusters

Communications

Communication Studies are often considered a part of both the social sciences and the humanities, drawing heavily on fields such as sociology, psychology, anthropology, information science, biology, political science, and economics as well as rhetoric, literary studies, and semiotics. Many communications concepts describe the transfer of information from one source to another, and can thus be conceived of in terms of a network.

Community

In J.A. Barnes' day, a "community" referred to a specific geographic location and studies of community ties had to do with who talked, associated, traded, and attended church with whom. Today, however, there are extended "online" communities developed through telecommunications devices and social network services. Such devices and services require extensive and ongoing maintenance and analysis, often using network science methods. Community development studies, today, also make extensive use of such methods.

Complex networks

Complex networks require methods specific to modelling and interpreting social complexity and complex adaptive systems, including techniques of dynamic network analysis.

Criminal networks

In criminology and urban sociology, much attention has been paid to the social networks among criminal actors. For example, Andrew Papachristos [*citation needed*] has studied gang murders as a series of exchanges between gangs. Murders can be seen to diffuse outwards from a single source, because weaker gangs cannot afford to kill members of stronger gangs in retaliation, but must commit other violent acts to maintain their reputation for strength.

Diffusion of innovations

Diffusion of ideas and innovations studies focus on the spread and use of ideas from one actor to another or one culture and another. This line of research seeks to explain why some become "early adopters" of ideas and innovations, and links social network structure with facilitating or impeding the spread of an innovation.

Demography

In demography, the study of social networks has led to new sampling methods for estimating and reaching populations that are hard to enumerate (for example, homeless people or intravenous drug users.) For example, respondent driven sampling is a network-based sampling technique that relies on respondents to a survey recommending further respondents.

Economic sociology

The field of sociology focuses almost entirely on networks of outcomes of social interactions. More narrowly, economic sociology considers behavioral interactions of individuals and groups through social capital and social "markets". Sociologists, such as Mark Granovetter, have developed core principles about the interactions of social structure, information, ability to punish or reward, and trust that frequently recur in their analyses of political, economic and other institutions. Granovetter examines how social structures and social networks can affect economic outcomes like hiring, price, productivity and innovation and describes sociologists' contributions to analyzing the impact of social structure and networks on the economy.[54]

Health care

Analysis of social networks is increasingly incorporated into heath care analytics, not only in epidemological studies but also in models of patient communication and education, disease prevention, mental health diagnosis and treatment, and in the study of health care organizations and systems.[55]

Human ecology

Human ecology is an interdisciplinary and transdisciplinary study of the relationship between humans and their natural, social, and built environments. The scientific philosophy of human ecology has a diffuse history with connections to geography, sociology, psychology, anthropology, zoology, and natural ecology.[56][57]

Language and linguistics

Studies of language and lingustics, particularly evolutionary linguistics, focus on the development of linguistic forms and transfer of changes, sounds or words, from one language system to another through networks of social interaction. Social networks are also important in language shift, as groups of people add and/or abandon languages to their repertoire.

Organizational studies

Research studies of formal or informal organizational relationships, organizational communication, economics, economic sociology, and other resource transfers. Social networks have also been used to examine how organizations interact with each other, characterizing the many informal connections that link executives together, as well as associations and connections between individual employees at different organizations.[58] Intra-organizational networks have been found to affect organizational commitment,[59] organizational identification,[34] interpersonal citizenship behaviour.[60]

Social capital

Social capital is a sociological concept which refers to the value of social relations and the role of cooperation and confidence to achieve positive outcomes. The term refers to the value one can get from their social ties. For example, newly arrived immigrants can make use of their social ties to established migrants to acquire jobs they may otherwise have trouble getting (e.g., because of lack of knowledge of language). Studies show that a positive relationship exists between social capital and the intensity of social network use.[61]

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[7] For a historical overview of the development of social network analysis, see:

[8] See also the diagram in

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- Scott, John (1991). *Social Network Analysis: a handbook*. SAGE. ISBN 978-0-7619-6338-7.
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- Rainie, Lee and Barry Wellman. 2012. *Networked: The New Social Operating System.* MIT Press. isbn=978-0-262-01719-0

- E. Estrada, "The Structure of Complex Networks: Theory and Applications", Oxford University Press, 2011, ISBN 978-0-199-59175-6

External links

Organizations

- International Network for Social Network Analysis (http://www.insna.org/)

Peer-reviewed journals

- *Social Networks* (http://www.sciencedirect.com/science/journal/03788733)
- *Network Science* (http://journals.cambridge.org/action/displaySpecialPage?pageId=3656)
- *Journal of Social Structure* (http://www.cmu.edu/joss/content/articles/volindex.html)
- *Journal of Mathematical Sociology* (http://www.tandfonline.com/toc/gmas20/current)
- *Social Network Analysis and Mining (SNAM)* (http://www.springer.com/computer/database+management+& +information+retrieval/journal/13278)

Textbooks and educational resources

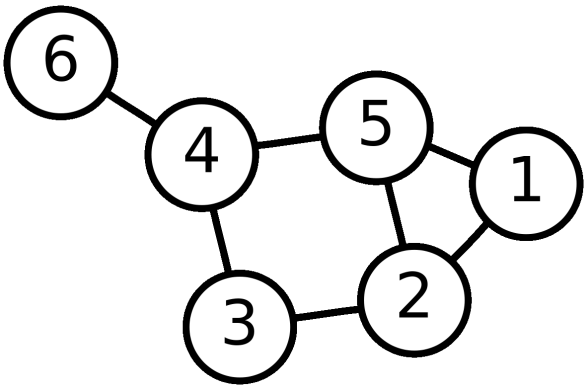
- *Networks, Crowds, and Markets* (http://www.cs.cornell.edu/home/kleinber/networks-book/) (2010) by D. Easley & J. Kleinberg
- *Introduction to Social Networks Methods* (http://faculty.ucr.edu/~hanneman/nettext/) (2005) by R. Hanneman & M. Riddle
- Social Network Analysis Instructional Web Site (http://www.analytictech.com/networks/) by S. Borgatti

Data sets

- Pajek's list of lists of datasets (http://pajek.imfm.si/doku.php?id=data:urls:index)
- UC Irvine Network Data Repository (http://networkdata.ics.uci.edu/index.html)
- Stanford Large Network Dataset Collection (http://snap.stanford.edu/data/)
- M.E.J. Newman datasets (http://www-personal.umich.edu/~mejn/netdata/)
- Pajek datasets (http://vlado.fmf.uni-lj.si/pub/networks/data/)
- Gephi datasets (http://wiki.gephi.org/index.php?title=Datasets#Social_networks)
- KONECT - Koblenz network collection (http://konect.uni-koblenz.de/networks)

Graph (mathematics)

In mathematics, a **graph** is a representation of a set of objects where some pairs of the objects are connected by links. The interconnected objects are represented by mathematical abstractions called *vertices*, and the links that connect some pairs of vertices are called *edges*.[1] Typically, a graph is depicted in diagrammatic form as a set of dots for the vertices, joined by lines or curves for the edges. Graphs are one of the objects of study in discrete mathematics.



A drawing of a labeled graph on 6 vertices and 7 edges.

The edges may be directed or undirected. For example, if the vertices represent people at a party, and there is an edge between two people if they shake hands, then this is an undirected graph, because if person A shook hands with person B, then person B also shook hands with person A. On the other hand, if the vertices represent people at a party, and there is an edge from person A to person B when person A knows of person B, then this graph is directed, because knowledge of someone is not necessarily a symmetric relation (that is, one person knowing another person does not necessarily imply the reverse; for example, many fans may know of a celebrity, but the celebrity is unlikely to know of all their fans). This latter type of graph is called a *directed* graph and the edges are called *directed edges* or *arcs*.

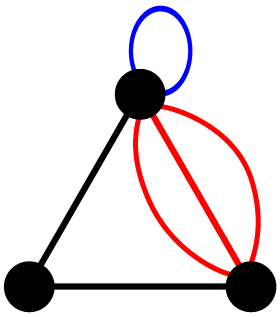
Vertices are also called *nodes* or *points*, and edges are also called *lines* or *arcs*. Graphs are the basic subject studied by graph theory. The word "graph" was first used in this sense by J.J. Sylvester in 1878.[2]

Definitions

Definitions in graph theory vary. The following are some of the more basic ways of defining graphs and related mathematical structures.

Graph

In the most common sense of the term,[3] a **graph** is an ordered pair $G = (V, E)$ comprising a set V of **vertices** or **nodes** together with a set E of **edges** or **lines**, which are 2-element subsets of V (i.e., an edge is related with two vertices, and the relation is represented as unordered pair of the vertices with respect to the particular edge). To avoid ambiguity, this type of graph may be described precisely as undirected and simple**.**


A general example of a graph (actually, a pseudograph) with three vertices and six edges.

Other senses of *graph* stem from different conceptions of the edge set. In one more generalized notion,[4] E is a set together with a relation of **incidence** that associates with each edge two vertices. In another generalized notion, E is a multiset of unordered pairs of (not necessarily distinct) vertices. Many authors call this type of object a multigraph or pseudograph.

All of these variants and others are described more fully below.

The vertices belonging to an edge are called the **ends**, **endpoints**, or **end vertices** of the edge. A vertex may exist in a graph and not belong to an edge.

V and E are usually taken to be finite, and many of the well-known results are not true (or are rather different) for **infinite graphs** because many of the arguments fail in the infinite case. The **order** of a graph is $|V|$ (the number of vertices). A graph's **size** is $|E|$, the number of edges. The **degree** of a vertex is the number of edges that connect to it, where an edge that connects to the vertex at both ends (a loop) is counted twice.
For an edge $\{u, v\}$, graph theorists usually use the somewhat shorter notation uv.

Adjacency relation

The edges E of an undirected graph G induce a symmetric binary relation ~ on V that is called the **adjacency** relation of G. Specifically, for each edge $\{u, v\}$ the vertices u and v are said to be **adjacent** to one another, which is denoted $u \sim v$.

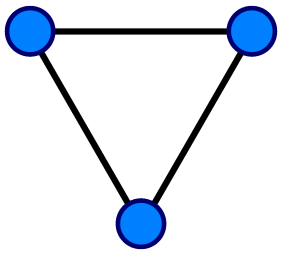
Types of graphs

Distinction in terms of the main definition

As stated above, in different contexts it may be useful to define the term *graph* with different degrees of generality. Whenever it is necessary to draw a strict distinction, the following terms are used. Most commonly, in modern texts in graph theory, unless stated otherwise, *graph* means "undirected simple finite graph" (see the definitions below).

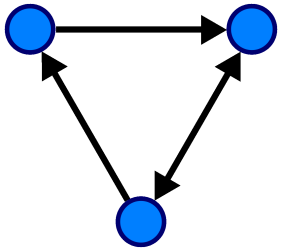
Undirected graph

An undirected graph is one in which edges have no orientation. The edge (a, b) is identical to the edge (b, a), i.e., they are not ordered pairs, but sets $\{u, v\}$ (or 2-multisets) of vertices.



A simple undirected graph with three vertices and three edges. Each vertex has degree two, so this is also a regular graph.

Directed graph



A directed graph

A **directed graph** or **digraph** is an ordered pair $D = (V, A)$ with

- V a set whose elements are called **vertices** or **nodes**, and
- A a set of ordered pairs of vertices, called **arcs**, **directed edges**, or **arrows**.

An arc $a = (x, y)$ is considered to be directed **from** x **to** y; y is called the **head** and x is called the **tail** of the arc; y is said to be a **direct successor** of x, and x is said to be a **direct predecessor** of y. If a path leads from x to y, then y is said to be a **successor** of x and **reachable** from x, and x is said to be a **predecessor** of y. The arc (y, x) is called the arc (x, y) **inverted**.

A directed graph D is called **symmetric** if, for every arc in D, the corresponding inverted arc also belongs to D. A symmetric loopless directed graph $D = (V, A)$ is equivalent to a simple undirected graph $G = (V, E)$, where the pairs of inverse arcs in A correspond 1-to-1 with the edges in E; thus the edges in G number $|E| = |A|/2$, or half the number

of arcs in D.

A variation on this definition is the **oriented graph**, in which at most one of (x, y) and (y, x) may be arcs.

Mixed graph

A **mixed graph** G is a graph in which some edges may be directed and some may be undirected. It is written as an ordered triple $G = (V, E, A)$ with V, E, and A defined as above. Directed and undirected graphs are special cases.

Multigraph

A loop is an edge (directed or undirected) which starts and ends on the same vertex; these may be permitted or not permitted according to the application. In this context, an edge with two different ends is called a **link**.

The term "multigraph" is generally understood to mean that multiple edges (and sometimes loops) are allowed. Where graphs are defined so as to *allow* loops and multiple edges, a multigraph is often defined to mean a graph *without* loops,[5] however, where graphs are defined so as to *disallow* loops and multiple edges, the term is often defined to mean a "graph" which can have both multiple edges *and* loops,[6] although many use the term "pseudograph" for this meaning.[7]

Quiver

A **quiver** or "multidigraph" is a directed graph which may have more than one arrow from a given source to a given target. A quiver may also have directed loops in it.

Simple graph

As opposed to a multigraph, a simple graph is an undirected graph that has no loops and no more than one edge between any two different vertices. In a simple graph the edges of the graph form a set (rather than a multiset) and each edge is a *distinct* pair of vertices. In a simple graph with n vertices every vertex has a degree that is less than n (the converse, however, is not true — there exist non-simple graphs with n vertices in which every vertex has a degree smaller than n).

Weighted graph

A graph is a weighted graph if a number (weight) is assigned to each edge.[8] Such weights might represent, for example, costs, lengths or capacities, etc. depending on the problem at hand. Some authors call such a graph a network.[9]

Half-edges, loose edges

In exceptional situations it is even necessary to have edges with only one end, called **half-edges**, or no ends (**loose edges**); see for example signed graphs and biased graphs.

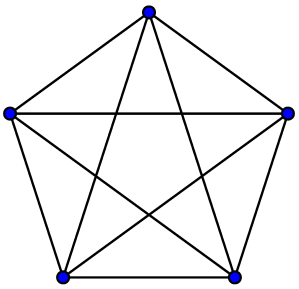
Important graph classes

Regular graph

A regular graph is a graph where each vertex has the same number of neighbours, i.e., every vertex has the same degree or valency. A regular graph with vertices of degree k is called a k-regular graph or regular graph of degree k.

Complete graph

Complete graphs have the feature that each pair of vertices has an edge connecting them.



A complete graph with 5 vertices. Each vertex has an edge to every other vertex.

Finite and infinite graphs

A finite graph is a graph $G = (V, E)$ such that V and E are finite sets. An infinite graph is one with an infinite set of vertices or edges or both.

Most commonly in graph theory it is implied that the graphs discussed are finite. If the graphs are infinite, that is usually specifically stated.

Graph classes in terms of connectivity

In an undirected graph G, two vertices u and v are called **connected** if G contains a path from u to v. Otherwise, they are called **disconnected**. A graph is called **connected** if every pair of distinct vertices in the graph is connected; otherwise, it is called **disconnected**.

A graph is called ***k*-vertex-connected** or ***k*-edge-connected** if no set of $k-1$ vertices (respectively, edges) exists that, when removed, disconnects the graph. A k-vertex-connected graph is often called simply ***k*-connected**.

A directed graph is called **weakly connected** if replacing all of its directed edges with undirected edges produces a connected (undirected) graph. It is **strongly connected** or **strong** if it contains a directed path from u to v and a directed path from v to u for every pair of vertices u, v.

Properties of graphs

Two edges of a graph are called **adjacent** (sometimes **coincident**) if they share a common vertex. Two arrows of a directed graph are called **consecutive** if the head of the first one is at the nock (notch end) of the second one. Similarly, two vertices are called **adjacent** if they share a common edge (**consecutive** if they are at the notch and at the head of an arrow), in which case the common edge is said to **join** the two vertices. An edge and a vertex on that edge are called **incident**.

The graph with only one vertex and no edges is called the **trivial graph**. A graph with only vertices and no edges is known as an **edgeless graph**. The graph with no vertices and no edges is sometimes called the **null graph** or **empty graph**, but the terminology is not consistent and not all mathematicians allow this object.

In a **weighted** graph or digraph, each edge is associated with some value, variously called its *cost*, *weight*, *length* or other term depending on the application; such graphs arise in many contexts, for example in optimal routing problems such as the traveling salesman problem.

Normally, the vertices of a graph, by their nature as elements of a set, are distinguishable. This kind of graph may be called **vertex-labeled**. However, for many questions it is better to treat vertices as indistinguishable; then the graph may be called **unlabeled**. (Of course, the vertices may be still distinguishable by the properties of the graph itself, e.g., by the numbers of incident edges). The same remarks apply to edges, so graphs with labeled edges are called **edge-labeled** graphs. Graphs with labels attached to edges or vertices are more generally designated as **labeled**. Consequently, graphs in which vertices are indistinguishable and edges are indistinguishable are called *unlabeled*. (Note that in the literature the term *labeled* may apply to other kinds of labeling, besides that which serves only to

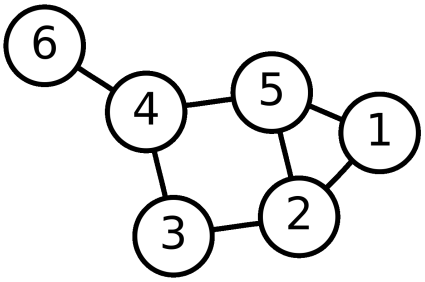
distinguish different vertices or edges.)

Examples

- The diagram at right is a graphic representation of the following graph:

  $V = \{1, 2, 3, 4, 5, 6\}$

  $E = \{\{1, 2\}, \{1, 5\}, \{2, 3\}, \{2, 5\}, \{3, 4\}, \{4, 5\}, \{4, 6\}\}$.



A graph with six nodes.

- In category theory a small category can be represented by a directed multigraph in which the objects of the category represented as vertices and the morphisms as directed edges. Then, the functors between categories induce some, but not necessarily all, of the digraph morphisms of the graph.
- In computer science, directed graphs are used to represent knowledge (e.g., Conceptual graph), finite state machines, and many other discrete structures.
- A binary relation R on a set X defines a directed graph. An element x of X is a direct predecessor of an element y of X iff xRy.

Important graphs

Basic examples are:

- In a complete graph, each pair of vertices is joined by an edge; that is, the graph contains all possible edges.
- In a bipartite graph, the vertex set can be partitioned into two sets, W and X, so that no two vertices in W are adjacent and no two vertices in X are adjacent. Alternatively, it is a graph with a chromatic number of 2.
- In a complete bipartite graph, the vertex set is the union of two disjoint sets, W and X, so that every vertex in W is adjacent to every vertex in X but there are no edges within W or X.
- In a *linear graph* or path graph of length n, the vertices can be listed in order, v_0, v_1, ..., v_n, so that the edges are $v_{i-1}v_i$ for each $i = 1, 2, ..., n$. If a linear graph occurs as a subgraph of another graph, it is a path in that graph.
- In a cycle graph of length $n \geq 3$, vertices can be named v_1, ..., v_n so that the edges are $v_{i-1}v_i$ for each $i = 2,...,n$ in addition to v_nv_1. Cycle graphs can be characterized as connected 2-regular graphs. If a cycle graph occurs as a subgraph of another graph, it is a *cycle* or *circuit* in that graph.
- A planar graph is a graph whose vertices and edges can be drawn in a plane such that no two of the edges intersect (i.e., *embedded* in a plane).
- A tree is a connected graph with no cycles.
- A *forest* is a graph with no cycles (i.e. the disjoint union of one or more *trees*).

More advanced kinds of graphs are:

- The Petersen graph and its generalizations
- Perfect graphs
- Cographs
- Chordal graphs
- Other graphs with large automorphism groups: vertex-transitive, arc-transitive, and distance-transitive graphs.
- Strongly regular graphs and their generalization distance-regular graphs.

Operations on graphs

There are several operations that produce new graphs from old ones, which might be classified into the following categories:

- Elementary operations, sometimes called "editing operations" on graphs, which create a new graph from the original one by a simple, local change, such as addition or deletion of a vertex or an edge, merging and splitting of vertices, etc.
- Graph rewrite operations replacing the occurrence of some pattern graph within the host graph by an instance of the corresponding replacement graph.
- Unary operations, which create a significantly new graph from the old one. Examples:
  - Line graph
  - Dual graph
  - Complement graph
- Binary operations, which create new graph from two initial graphs. Examples:
  - Disjoint union of graphs
  - Cartesian product of graphs
  - Tensor product of graphs
  - Strong product of graphs
  - Lexicographic product of graphs

Generalizations

In a hypergraph, an edge can join more than two vertices.

An undirected graph can be seen as a simplicial complex consisting of 1-simplices (the edges) and 0-simplices (the vertices). As such, complexes are generalizations of graphs since they allow for higher-dimensional simplices.

Every graph gives rise to a matroid.

In model theory, a graph is just a structure. But in that case, there is no limitation on the number of edges: it can be any cardinal number, see continuous graph.

In computational biology, power graph analysis introduces power graphs as an alternative representation of undirected graphs.

In geographic information systems, geometric networks are closely modeled after graphs, and borrow many concepts from graph theory to perform spatial analysis on road networks or utility grids.

Notes

[3] See, for instance, Iyanaga and Kawada, **69 J**, p. 234 or Biggs, p. 4.
[4] See, for instance, Graham et al., p. 5.
[5] For example, see Balakrishnan, p. 1, Gross (2003), p. 4, and Zwillinger, p. 220.
[6] For example, see. Bollobás, p. 7 and Diestel, p. 25.
[7] Gross (1998), p. 3, Gross (2003), p. 205, Harary, p.10, and Zwillinger, p. 220.

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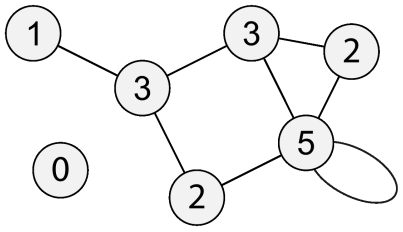
External links

- A searchable database of small connected graphs (http://www.gfredericks.com/main/sandbox/graphs)
- VisualComplexity.com (http://www.visualcomplexity.com) — A visual exploration on mapping complex networks
- Weisstein, Eric W., " Graph (http://mathworld.wolfram.com/Graph.html)" from MathWorld.
- Intelligent Graph Visualizer (https://sourceforge.net/projects/igv-intelligent/) — IGV create and edit graph, automatically places graph, search shortest path (+coloring vertices), center, degree, eccentricity, etc.
- Visual Graph Editor 2 (http://code.google.com/p/vge2/) — VGE2 designed for quick and easy creation, editing and saving of graphs and analysis of problems connected with graphs.
- GraphsJ (http://gianlucacosta.info/software/graphsj/) — GraphsJ is an open source didactic Java software which features an easy-to-use GUI and interactively solves step-by-step many graph problems. Extensible via its Java SDK.
- GraphClasses (http://graphclasses.org) — Information System on Graph Classes and their Inclusions.

Degree (graph theory)

In graph theory, the **degree** (or **valency**) of a vertex of a graph is the number of edges incident to the vertex, with loops counted twice.[1] The degree of a vertex v is denoted $\deg(v)$. The maximum degree of a graph G, denoted by $\Delta(G)$, and the minimum degree of a graph, denoted by $\delta(G)$, are the maximum and minimum degree of its vertices. In the graph on the right, the maximum degree is 5 and the minimum degree is 0. In a regular graph, all degrees are the same, and so we can speak of *the* degree of the graph.



A graph with vertices labeled by degree

Handshaking lemma

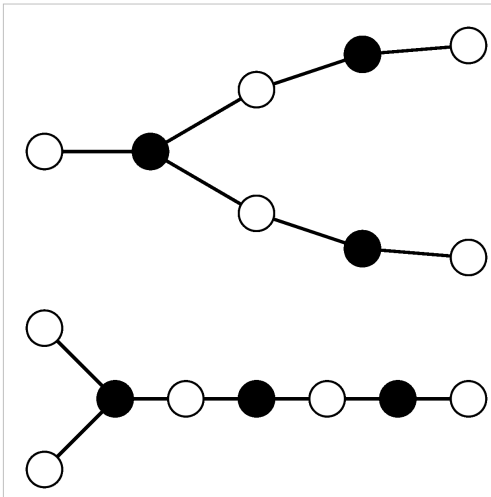
The **degree sum formula** states that, given a graph $G = (V, E)$,

$$\sum_{v \in V} \deg(v) = 2|E|\,.$$

The formula implies that in any graph, the number of vertices with odd degree is even. This statement (as well as the degree sum formula) is known as the handshaking lemma. The latter name comes from a popular mathematical problem, to prove that in any group of people the number of people who have shaken hands with an odd number of other people from the group is even.

Degree sequence

The **degree sequence** of an undirected graph is the non-increasing sequence of its vertex degrees;[2] for the above graph it is (5, 3, 3, 2, 2, 1, 0). The degree sequence is a graph invariant so isomorphic graphs have the same degree sequence. However, the degree sequence does not, in general, uniquely identify a graph; in some cases, non-isomorphic graphs have the same degree sequence.


Two non-isomorphic graphs with the same degree sequence (3, 2, 2, 2, 2, 1, 1, 1).

The **degree sequence problem** is the problem of finding some or all graphs with the degree sequence being a given non-increasing sequence of positive integers. (Trailing zeroes may be ignored since they are trivially realized by adding an appropriate number of isolated vertices to the graph.) A sequence which is the degree sequence of some graph, i.e. for which the degree sequence problem has a solution, is called a **graphic** or **graphical sequence**. As a consequence of the degree sum formula, any sequence with an odd sum, such as (3, 3, 1), cannot be realized as the degree sequence of a graph. The converse is also true: if a sequence has an even sum, it is the degree sequence of a multigraph. The construction of such a graph is straightforward: connect vertices with odd degrees in pairs by a matching, and fill out the remaining even degree counts by self-loops. The question of whether a given degree sequence can be realized by a simple graphs is more challenging. The Erdős–Gallai theorem states that a non-increasing sequence of n numbers d_i (for $i = 1,\ldots,n$) is the degree sequence of a simple graph if and only if the sum of the sequence is even and

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k) \quad \text{for } k \in \{1, \ldots, n\}.$$

For instance, the sequence (3, 3, 3, 1) is not the degree sequence of a simple graph; it satisfies the Erdős–Gallai inequality when k is 1, 2, or 4 but not when $k = 3$.

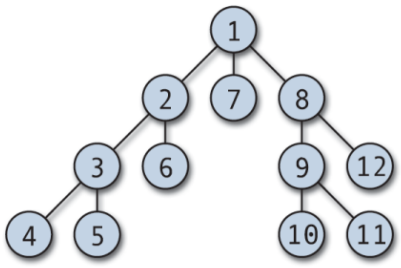
Havel (1955), and later, Hakimi (1962) proved that $(d_1, d_2, ..., d_n)$ is a degree sequence of a simple graph if and only if $(d_2 - 1, d_3 - 1, ..., d_{d_1+1} - 1, d_{d_1+2}, d_{d_1+3}, ..., d_n)$ is. This fact leads to a simple algorithm for finding a simple graph that has a given realizable degree sequence:

1. Begin with a graph with no edges.
2. Maintain a list of vertices whose degree requirement has not yet been met in non-increasing order of residual degree requirement.
3. Connect the first vertex to the next d_1 vertices in this list, and then remove it from the list. Re-sort the list and repeat until all degree requirements are met.

The problem of finding or estimating the number of graphs with a given degree sequence is a problem from the field of graph enumeration.

Special values

- A vertex with degree 0 is called an isolated vertex.
- A vertex with degree 1 is called a leaf vertex or end vertex, and the edge incident with that vertex is called a pendant edge. In the graph on the right, {3,5} is a pendant edge. This terminology is common in the study of trees in graph theory and especially trees as data structures.
- A vertex with degree *n-1* in a graph on *n* vertices is called a dominating vertex.



An undirected graph with leaf nodes 4, 5, 6, 7, 10, 11, and 12

Global properties

- If each vertex of the graph has the same degree k the graph is called a k-regular graph and the graph itself is said to have degree k. Similarly, a bipartite graph in which every two vertices on the same side of the bipartition as each other have the same degree is called a biregular graph.
- An undirected, connected graph has an Eulerian path if and only if it has either 0 or 2 vertices of odd degree. If it has 0 vertices of odd degree, the Eulerian path is an Eulerian circuit.
- A directed graph is a pseudoforest if and only if every vertex has outdegree at most 1. A functional graph is a special case of a pseudoforest in which every vertex has outdegree exactly 1.
- By Brooks' theorem, any graph other than a clique or an odd cycle has chromatic number at most Δ, and by Vizing's theorem any graph has chromatic index at most $\Delta + 1$.
- A k-degenerate graph is a graph in which each subgraph has a vertex of degree at most k.

Notes

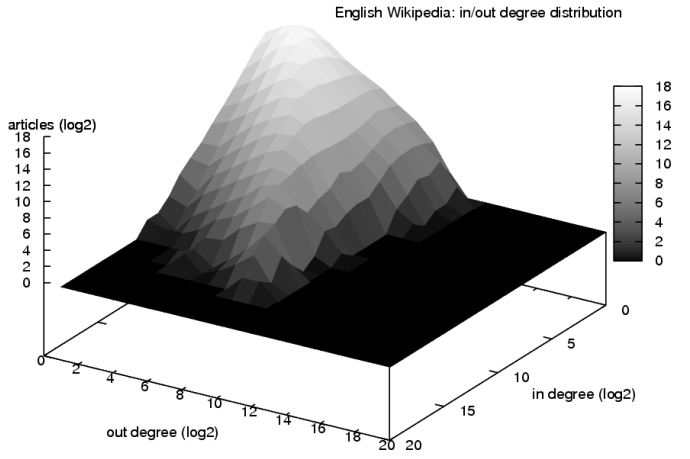
[1] Diestel p.5
[2] Diestel p.278

References

- Diestel, Reinhard (2005), *Graph Theory* (http://www.math.uni-hamburg.de/home/diestel/books/graph.theory/) (3rd ed.), Berlin, New York: Springer-Verlag, ISBN 978-3-540-26183-4.
- Erdős, P.; Gallai, T. (1960), "Gráfok előírt fokszámú pontokkal" (http://www.renyi.hu/~p_erdos/1961-05.pdf) (in Hungarian), *Matematikai Lapok* **11**: 264–274.
- Havel, Václav (1955), "A remark on the existence of finite graphs" (http://eudml.org/doc/19050) (in Czech), *Časopis pro pěstování matematiky* **80**: 477–480
- Hakimi, S. L. (1962), "On realizability of a set of integers as degrees of the vertices of a linear graph. I", *Journal of the Society for Industrial and Applied Mathematics* **10**: 496–506, MR 0148049 (http://www.ams.org/mathscinet-getitem?mr=0148049).
- Sierksma, Gerard; Hoogeveen, Han (1991), "Seven criteria for integer sequences being graphic", *Journal of Graph Theory* **15** (2): 223–231, doi: 10.1002/jgt.3190150209 (http://dx.doi.org/10.1002/jgt.3190150209), MR 1106533 (http://www.ams.org/mathscinet-getitem?mr=1106533).

Degree distribution

In the study of graphs and networks, the degree of a node in a network is the number of connections it has to other nodes and the **degree distribution** is the probability distribution of these degrees over the whole network.



In/out degree distribution for Wikipedia's hyperlink graph (logarithmic scales)

Definition

The degree of a node in a network (sometimes referred to incorrectly as the connectivity) is the number of connections or edges the node has to other nodes. If a network is directed, meaning that edges point in one direction from one node to another node, then nodes have two different degrees, the in-degree, which is the number of incoming edges, and the out-degree, which is the number of outgoing edges.

The degree distribution $P(k)$ of a network is then defined to be the fraction of nodes in the network with degree k. Thus if there are n nodes in total in a network and n_k of them have degree k, we have $P(k) = n_k/n$.

The same information is also sometimes presented in the form of a *cumulative degree distribution*, the fraction of nodes with degree greater than or equal to k.

Observed degree distributions

The degree distribution is very important in studying both real networks, such as the Internet and social networks, and theoretical networks. The simplest network model, for example, the (Bernoulli) random graph, in which each of n nodes is connected (or not) with independent probability p (or $1 - p$), has a binomial distribution of degrees:

$$P(k) = \binom{n-1}{k} p^k (1-p)^{n-1-k},$$

(or Poisson in the limit of large n). Most networks in the real world, however, have degree distributions very different from this. Most are highly right-skewed, meaning that a large majority of nodes have low degree but a small number, known as "hubs", have high degree. Some networks, notably the Internet, the world wide web, and some social networks are found to have degree distributions that approximately follow a power law: $P(k) \sim k^{-\gamma}$, where γ is a constant. Such networks are called scale-free networks and have attracted particular attention for their structural and dynamical properties.

References

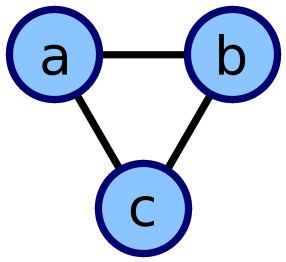
- Albert, R.; Barabasi, A.-L. (2002). "Statistical mechanics of complex networks". *Reviews of Modern Physics* **74**: 47–97. arXiv:cond-mat/0106096 [1]. Bibcode 2002RvMP...74...47A [2]. doi:10.1103/RevModPhys.74.47 [3].
- Dorogovtsev, S.; Mendes, J. F. F. (2002). "Evolution of networks". *Advances in Physics* **51** (4): 1079–1187. arXiv:cond-mat/0106144 [4]. Bibcode 2002AdPhy..51.1079D [5]. doi:10.1080/00018730110112519 [6].
- Newman, M. E. J. (2003). "The structure and function of complex networks" [7]. *SIAM Review* **45** (2): 167–256. arXiv:cond-mat/0303516 [8]. Bibcode 2003SIAMR..45..167N [9]. doi:10.1137/S003614450342480 [10].
- Shlomo Havlin and Reuven Cohen (2010). *Complex Networks: Structure, Robustness and Function* [11]. Cambridge University Press.

References

[1] http://arxiv.org/abs/cond-mat/0106096
[2] http://adsabs.harvard.edu/abs/2002RvMP...74...47A
[3] http://dx.doi.org/10.1103%2FRevModPhys.74.47
[4] http://arxiv.org/abs/cond-mat/0106144
[5] http://adsabs.harvard.edu/abs/2002AdPhy..51.1079D
[6] http://dx.doi.org/10.1080%2F00018730110112519
[7] http://siamdl.aip.org/getabs/servlet/GetabsServlet?prog=normal&id=SIREAD000045000002000167000001
[8] http://arxiv.org/abs/cond-mat/0303516
[9] http://adsabs.harvard.edu/abs/2003SIAMR..45..167N
[10] http://dx.doi.org/10.1137%2FS003614450342480
[11] http://havlin.biu.ac.il/Shlomo%20Havlin%20books_com_net.php

Adjacency list

In graph theory and computer science, an **adjacency list** representation of a graph is a collection of unordered lists, one for each vertex in the graph. Each list describes the set of neighbors of its vertex.



This undirected cyclic graph can be described by the list {a,b}, {a,c}, {b,c}.

Implementation details

The graph pictured above has this adjacency list representation:		
a	adjacent to	b,c
b	adjacent to	a,c
c	adjacent to	a,b

An adjacency list representation for a graph associates each vertex in the graph with the collection of its neighboring vertices or edges. There are many variations of this basic idea, differing in the details of how they implement the association between vertices and collections, in how they implement the collections, in whether they include both vertices and edges or only vertices as first class objects, and in what kinds of objects are used to represent the vertices and edges.

- An implementation suggested by Guido van Rossum uses a hash table to associate each vertex in a graph with an array of adjacent vertices. In this representation, a vertex may be represented by any hashable object. There is no explicit representation of edges as objects.[1]
- Cormen et al. suggest an implementation in which the vertices are represented by index numbers.[2] Their representation uses an array indexed by vertex number, in which the array cell for each vertex points to a singly linked list of the neighboring vertices of that vertex. In this representation, the nodes of the singly linked list may be interpreted as edge objects; however, they do not store the full information about each edge (they only store one of the two endpoints of the edge) and in undirected graphs there will be two different linked list nodes for each edge (one within the lists for each of the two endpoints of the edge).
- The object oriented **incidence list** structure suggested by Goodrich and Tamassia has special classes of vertex objects and edge objects. Each vertex object has an instance variable pointing to a collection object that lists the neighboring edge object. In turn, each edge object points to the two vertex objects at its endpoints.[] This version of the adjacency list uses more memory than the version in which adjacent vertices are listed directly, but the existence of explicit edge objects allows it extra flexibility in storing additional information about edges.

Operations

The main operation performed by the adjacency list data structure is to report a list of the neighbors of a given vertex. Using any of the implementations detailed above, this can be performed in constant time per neighbor. In other words, the total time to report all of the neighbors of a vertex *v* is proportional to the degree of *v*.

It is also possible, but not as efficient, to use adjacency lists to test whether an edge exists or does not exist between two specified vertices. In an adjacency list in which the neighbors of each vertex are unsorted, testing for the existence of an edge may be performed in time proportional to the degree of one of the two given vertices, by using a sequential search through the neighbors of this vertex. If the neighbors are represented as a sorted array, binary search may be used instead, taking time proportional to the logarithm of the degree.

Trade-offs

The main alternative to the adjacency list is the adjacency matrix, a matrix whose rows and columns are indexed by vertices and whose cells contain a Boolean value that indicates whether an edge is present between the vertices corresponding to the row and column of the cell. For a sparse graph (one in which most pairs of vertices are not connected by edges) an adjacency list is significantly more space-efficient than an adjacency matrix (stored as an array): the space usage of the adjacency list is proportional to the number of edges and vertices in the graph, while for an adjacency matrix stored in this way the space is proportional to the square of the number of vertices. However, it is possible to store adjacency matrices more space-efficiently, matching the linear space usage of an adjacency list, by using a hash table indexed by pairs of vertices rather than an array.

The other significant difference between adjacency lists and adjacency matrices is in the efficiency of the operations they perform. In an adjacency list, the neighbors of each vertex may be listed efficiently, in time proportional to the degree of the vertex. In an adjacency matrix, this operation takes time proportional to the number of vertices in the graph, which may be significantly higher than the degree. On the other hand, the adjacency matrix allows testing whether two vertices are adjacent to each other, in constant time; the adjacency list is slower to support this operation.

References

Additional reading

- David Eppstein (1996). "ICS 161 Lecture Notes: Graph Algorithms" (http://www.ics.uci.edu/~eppstein/161/960201.html).

External links

- The Boost Graph Library implements an efficient adjacency list (http://www.boost.org/doc/libs/1_43_0/libs/graph/doc/index.html)
- Open Data Structures - Section 12.2 - AdjacencyList: A Graph as a Collection of Lists (http://opendatastructures.org/versions/edition-0.1e/ods-java/12_2_AdjacencyLists_Graph_a.html)

Adjacency matrix

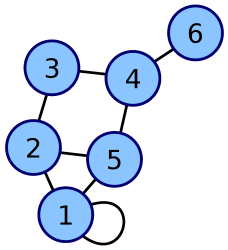
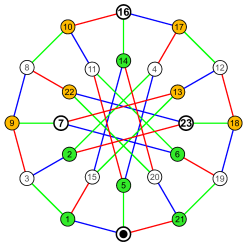
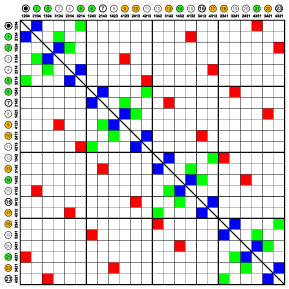
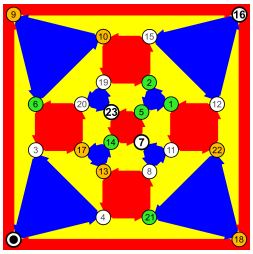
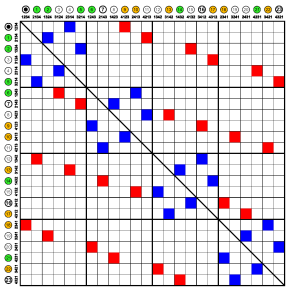
In mathematics and computer science, an **adjacency matrix** is a means of representing which vertices (or nodes) of a graph are adjacent to which other vertices. Another matrix representation for a graph is the incidence matrix.

Specifically, the adjacency matrix of a finite graph **G** on n vertices is the $n \times n$ matrix where the non-diagonal entry a_{ij} is the number of edges from vertex i to vertex j, and the diagonal entry a_{ii}, depending on the convention, is either once or twice the number of edges (loops) from vertex i to itself. Undirected graphs often use the latter convention of counting loops twice, whereas directed graphs typically use the former convention. There exists a unique adjacency matrix for each isomorphism class of graphs (up to permuting rows and columns), and it is not the adjacency matrix of any other isomorphism class of graphs. In the special case of a finite simple graph, the adjacency matrix is a (0,1)-matrix with zeros on its diagonal. If the graph is undirected, the adjacency matrix is symmetric.

The relationship between a graph and the eigenvalues and eigenvectors of its adjacency matrix is studied in spectral graph theory.

Examples

The convention followed here is that an adjacent edge counts 1 in the matrix for an undirected graph.

Labeled graph	Adjacency matrix
	$\begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$ Coordinates are 1-6.
 The Nauru graph	 Coordinates are 0-23. White fields are zeros, colored fields are ones.
 Directed Cayley graph of S_4	 As the graph is directed, the matrix is not symmetric.

- The adjacency matrix of a complete graph is all 1's except for 0's on the diagonal.
- The adjacency matrix of an empty graph is a zero matrix.

Adjacency matrix of a bipartite graph

The adjacency matrix A of a bipartite graph whose parts have r and s vertices has the form

$$A = \begin{pmatrix} O & B \\ B^T & O \end{pmatrix},$$

where B is an $r \times s$ matrix and O is an all-zero matrix. Clearly, the matrix B uniquely represents the bipartite graphs. It is sometimes called the biadjacency matrix. Formally, let $G = (U, V, E)$ be a bipartite graph with parts $U = u_1, ..., u_r$ and $V = v_1, ..., v_s$. The **biadjacency matrix** is the r x s 0-1 matrix B in which $b_{i,j} = 1$ iff $(u_i, v_j) \in E$.

If G is a bipartite multigraph or weighted graph then the elements $b_{i,j}$ are taken to be the number of edges between the vertices or the weight of the edge (u_i, v_j), respectively.

Properties

The adjacency matrix of an undirected simple graph is symmetric, and therefore has a complete set of real eigenvalues and an orthogonal eigenvector basis. The set of eigenvalues of a graph is the **spectrum** of the graph.

Suppose two directed or undirected graphs G_1 and G_2 with adjacency matrices A_1 and A_2 are given. G_1 and G_2 are isomorphic if and only if there exists a permutation matrix P such that

$$PA_1P^{-1} = A_2.$$

In particular, A_1 and A_2 are similar and therefore have the same minimal polynomial, characteristic polynomial, eigenvalues, determinant and trace. These can therefore serve as isomorphism invariants of graphs. However, two graphs may possess the same set of eigenvalues but not be isomorphic. [1]

If A is the adjacency matrix of the directed or undirected graph G, then the matrix A^n (i.e., the matrix product of n copies of A) has an interesting interpretation: the entry in row i and column j gives the number of (directed or undirected) walks of length n from vertex i to vertex j. This implies, for example, that the number of triangles in an undirected graph G is exactly the trace of A^3 divided by 6.

The main diagonal of every adjacency matrix corresponding to a graph without loops has all zero entries. Note that here 'loops' means, for example A->A, not 'cycles' such as A->B->A.

For (d)-regular graphs, d is also an eigenvalue of A for the vector $v = (1, \ldots, 1)$, and G is connected if and only if the multiplicity of d is 1. It can be shown that $-d$ is also an eigenvalue of A if G is a connected bipartite graph. The above are results of Perron–Frobenius theorem.

Variations

An **(a, b, c)-adjacency matrix** A of a simple graph has $A_{ij} = a$ if ij is an edge, b if it is not, and c on the diagonal. The Seidel adjacency matrix is a **(–1,1,0)-adjacency matrix**. This matrix is used in studying strongly regular graphs and two-graphs.[2]

The **distance matrix** has in position (i,j) the distance between vertices v_i and v_j. The distance is the length of a shortest path connecting the vertices. Unless lengths of edges are explicitly provided, the length of a path is the number of edges in it. The distance matrix resembles a high power of the adjacency matrix, but instead of telling only whether or not two vertices are connected (i.e., the connection matrix, which contains boolean values), it gives the exact distance between them.

Data structures

For use as a data structure, the main alternative to the adjacency matrix is the adjacency list. Because each entry in the adjacency matrix requires only one bit, it can be represented in a very compact way, occupying only $n^2/8$ bytes of contiguous space, where n is the number of vertices. Besides avoiding wasted space, this compactness encourages locality of reference.

However, if the graph is sparse, adjacency lists require less storage space, because they do not waste any space to represent edges that are *not* present. Using a naïve array implementation on a 32-bit computer, an adjacency list for an undirected graph requires about $8e$ bytes of storage, where e is the number of edges.

Noting that a simple graph can have at most n^2 edges, allowing loops, we can let $d = e/n^2$ denote the *density* of the graph. Then, $8e > n^2/8$, or the adjacency list representation occupies more space precisely when $d > 1/64$.

Thus a graph must be sparse indeed to justify an adjacency list representation.

Besides the space tradeoff, the different data structures also facilitate different operations. Finding all vertices adjacent to a given vertex in an adjacency list is as simple as reading the list. With an adjacency matrix, an entire row must instead be scanned, which takes O(n) time. Whether there is an edge between two given vertices can be determined at once with an adjacency matrix, while requiring time proportional to the minimum degree of the two vertices with the adjacency list.

References

[1] Godsil, Chris; Royle, Gordon *Algebraic Graph Theory*, Springer (2001), ISBN 0-387-95241-1, p.164

Further reading

- Cormen, Thomas H.; Leiserson, Charles E.; Rivest, Ronald L.; Stein, Clifford (2001). "Section 22.1: Representations of graphs". *Introduction to Algorithms* (Second ed.). MIT Press and McGraw-Hill. pp. 527–531. ISBN 0-262-03293-7.
- Godsil, Chris; Royle, Gordon (2001). *Algebraic Graph Theory*. New York: Springer. ISBN 0-387-95241-1.

External links

- Fluffschack (http://www.x2d.org/java/projects/fluffschack.jnlp) — an educational Java web start game demonstrating the relationship between adjacency matrices and graphs.
- Open Data Structures - Section 12.1 - AdjacencyMatrix: Representing a Graph by a Matrix (http://opendatastructures.org/versions/edition-0.1e/ods-java/12_1_AdjacencyMatrix_Repres.html)
- McKay, Brendan. "Description of graph6 and sparse6 encodings" (http://cs.anu.edu.au/~bdm/data/formats.txt).
- Café math : Adjacency Matrices of Graphs (http://www.cafemath.fr/mathblog/article.php?page=GoodWillHunting.php) : Application of the adjacency matrices to the computation generating series of walks.

Distance (graph theory)

In the mathematical field of graph theory, the **distance** between two vertices in a graph is the number of edges in a shortest path connecting them. This is also known as the **geodesic distance** [1] because it is the length of the graph geodesic between those two vertices.[2] If there is no path connecting the two vertices, i.e., if they belong to different connected components, then conventionally the distance is defined as infinite.

Related concepts

A metric space defined over a set of points in terms of distances in a graph defined over the set is called a **graph metric**. The vertex set (of an undirected graph) and the distance function form a metric space, if and only if the graph is connected.

The **eccentricity** ϵ of a vertex v is the greatest geodesic distance between v and any other vertex. It can be thought of as how far a node is from the node most distant from it in the graph.

The **radius** of a graph is the minimum eccentricity of any vertex.

The **diameter** of a graph is the maximum eccentricity of any vertex in the graph. That is, it is the greatest distance between any pair of vertices. To find the diameter of a graph, first find the shortest path between each pair of vertices. The greatest length of any of these paths is the diameter of the graph.

A **central vertex** in a graph of radius r is one whose eccentricity is r—that is, a vertex that achieves the radius.

A **peripheral vertex** in a graph of diameter d is one that is distance d from some other vertex—that is, a vertex that achieves the diameter.

A **pseudo-peripheral vertex** v has the property that for any vertex u, if v is as far away from u as possible, then u is as far away from v as possible. Formally, a vertex u is pseudo-peripheral, if for each vertex v with $d(u,v) = \epsilon(u)$ holds $\epsilon(u) = \epsilon(v)$.

The partition of a graphs vertices into subsets by their distances from a given starting vertex is called the level structure of the graph.

Algorithm for finding pseudo-peripheral vertices

Often peripheral sparse matrix algorithms need a starting vertex with a high eccentricity. A peripheral vertex would be perfect, but is often hard to calculate. In most circumstances a pseudo-peripheral vertex can be used. A pseudo-peripheral vertex can easily be found with the following algorithm:

1. Choose a vertex u.
2. Among all the vertices that are as far from u as possible, let v be one with minimal degree.
3. If $\epsilon(v) > \epsilon(u)$ then set $u = v$ and repeat with step 2, else v is a pseudo-peripheral vertex.

Notes

Preferential attachment

A **preferential attachment process** is any of a class of processes in which some quantity, typically some form of wealth or credit, is distributed among a number of individuals or objects according to how much they already have, so that those who are already wealthy receive more than those who are not. "Preferential attachment" is only the most recent of many names that have been given to such processes. They are also referred to under the names "Yule process", "cumulative advantage", "the rich get richer", and, less correctly, the "Matthew effect". It is related to Gibrat's law. The principal reason for scientific interest in preferential attachment is that it can, under suitable circumstances, generate power law distributions.

Definition

A preferential attachment process is a stochastic urn process, meaning a process in which discrete units of wealth, usually called "balls", are added in a random or partly random fashion to a set of objects or containers, usually called "urns". A preferential attachment process is an urn process in which additional balls are added continuously to the system and are distributed among the urns as an increasing function of the number of balls the urns already have. In the most commonly studied examples, the number of urns also increases continuously, although this is not a necessary condition for preferential attachment and examples have been studied with constant or even decreasing numbers of urns.

A classic example of a preferential attachment process is the growth in the number of species per genus in some higher taxon of biotic organisms.[] New genera ("urns") are added to a taxon whenever a newly appearing species is considered sufficiently different from its predecessors that it does not belong in any of the current genera. New species ("balls") are added as old ones speciate (i.e., split in two) and, assuming that new species belong to the same genus as their parent (except for those that start new genera), the probability that a species is added to a new genus will be proportional to the number of species the genus already has. This process, first studied by Yule, is a *linear* preferential attachment process, since the rate at which genera accrue new species is linear in the number they already have.

Linear preferential attachment processes in which the number of urns increases are known to produce a distribution of balls over the urns following the so-called Yule distribution. In the most general form of the process, balls are added to the system at an overall rate of m new balls for each new urn. Each newly created urn starts out with k_0 balls and further balls are added to urns at a rate proportional to the number k that they already have plus a constant $a > -k_0$. With these definitions, the fraction $P(k)$ of urns having k balls in the limit of long time is given by[1]

$$P(k) = \frac{\mathrm{B}(k+a, \gamma)}{\mathrm{B}(k_0+a, \gamma-1)},$$

for $k \geq k_0$ (and zero otherwise), where $\mathrm{B}(x, y)$ is the Euler beta function:

$$\mathrm{B}(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)},$$

with $\Gamma(x)$ being the standard gamma function, and

$$\gamma = 2 + \frac{k_0 + a}{m}.$$

The beta function behaves asymptotically as $\mathrm{B}(x, y) \sim x^{-y}$ for large x and fixed y, which implies that for large values of k we have

$$P(k) \propto k^{-\gamma}.$$

In other words, the preferential attachment process generates a "long-tailed" distribution following a Pareto distribution or power law in its tail. This is the primary reason for the historical interest in preferential attachment:

the species distribution and many other phenomena are observed empirically to follow power laws and the preferential attachment process is a leading candidate mechanism to explain this behavior. Preferential attachment is considered a possible candidate for, among other things, the distribution of the sizes of cities,[] the wealth of extremely wealthy individuals,[] the number of citations received by learned publications,[] and the number of links to pages on the World Wide Web.[]

The general model described here includes many other specific models as special cases. In the species/genus example above, for instance, each genus starts out with a single species ($k_0 = 1$) and gains new species in direct proportion to the number it already has ($a = 0$), and hence $P(k) = \mathrm{B}(k, \gamma)/\mathrm{B}(k_0, \gamma - 1)$ with $\gamma = 2 + 1/m$. Similarly the Price model for scientific citations[] corresponds to the case $k_0 = 0$, $a = 1$ and the widely studied Barabási-Albert model[] corresponds to $k_0 = m$, $a = 0$.

Preferential attachment is sometimes referred to as the Matthew effect, but the two are not precisely equivalent. The Matthew effect, first discussed by Robert Merton,[2] is named for a passage in the biblical Gospel of Matthew: "For everyone who has will be given more, and he will have an abundance. Whoever does not have, even what he has will be taken from him." (Matthew 25:29, New International Version.) The preferential attachment process does not incorporate the taking away part. An urn process that includes both the giving and the taking away would produce a log-normal distribution rather than a power law [*citation needed*]. This point may be moot, however, since the scientific insight behind the Matthew effect is in any case entirely different. Qualitatively it is intended to describe not a mechanical multiplicative effect like preferential attachment but a specific human behavior in which people are more likely to give credit to the famous than to the little known. The classic example of the Matthew effect is a scientific discovery made simultaneously by two different people, one well known and the other little known. It is claimed that under these circumstances people tend more often to credit the discovery to the well-known scientist. Thus the real-world phenomenon the Matthew effect is intended to describe is quite distinct from (though certainly related to) preferential attachment.

History

The first rigorous consideration of preferential attachment seems to be that of Yule in 1925, who used it to explain the power-law distribution of the number of species per genus of flowering plants.[] The process is sometimes called a "Yule process" in his honor. Yule was able to show that the process gave rise to a distribution with a power-law tail, but the details of his proof are, by today's standards, contorted and difficult, since the modern tools of stochastic process theory did not yet exist and he was forced to use more cumbersome methods of proof.

Most modern treatments of preferential attachment make use of the master equation method, whose use in this context was pioneered by Simon in 1955, in work on the distribution of sizes of cities and other phenomena.[]

The first application of preferential attachment to learned citations was given by Price in 1976.[] (He referred to the process as a "cumulative advantage" process.) His was also the first application of the process to the growth of a network, producing what would now be called a scale-free network. It is in the context of network growth that the process is most frequently studied today. Price also promoted preferential attachment as a possible explanation for power laws in many other phenomena, including Lotka's law of scientific productivity and Bradford's law of journal use.

The application of preferential attachment to the growth of the World Wide Web was proposed by Barabási and Albert in 1999.[] Barabási and Albert also coined the name "preferential attachment" by which the process is best known today and suggested that the process might apply to the growth of other networks as well.

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Balance theory

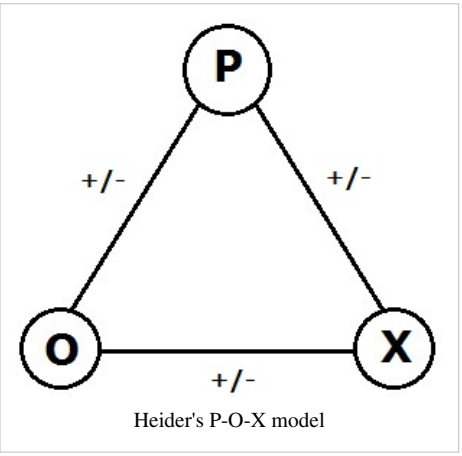
Balance Theory is a motivational theory of attitude change, proposed by Fritz Heider.[1] It conceptualizes the cognitive consistency motive as a drive toward psychological balance. The consistency motive is the urge to maintain one's values and beliefs over time. Heider proposed that "sentiment" or liking relationships are balanced if the affect valence in a system multiplies out to a positive result.

P-O-X model

For example: a Person (**P**) who likes an Other (**O**) person will be balanced by the same valence attitude on behalf of the other. Symbolically, P (+) > O and P < (+) O results in psychological balance.

This can be extended to objects (**X**) as well, thus introducing triadic relationships. If a person P likes object X but dislikes other person O, what does P feel upon learning that O created X? This is symbolized as such:

- P (+) > X
- P (-) > O
- O (+) > X



Heider's P-O-X model

Multiplying the signs shows that the person will perceive imbalance (a negative multiplicative product) in this relationship, and will be motivated to correct the imbalance somehow. The Person can either:

- Decide that O isn't so bad after all,
- Decide that X isn't as great as originally thought, or
- Conclude that O couldn't really have made X.

Any of these will result in psychological balance, thus resolving the dilemma and satisfying the drive. (Person P could also avoid object X and other person O entirely, lessening the stress created by psychological imbalance.)

To predict the outcome of a situation using Heider's Balance Theory, one must weigh the effects of all the potential results, and the one requiring the least amount of effort will be the likely outcome.

Examples

Balance Theory is also useful in examining how celebrity endorsement affects consumers' attitudes toward products.[2] If a person likes a celebrity and perceives (due to the endorsement) that said celebrity likes a product, said person will tend to liking the product more, in order to achieve psychological balance.

However, if the person already had a dislike for the product being endorsed by the celebrity, she may like the celebrity less in addition to liking the product more, again to achieve psychological balance.

Heider's balance theory can explain why holding the same negative attitudes of others promotes closeness (see The enemy of my enemy is my friend).

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Social comparison theory

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Social comparison theory was initially proposed by social psychologist Leon Festinger in 1954. Social comparison theory is centered on the belief that there is a drive within individuals to gain accurate self-evaluations. The theory explains how individuals evaluate their own opinions and abilities by comparing themselves to others in order to reduce uncertainty in these domains, and learn how to define the self.

Following the initial theory, research began to focus on social comparison as a way of self-enhancement (Gruder, 1971; Wills, 1981), introducing the concepts of downward and upward comparisons and expanding the motivations of social comparisons (e.g. Schachter, 1959).

Initial framework

In the initial theory, Festinger (1954) hypothesized several things. First, he stated that individuals are motivated to gain accurate evaluations of themselves by examining their opinions and abilities in comparison to others. Such comparisons provide an objective benchmark against which an individual can compare themselves in relevant domains, providing a sense of validity and cognitive clarity. He hypothesized that people who are similar to an individual are especially good in generating accurate evaluations of abilities and opinions (Suls, Martin, & Wheeler, 2002).To this, he added that the tendency to compare oneself with some other specific person decreases as the difference between their opinions and abilities become more divergent. He also hypothesized that there is an upward drive towards achieving greater abilities (Festinger, 1954).

He further theorized that comparing the self with others leads to pressures of uniformity. If discrepancies arise between the evaluator and comparison group there is a tendency to reduce the divergence by either attempting to persuade others, or changing their personal views to attain uniformity. However, the importance, relevance and attraction to a comparison group that affects the original motivation for comparison, mediates the pressures towards uniformity (Festinger, 1954).

Main criticisms and further development

Many criticisms arose regarding Festinger's similarity hypothesis. Deutsch and Krauss (1965) argued that people actually seek out dissimilar others in their comparisons maintaining that this is important for providing valuable self-knowledge, as demonstrated in research (Goethals & Nelson, 1973; Mettee & Smith, 1977). Ambiguity also circulated about the important dimensions for similarity. Goethals and Darley (1977) clarified the role of similarity suggesting that people prefer to compare those who are similar on related attributes such as opinions, characteristics or abilities to increase confidence for value judgments, however those dissimilar in related attributes are preferred when validating one's beliefs.

Since its introduction to communication and social psychology, research has shown that social comparisons are more complex than initially thought, and that people play a more active role in comparisons (Suls, Martin & Wheeler 2002). A number of revisions, including new domains for comparison and motives, have also been made since 1954. Motives that are relevant to comparison include self-enhancement(Gruder, 1971; Wills, 1981), maintenance of a positive self-evaluation (Tesser & Campbell, 1982), components of attributions and validation (Goethals & Darley, 1977) and the avoidance of closure (Kruglanski & Mayseless, 1990; Suls, Martin, & Wheeler, 2002).

Thornton and Arrowood (1996) recognized two separate motivations for social comparison: Self-evaluation as described by Festinger (1954) and self-enhancement which they proposed could be gained by identifying with a superior other. Alternatively, Hakmiller (1966) suggested self-enhancement comes from comparing with others who are inferior. The recognition of this motivation led to the rise of downward and upward comparisons.

Wills (1981) introduced the concept of downward comparison. **Downward social comparison** is a defensive tendency that people use as a means of self-evaluation. These individuals will look to another individual or comparison group who are considered to be worse off in order to dissociate themselves from perceived similarities and to make themselves feel better about their self or personal situation. Social comparison research has suggested that comparisons with others who are better off or superior on an upward comparison can lower self-regard (Tesser et al., 1988) whereas downward comparisons can elevate self-regard (Gibbons, 1986). Downward comparison theory emphasizes the positive effects of comparisons in increasing one's subjective well-being (Wills, 1981). For example, Wood, Taylor and Lichtman (1985) found breast cancer patients made the majority of comparisons with patients less fortunate than themselves.

People make *upward* comparisons, both consciously and subconsciously, with other individuals they perceive to be better than themselves in order to improve their views of self or to create a more positive perception of their personal reality. In an **upward social comparison**, people want to believe themselves to be part of the elite or superior, and make comparisons showing the similarities in themselves and the comparison group (Suls, Martin & Wheeler 2002). Taylor and Lobel (1989) suggested upward comparisons provide inspiration to improve. They maintained whilst breast cancer patients may make more downward comparisons, patients preferred information about more fortunate others.

A good example of upward social comparison relates to women and their perceptions of the self and others. For example, a woman looks at images of idealized others, and feels as though she is not equal to what she sees. Although men do make upward comparisons, research finds that more women make upward comparisons and are comparing themselves with unrealistically high standards presented in the media (Strahan, Wilson, Cressman, & Buote, 2006). As women are shown more mainstream media images of powerful, successful and thin women, they perceive the "ideal" to be the norm for societal views of attractive. Some women have reported making upward comparisons in a positive manner for the purposes of self-motivation, but the majority of upward comparisons are made when the individual is feeling lesser and therefore evoke a negative connotation.

The media has been found to play a large role in social comparisons. Researchers examining the social effects of the media have used social comparison theory have found that in most cases women tend to engage in upward social comparisons with a target other, which results in more negative feelings about the self. The majority of women have a daily opportunity to make upward comparison by measuring themselves against some form of societal ideal. Social comparisons have become a relevant mechanism for learning about the appearance-related social expectations among peers and for evaluating the self in terms of those standards" (Jones, 2001, P. 647).

Several models have been introduced to social comparison, including the **Self-Evaluation Maintenance Model (SEM)** (Tesser, 1988), Proxy Model (Wheeler et al., 1997), the Triadic Model and the Three-Selves Model. The SEM model proposes that we make comparisons to maintain or enhance our self-evaluations, focusing on the antagonistic processes of comparison and reflection (Tesser, 1988).**The Proxy model** anticipates the success of something that is unfamiliar. The model proposes that if a person is successful or familiar with a task, then he or she would also be successful at a new similar task. The proxy is evaluated based on ability and is concerned with the

question "Can I do X?" A proxy's comparison is based previous attributes. The opinion of the comparer and whether the proxy exerted maximum effort on a preliminary task are variables influencing his or her opinion (Suls, Martin, & Wheeler, 2002). **The Triadic Model** builds on the attribution elements of social comparison, proposing that opinions of social comparison are best considered in terms of 3 different evaluative questions: preference assessment (i.e., "Do I like X?"), belief assessment (i.e., "Is X correct?"), and preference prediction (i.e., "Will I like X?"). In the Triadic Model the most meaningful comparisons are with a person who has already experienced a proxy and exhibits consistency in related attributes or past preferences (Suls, Martin & Wheeler, 2002). **The Three-Selves Model** proposes that social comparison theory is a combination of two different theories. One theory is developed around motivation and the factors that influence the type of social comparison information people seek from their environment and the second is about self-evaluation and the factors that influence the effects of social comparisons on the judgments of self (Blanton, 2000). While there has been much research in the area of comparison motives, there has been little in the area of comparative evaluation. Explaining that the self is conceived as interrelated conceptions accessible depending upon current judgment context (Markus & Wurf, 1987) and taking a cue from Social Cognitive Theory, this model examines the Assimilation effect and distinguishes three classes of working Self-concept ideas: individual selves, possible selves and collective selves.

While there have been changes in Festinger's original concept, many fundamental aspects remain, including similarity, the tendency towards social comparison and the general process that is social comparison (Kruglanski, & Mayseless, 1990).

History

In the 1950s, Festinger was given a grant from the Behavioral Sciences Division of the Ford Foundation. This grant was part of the research program of the Laboratory for Research in Social Relations, which developed the Social Comparison Theory (Festinger, 1954). The development of social comparison hinged on several socio-psychological processes, and in order to create this theory, Festinger was influenced by research that focused on social communication (Festinger, 1950), group dynamics, conformity and the autokinetic effect (Sherif, 1936), compliant behavior, social groups, independence and dependence in response to unanimous majority (Asch, 1956) and level of aspiration (Festinger, 1942; Kruglanski & Mayseless, 1990). In his article, he sourced various experiments with children and adults, however, much of his theory was based on his own research (Festinger, 1954). After the 1966 supplement work by Pettigrew, Brickman, and Wheeler and the linkage of social comparison to Attribution Theory, interest was rekindled in comparison processes.

When understanding the basis of social comparison, it is imperative to understand that no one thought process created the theory, but rather, a compilation of experiments, historical evidence and philosophical thought. While Festinger was the first social psychologist to coin the term "Social Comparison", the general concept cannot be claimed exclusively by him (Suls & Wheeler, 2000). In fact, this theory's origins can be dated back to Aristotle and Plato. Plato spoke of comparisons of self-understanding and absolute standards. Aristotle was concerned with comparisons between people. Later, philosophers such as Kant, Marx and Rousseau spoke on moral reasoning and social inequality. (Suls, Martin, & Wheeler, 2002).

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Social identity approach

The term **social identity approach** refers to research and theory pertaining to two intertwined, but distinct, social psychological theories.[1][2][3] These being: social identity theory and self-categorization theory. The social identity approach has been applied to a wide variety of fields and continues to be very influential. There is a high citation rate for key social identity papers and that rate continues to increase.[3]

Aspects of the social identity approach

The term "social identity approach" arose as an attempt to mitigate against the tendency to conflate the two theories,[][] as well as the tendency to mistakenly believe one theory to be a component of the other. Instead these theories should be thought of as overlapping in the manner demonstrated in Fig 1.[1] That is, while there are similarities, self categorisation theory has greater explanatory scope (i.e. is less focused on intergroup relationships specifically) and has been investigated in a broader range of empirical conditions. Self-categorization theory can also be thought of as developed to address limitations of social identity theory. Specifically the limited manner in which social identity theory deals with the cognitive processes that underpin the behaviour it describes.

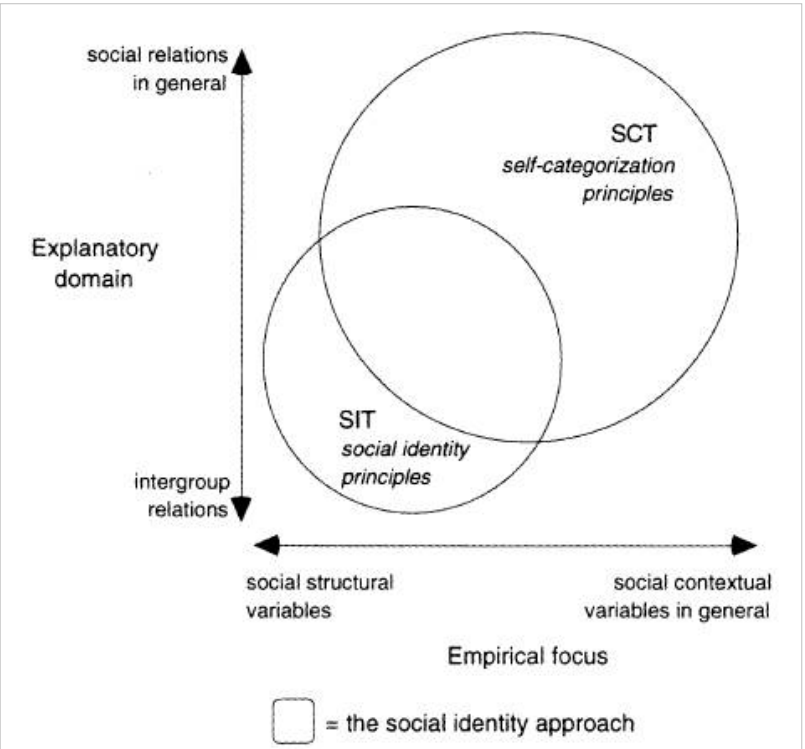

Figure 1. The explanatory profiles of social identity and self-categorization theories.

Although this term may be useful when contrasting broad social psychological movements, when applying either theory it is thought of as beneficial to distinguish carefully between the two theories in such a way that their specific characteristics can be retained.[1]

Implications

Social Groups

The social identity approach has been contrasted with the social cohesion approach when it comes to defining social groups. The social identity approach posits that the necessary and sufficient conditions for the formation of social groups is “awareness of a common category membership” and that a social group can be "usefully conceptualized as a number of individuals who have internalized the same social category membership as a component of their self concept".[]

On the topic of social groups, some social psychologists draw a distinction between different types of group phenomenon. Specifically, “those that derive from interpersonal relationships and interdependence with specific others and those that derive from membership in larger, more impersonal collectives or social categories”.[] The

social identity approach however does not anticipate this distinction. Instead it anticipates that the same psychological processes underlie intergroup and intragroup phenomenon involving both small and large groups. Relatedly, the persistent perception that the social identity approach is only relevant to large group phenomenon has led some social identity theorists to specifically reassert (both theoretically and empirically) the relevance of the social identity approach to small group interactions.[]

Applications

Leadership

According to the social identity approach, leadership is a function of the group instead of the individual.[1][] Individuals who are leaders in their groups tend to be closer to the prototypical group member than are followers.[4] Additionally, they tend to be more socially attractive, which makes it easier for group members to accept their authority and comply with their decisions. Finally, leaders tend to be viewed by others as the leader. In this final distinction, group members attribute leadership traits to the person and not the situation, furthering the distinction between the leader and others in the group by viewing him or her as special.[5] Consistent with this view of leadership, researchers have found that individuals can manipulate their own leadership status in groups by portraying themselves as prototypical to the group.[6]

Economics

Social identity concepts have been applied to economics resulting in what is now known as identity economics.[7][8] For example, two separate papers and a book by Akerlof and Kranton incorporate social identity as a factor in the principal–agent model. The main conclusion is that when agents consider themselves insiders, they will maximize their identity utility by exerting greater effort compared to the prescription behavior. On the other hand, if they consider themselves outsiders, they will require a higher wage to compensate their loss for behavior difference with prescribed behaviors.[9][10][8]

Related theoretical work

Social identity model of deindividuation effects

The social identity model of deindividuation effects (SIDE) was developed from further research on the social identity theory and the self-categorization theory, further specifying the effects of situational factors on the functioning of processes proposed by the two theories. The SIDE model uses this framework to explain cognitive effects of visibility and anonymity in intra-group and inter-group contexts. The model is based on the idea that the self-concept is flexible and different in different situations or contexts. The theory consists of a range of different self-categories that define people as unique individuals or in terms of their membership to specific social groups and other, broader social categories based on the context of the situation. The SIDE model proposes that anonymity shifts both the focus of self-awareness from the individual self to the group self and the perceptions of others from being mostly interpersonal to being group-based (stereotyping).[]

Research has suggested that visual anonymity not only increases negative behavior towards others, but also can also promote positive social relations. In one study, all volunteers participated individually in group discussion based on three different topics. In the visually anonymous condition, all communications between participants were text-based while in the visually identifiable condition, the communication was also supplemented by two-way video cameras. The study resulted in the findings that showed anonymity significantly increased group attraction.[]

Intergroup emotion theory

Intergroup emotion theory further expands on the concept of personally significant group memberships as posed by social identity and self-categorization theories. This theory is primarily based on the concept of depersonalization and the interchangeability of the self with other ingroup members. This causes cognitive representations of the self and the group to become inevitably connected, and therefore the group obtains an emotional significance. This means that individuals not only categorize themselves as members of the ingroup but also "react emotionally when situations or events affect the ingroup".[11] For example, people often report that their group is being discriminated against, even though they feel that they personally are not subject to that discrimination.[12]

Controversies

Social identity vs. interdependence

Some researchers have claimed that the majority of results in research using the minimal group paradigm can be derived from self-interest and interdependence and that this poses a serious problem for social identity theory and self-categorization theory, and in particular self-categorization theory's account of social groups.[13][] Social identity researchers have responded by suggesting that the interdependence centric analysis that has been proposed as an alternative is inconsistent and still relies heavily on the social categorization processes detailed in self-categorization theory.[14][][] Moreover, they argue that researchers making the above criticisms have also significantly misinterpreted the role of sociological categories in the two theories.[14][]

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Assortativity

Assortativity, or assortative mixing is a preference for a network's nodes to attach to others that are similar in some way. Though the specific measure of similarity may vary, network theorists often examine assortativity in terms of a node's degree.[1] The addition of this characteristic to network models more closely approximates the behaviors of many real world networks.

Correlations between nodes of similar degree are often found in the mixing patterns of many observable networks. For instance, in social networks, highly connected nodes tend to be connected with other high degree nodes. This tendency is referred to as assortative mixing, or *assortativity*. On the other hand, technological and biological networks typically show disassortative mixing, or *dissortativity*, as high degree nodes tend to attach to low degree nodes.[2]

Measuring assortativity

Assortativity is often operationalized as a correlation between two nodes. However, there are several ways to capture such a correlation. The two most prominent measures are the *assortativity coefficient* and the *neighbor connectivity*. These measures are outlined in more detail below.

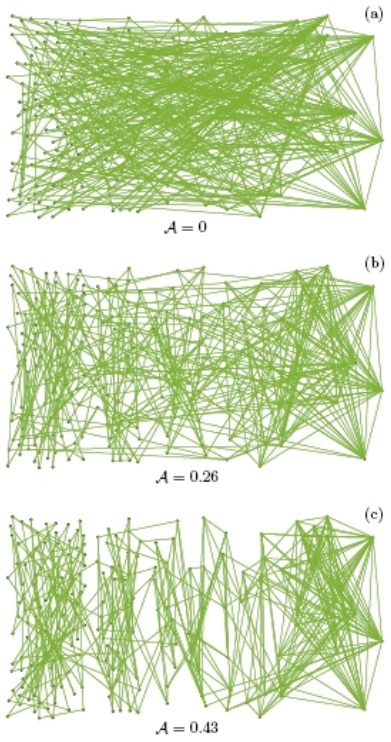

Fig. 1: Scale-free networks for different degrees of assortativity: (a) A = 0 (uncorrelated network), (b) *A* = 0.26, (c) *A* = 0.43, where *A* indicates *r*.[3]

Assortativity coefficient

The *assortativity coefficient* is the Pearson correlation coefficient of degree between pairs of linked nodes.[2] Positive values of *r* indicate a correlation between nodes of similar degree, while negative values indicate relationships between nodes of different degree. In general, *r* lies between −1 and 1. When *r* = 1, the network is said to have perfect assortative mixing patterns, while at *r* = −1 the network is completely disassortative.

The *assortativity coefficient* is given by $r = \frac{\sum_{jk} jk(e_{jk} - q_j q_k)}{\sigma_q^2}$.

The term q_k is the distribution of the *remaining degree*. This captures the number of edges leaving the node, other than the one that connects the pair. The distribution of this term is derived from the degree distribution p_k as $q_k = \frac{(k+1)p_{k+1}}{\sum_j j p_j}$. Finally, e_{jk} refers to the joint probability distribution of the remaining degrees of the two vertices. This quantity is symmetric on an undirected graph, and follows the sum rules $\sum_{jk} e_{jk} = 1$ and $\sum_j e_{jk} = q_k$.

For a directed graph, one has four types of assortativity to consider (see[4]). Adopting the notation of that article, we define four statistics $r(\text{in}, \text{in})$, $r(\text{in}, \text{out})$, $r(\text{out}, \text{in})$, and $r(\text{out}, \text{out})$. Let (α, β), be one of the *in*/*out* word pairs (e.g. $(\alpha,$ Suppose we label the edges of the network $1, \ldots, E$. Given edge i, let j_i^α be the α-degree of the source (i.e. *tail*) node vertex of edge,

and k_i^β be the β-degree of the target (i.e. *head*) node of edge i. We indicate average values with bars, so that $\bar{j^\alpha}$, and $\bar{k^\beta}$ are the average α-degree of sources, and β-degree of targets, respectively; averages being taken over the edges of the network.

Finally, we have

$$r(\alpha,\beta)=\frac{\sum_i(j_i^\alpha-\bar{j^\alpha})(k_i^\beta-\bar{k^\beta})}{\sqrt{\sum_i(j_i^\alpha-\bar{j^\alpha})^2}\sqrt{\sum_i(k_i^\beta-\bar{k^\beta})^2}}.$$

Neighbor connectivity

Another means of capturing the degree correlation is by examining the properties of $\langle k_{nn}\rangle$, or the average degree of neighbors of a node with degree *k*.[5] This term is formally defined as: $\langle k_{nn}\rangle=\sum_{k'}k'P(k'|k)$, where $P(k'|k)$ is the conditional probability that an edge of node degree *k* points to a node with degree *k*'. If this function is increasing, the network is assortative, since it shows that nodes of high degree connect, on average, to nodes of high degree. Alternatively, if the function is decreasing, the network is dissortative, since nodes of high degree tend to connect to nodes of lower degree. The function can be plotted on a graph (see Fig. 2) to depict the overall assortativity trend for a network.

Assortative mixing patterns of real networks

The assortative patterns of a variety of real world networks have been examined. For instance, Fig. 3 lists values of *r* for a variety of networks. Note that the social networks (the first five entries) have apparent assortative mixing. On the other hand, the technological and biological networks (the middle six entries) all appear to be disassortative. It has been suggested that this is because most networks have a tendency to evolve, unless otherwise constrained, towards their maximum entropy state—which is usually disassortative.[6]

The table also has the value of r calculated analytically for two models of networks:

1. the random graph of Erdos and Renyi
2. BA Model (Barabási and Albert model)

In the ER model, since edges are placed at random without regard to vertex degree, it follows that r = 0 in the limit of large graph size. Interestingly, the scale-free BA model also holds this property. For the BA model, $r\to 0$ as $(\log^2 N)/N$ in the limit of large N.[2]

Application

The properties of assortativity are useful in the field of epidemiology, since they can help understand the spread of disease or cures. For instance, the removal of a portion of a network's vertices may correspond to curing, vaccinating, or quarantining individuals or cells. Since social networks demonstrate assortative mixing, diseases targeting high degree individuals are likely to spread to other high degree nodes. Alternatively, within the cellular network—which, as a biological network is likely dissortative—vaccination strategies that specifically target the high degree vertices may quickly destroy the epidemic network.

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Homophily

Homophily (i.e., "love of the same") is the tendency of individuals to associate and bond with similar others. The presence of homophily has been discovered in a vast array of network studies. More than 100 studies that have observed homophily in some form or another and they establish that similarity breeds connection.[1] These include age, gender, class, and organizational role.[]

This is often expressed in the adage "birds of a feather flock together".

Individuals in homophilic relationships share common characteristics (beliefs, values, education, etc.) that make communication and relationship formation easier. Homophily often leads to homogamy—marriage between people with similar characteristics.[1]

Types of homophily

In their original formulation of homophily, Lazarsfeld and Merton (1954) distinguished between *status homophily*, which contends that individuals with similar social status characteristics are more likely to associate with each other than by chance, and *value homophily*, which refers to tendency to associate with others who think in similar ways, regardless of differences in status.[2]

To test the relevance of homophily researchers have distinguished between *baseline homophily* and *inbreeding homophily*. The former is simply the amount of homophily that would be expected by chance and the second is the amount of homophily over and above this expected value.

The opposite of homophily is heterophily.

Twitter

Sociologist Dhiraj Murthy in his book *Twitter: Social Communication in the Twitter Age* (2012) extends the topic of homophily to Twitter, exploring whether the social platform contributes to “cross-talk” by exposing users to differing worldviews or whether it reinforces existing social structures.[] In the chapter called “Theorizing Twitter”, he notes that while Twitter does promote the exchange of differing perspective ideas across cultures people tend to follow tweets of people similar to themselves, their thoughts and values, rather than following tweets of people dissimilar to them.[]

Murthy notes that while research has shown a positive correlation between homophily and information diffusion, Twitter has perhaps broken the “grip” homophily with hashtags being able to link people together.

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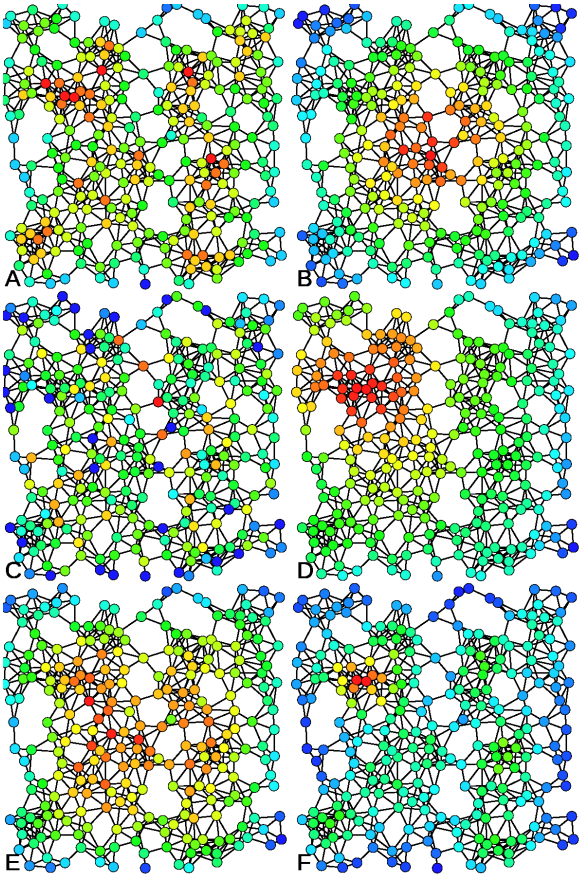
Centrality

Within the scope of graph theory and network analysis, there are various types of measures of the **centrality** of a vertex within a graph that determine the relative importance of a vertex within the graph (i.e. how influential a person is within a social network, or, in the theory of space syntax, how important a room is within a building or how well-used a road is within an urban network). Many of the centrality concepts were first developed in social network analysis, and many of the terms used to measure centrality reflect their sociological origin.[1]

There are four measures of centrality that are widely used in network analysis: degree centrality, betweenness, closeness, and eigenvector centrality. For a review as well as generalizations to weighted networks, see Opsahl et al. (2010).[2]



Examples of A) Degree centrality, B) Closeness centrality, C) Betweenness centrality, D) Eigenvector centrality, E) Katz centrality and F) Alpha centrality of the same graph.

Degree centrality

Historically first and conceptually simplest is **degree centrality**, which is defined as the number of links incident upon a node (i.e., the number of ties that a node has). The degree can be interpreted in terms of the immediate risk of a node for catching whatever is flowing through the network (such as a virus, or some information). In the case of a directed network (where ties have direction), we usually define two separate measures of degree centrality, namely indegree and outdegree. Accordingly, indegree is a count of the number of ties directed to the node and outdegree is the number of ties that the node directs to others. When ties are associated to some positive aspects such as friendship or collaboration, indegree is often interpreted as a form of popularity, and outdegree as gregariousness.

The degree centrality of a vertex v, for a given graph $G := (V, E)$ with $|V|$ vertices and $|E|$ edges, is defined as

$$C_D(v) = \deg(v)$$

Calculating degree centrality for all the nodes in a graph takes $\Theta(V^2)$ in a dense adjacency matrix representation of the graph, and for edges takes $\Theta(E)$ in a sparse matrix representation.

Sometimes the interest is in finding the centrality of a graph within a graph. The definition of centrality on the node level can be extended to the whole graph. Let $v*$ be the node with highest degree centrality in G. Let

$X := (Y, Z)$ be the Y node connected graph that maximizes the following quantity (with $y*$ being the node with highest centrality in X):

$$H = \sum_{j=1}^{|Y|} C_D(y*) - C_D(y_j)$$

Correspondingly, the degree centrality of the graph G is as follows:

$$C_D(G) = \frac{\sum_{i=1}^{|V|} [C_D(v*) - C_D(v_i)]}{H}$$

The value of H is maximized when the graph X contains one central node to which all other nodes are connected (a star graph), and in this case $H = (n-1)(n-2)$.

Closeness centrality

In connected graphs there is a natural distance metric between all pairs of nodes, defined by the length of their shortest paths. The **farness** of a node *s* is defined as the sum of its distances to all other nodes, and its closeness is defined as the inverse of the farness.[3] Thus, the more central a node is the lower its total distance to all other nodes. Closeness can be regarded as a measure of how fast it will take to spread information from *s* to all other nodes sequentially.[4]

In the classic definition of the closeness centrality, the spread of information is modeled by the use of shortest paths. This model might not be the most realistic for all types of communication scenarios. Thus, related definitions have been discussed to measure closeness, like the random walk closeness centrality introduced by Noh and Rieger (2004). It measures the speed with which randomly walking messages reach a vertex from elsewhere in the network—a sort of random-walk version of closeness centrality.[5]

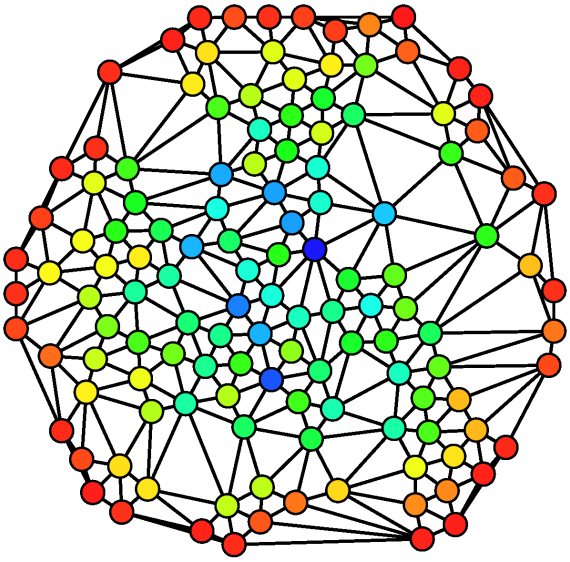
The *information centrality* of Stephenson and Zelen (1989) is another closeness measure, which bears some similarity to that of Noh and Rieger. In essence it measures the harmonic mean length of paths ending at a vertex $\mathbf{i}$, which is smaller if $\mathbf{i}$ has many short paths connecting it to other vertices.[6]

Note that by definition of graph theoretic distances, the classic closeness centrality of all nodes in an unconnected graph would be *0*. In a work by Dangalchev (2006) relating network vulnerability, the definition for closeness is modified such that it can be calculated more easily and can be also applied to graphs which lack connectivity [7]:

$$C_C(v) = \sum_{t \in V \setminus v} 2^{-d_G(v,t)}.$$

Another extension to networks with disconnected components has been proposed by Opsahl (2010).[]

Betweenness centrality



Hue (from red=0 to blue=max) shows the node betweenness.

Betweenness is a centrality measure of a vertex within a graph (there is also edge betweenness, which is not discussed here). Betweenness centrality quantifies the number of times a node acts as a bridge along the shortest path between two other nodes. It was introduced as a measure for quantifying the control of a human on the communication between other humans in a social network by Linton Freeman.[] In his conception, vertices that have a high probability to occur on a randomly chosen shortest path between two randomly chosen vertices have a high betweenness.

The betweenness of a vertex v in a graph $G := (V, E)$ with V vertices is computed as follows:

1. For each pair of vertices (*s*,*t*), compute the shortest paths between them.

2. For each pair of vertices (*s*,*t*), determine the fraction of shortest paths that pass through the vertex in question (here, vertex *v*).

3. Sum this fraction over all pairs of vertices (*s*,*t*).

More compactly the betweenness can be represented as [8]:

$$C_B(v) = \sum_{s \neq v \neq t \in V} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

where σ_{st} is total number of shortest paths from node s to node t and $\sigma_{st}(v)$ is the number of those paths that pass through v. The betweenness may be normalised by dividing through the number of pairs of vertices not including *v*, which for directed graphs is $(n-1)(n-2)$ and for undirected graphs is $(n-1)(n-2)/2$. For example, in an undirected star graph, the center vertex (which is contained in every possible shortest path) would have a betweenness of $(n-1)(n-2)/2$ (1, if normalised) while the leaves (which are contained in no shortest paths) would have a betweenness of 0.

From a calculation aspect, both betweenness and closeness centralities of all vertices in a graph involve calculating the shortest paths between all pairs of vertices on a graph, which requires $\Theta(V^3)$ time with the Floyd–Warshall algorithm. However, on sparse graphs, Johnson's algorithm may be more efficient, taking $O(V^2 \log V + VE)$ time. In the case of unweighted graphs the calculations can be done with Brandes' algorithm[8] which takes $O(VE)$ time. Normally, these algorithms assume that graphs are undirected and connected with the allowance of loops and multiple edges. When specifically dealing with network graphs, oftentimes graphs are without loops or multiple edges to maintain simple relationships (where edges represent connections between two people or vertices). In this case, using Brandes' algorithm will divide final centrality scores by 2 to account for each shortest path being counted twice.[8]

Eigenvector centrality

Eigenvector centrality is a measure of the influence of a node in a network. It assigns relative scores to all nodes in the network based on the concept that connections to high-scoring nodes contribute more to the score of the node in question than equal connections to low-scoring nodes. Google's PageRank is a variant of the Eigenvector centrality measure.[9] Another closely related centrality measure is Katz centrality.

Using the adjacency matrix to find eigenvector centrality

For a given graph $G := (V, E)$ with |V| number of vertices let $A = (a_{v,t})$ be the adjacency matrix, i.e. $a_{v,t} = 1$ if vertex v is linked to vertex t, and $a_{v,t} = 0$ otherwise. The centrality score of vertex v can be defined as:

$$x_v = \frac{1}{\lambda} \sum_{t \in M(v)} x_t = \frac{1}{\lambda} \sum_{t \in G} a_{v,t} x_t$$

where $M(v)$ is a set of the neighbors of v and λ is a constant. With a small rearrangement this can be rewritten in vector notation as the eigenvector equation

$$\mathbf{A}\mathbf{x} = \lambda \mathbf{x}$$

In general, there will be many different eigenvalues λ for which an eigenvector solution exists. However, the additional requirement that all the entries in the eigenvector be positive implies (by the Perron–Frobenius theorem) that only the greatest eigenvalue results in the desired centrality measure.[10] The v^{th} component of the related eigenvector then gives the centrality score of the vertex v in the network. Power iteration is one of many eigenvalue algorithms that may be used to find this dominant eigenvector.[9] Furthermore, this can be generalized so that the entries in *A* can be real numbers representing connection strengths, as in a stochastic matrix.

Katz centrality and PageRank

Katz centrality [11] is a generalization of degree centrality. Degree centrality measures the number of direct neighbors, and Katz centrality measures the number of all nodes that can be connected through a path, while the contribution of a distant node is penalized by an attenuation factor $\alpha \in (0, 1)$. Mathematically, it is defined as $x_i = \sum_{k=1}^{\infty} \sum_{j=1}^{N} \alpha^k (A^k)_{ij}$.

Katz centrality can be viewed as a variant of eigenvector centrality. Another form of Katz centrality is $x_i = \alpha \sum_{j=1}^{N} a_{ij}(x_j + 1)$. Compared to the expression of eigenvector centrality, x_j is replaced by $x_j + 1$.

It is shown that [12] the principal eigenvector (associated with the largest eigenvalue of A, the adjacency matrix) is the limit of Katz centrality as α approaches $1/\lambda$ from below.

PageRank satisfies the following equation $x_i = \alpha \sum_j a_{ji} \frac{x_j}{L(j)} + \frac{1-\alpha}{N}$, where $L(j) = \sum_j a_{ij}$ is the number of neighbors of node j (or number of outbound links in a directed graph). Compared to eigenvector centrality and Katz centrality, one major difference is the scaling factor $L(j)$. Another difference between PageRank and eigenvector centrality is that the PageRank vector is a left hand eigenvector (note the factor a_{ji} has indices reversed). [13]

Definition and characterization of centrality indices

Next to the above named classic centrality indices, there are dozens of other more specialized centrality indices. Despite its intuitive notion there is not yet a definition or characterization of centrality indices which captures all of them.[] A very loose definition of a centrality index is the following:

A centrality index is a real-valued function on the nodes of a graph. It is a structural index, i.e., if G and H are two isomorphic graphs and Φ is the mapping from the vertex set $V(G)$ of G to *V*(*H*), then the centrality of a vertex v of G must be the same as the centrality of $\Phi(v)$ in H . Conventionally, the higher the centrality index of a node, the higher its perceived centrality in the graph.[] This definition comprises all classic centrality measures but not all measures that fulfill this definition would be accepted as centrality indices.

Borgatti and Everett summarize that centrality indices measure the position of a node along a predefined set of walks. They characterize centrality indices along four dimensions: the set of walks, whether the length or the number of these walks is considered, the position of the node on the walks (at the start=radial; in the middle=medial), and how the numbers assigned to the paths are summarized in the measure (average, median, weighted sum, ...).[] This leads to a characterization by the way a centrality index is calculated. In a different characterization, Borgatti differentiates the centrality indices by what type of paths they consider and which type of network flow they imply.[14] The latter characterizes the centrality indices by the quality with which they predict which node is most central for a given network flow process. This characterization thus provides guidance on when to use which centrality index.

Centralization

The *centralization* of any network is a measure of how central its most central node is in relation to how central all the other nodes are.[15] The general definition of centralization for non-weighted networks was proposed by Linton Freeman (1979). Centralization measures then (a) calculate the sum in differences in centrality between the most central node in a network and all other nodes; and (b) divide this quantity by the theoretically largest such sum of differences in any network of the same degree.[15] Thus, every centrality measure can have its own centralization measure. Defined formally, if $C_x(p_i)$ is any centrality measure of point i , if $C_x(p_*)$ is the largest such measure in the network, and if $max \sum_{j=1}^{N} C_x(p_*) - C_x(p_i)$ is the largest sum of differences in point centrality C_x for any graph of with the same number of nodes, then the centralization of the network is:[15]

$$C_x = \frac{\sum_{j=1}^{N} C_x(p_*) - C_x(p_i)}{max \sum_{j=1}^{N} C_x(p_*) - C_x(p_i)}$$

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External links

- https://networkx.lanl.gov/trac/attachment/ticket/119/page_rank.py
- http://www.faculty.ucr.edu/~hanneman/nettext/C10_Centrality.html

Betweenness centrality

Betweenness centrality is a measure of a node's centrality in a network equal to the number of shortest paths from all vertices to all others that pass through that node. Betweenness centrality is a more useful measure of the load placed on the given node in the network as well as the node's importance to the network than just connectivity. The latter is only a local effect while the former is more global to the network. Development of betweenness centrality is generally attributed to sociologist Linton Freeman, who has also developed a number of other centrality measures.[] The same idea was also earlier proposed by mathematician J. Anthonisse, but his work was never published.[1] Over the past few years, betweenness centrality has become a popular stategy to deal with complex networks. Applications include computer and social networks,[][] biology,[][] transport,[][] scientific cooperation[] and so forth.

Definition

The betweenness centrality of a node v is given by the expression:

$$g(v) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

where σ_{st} total number of shortest paths from node s to node t and $\sigma_{st}(v)$ is the number of those paths that pass through v.

Note that the betweenness centrality of a node scales with the number of pairs of nodes as implied by the summation indices. Therefore the calculation may be rescaled by dividing through by the number of pairs of nodes not including v, so that $g \in [0, 1]$. The division is done by $(N-1)(N-2)$ for directed graphs and $(N-1)(N-2)/2$ for undirected graphs, where N is the number of nodes in the giant component. Note that this scales for the highest possible value, where one node is crossed by every single shortest path. This is often not the case, and a normalization can be performed without a loss of precision

$$normal(g(v)) = \frac{g(v) - min(g)}{max(g) - min(g)}$$

which results in:

$$max(normal) = 1$$
$$min(normal) = 0$$

Note that this will always be a scaling from a smaller range into a larger range, so no precision is lost.

The load distribution in real and model networks

Model networks

It has been shown that the load distribution of a scale free network follows a power law given by a load exponent δ [2],

$$P(g) \approx g^{-\delta} \quad (1)$$

this implies the scaling relation to the degree of the node,

$$g(k) \approx k^{\eta}.$$

Where $g(k)$ is the average load of vertices with degree k. The exponents δ and η are not independent since equation (1) implies [3]

$$P(g) = \int dk P(k) \delta(g - k^{\eta})$$

For large g, and therefore large k, the expression becomes

$$P(g \gg 1) = \int dk k^{-\gamma} \delta(g - k^{\eta})$$

$$\sim g^{-1-\frac{\gamma-1}{\eta}}$$

which proves the following equality:

$$\eta = \frac{\gamma - 1}{\delta - 1}$$

The important exponent appears to be η which describes how the betweenness centrality depends on the connectivity. The situation which maximizes the betweenness centrality for a vertex is when all shortest paths are going through it, which corresponds to a tree structure (a network with no clustering). In the case of a tree network the maximum value of $\eta = 2$ is reached.[3]

$$\eta = 2 \rightarrow \delta = \frac{\gamma + 1}{2}$$

This maximal value of η (and hence minimum of δ) puts bounds on the load exponents for networks with non-vanishing clustering.

$$\eta \leq 2 \rightarrow \delta \geq \frac{\gamma + 1}{2}$$

In this case, the exponents δ, η are not universal and depend on the different details (average connectivity, correlations, etc.)

Real networks

Real world scale free networks, such as the internet, also follow a power law load distribution.[4] This is an intuitive result. Scale free networks arrange themselves to create short path lengths across the network by creating a few hub nodes with much higher connectivity than the majority of the network. These hubs will naturally experience much higher loads because of this added connectivity.

Weighted networks

In a weighted network the links connecting the nodes are no longer treated as binary interactions, but are weighted in proportion to their capacity, influence, frequency, etc., which adds another dimension of heterogeneity within the network beyond the topological effects. A node's strength in a weighted network is given by the sum of the weights of its adjacent edges.

$$s_i = \sum_{j=1}^{N} a_{ij} w_{ij}$$

With a_{ij} and w_{ij} being adjacency and weight matricies between nodes i and j, respectively. Analogous to the power law distribution of degree found in scale free networks, the strength of a given node follows a power law distribution as well.

$$s(k) \approx k^{\beta}$$

A study of the average value $s(b)$ of the strength for vertices with betweenness b shows that the functional behavior can be approximated by a scaling form [5]

$$s(b) \approx b^{\alpha}$$

Algorithms

Calculating the betweenness and closeness centralities of all the vertices in a graph involves calculating the shortest paths between all pairs of vertices on a graph. This takes $\Theta(|V|^3)$ time with the Floyd–Warshall algorithm, modified to not only find one but count all shortest paths between two nodes. On a sparse graph, Johnson's algorithm may be more efficient, taking $O(|V|^2 \log |V| + |V||E|)$ time. On unweighted graphs, calculating betweenness centrality takes $O(|V||E|)$ time using Brandes' algorithm.[8]
In calculating betweenness and closeness centralities of all vertices in a graph, it is assumed that graphs are undirected and connected with the allowance of loops and multiple edges. When specifically dealing with network graphs, oftentimes graphs are without loops or multiple edges to maintain simple relationships (where edges represent connections between two people or vertices). In this case, using Brandes' algorithm will divide final centrality scores by 2 to account for each shortest path being counted twice.[8]

Another algorithm generalizes the Freeman's betweenness computed on geodesics and Newman's betweenness computed on all paths, by introducing an hyper-parameter controlling the trade-off between exploration and exploitation. The time complexity is the number of edges times the number of nodes in the graph.[]

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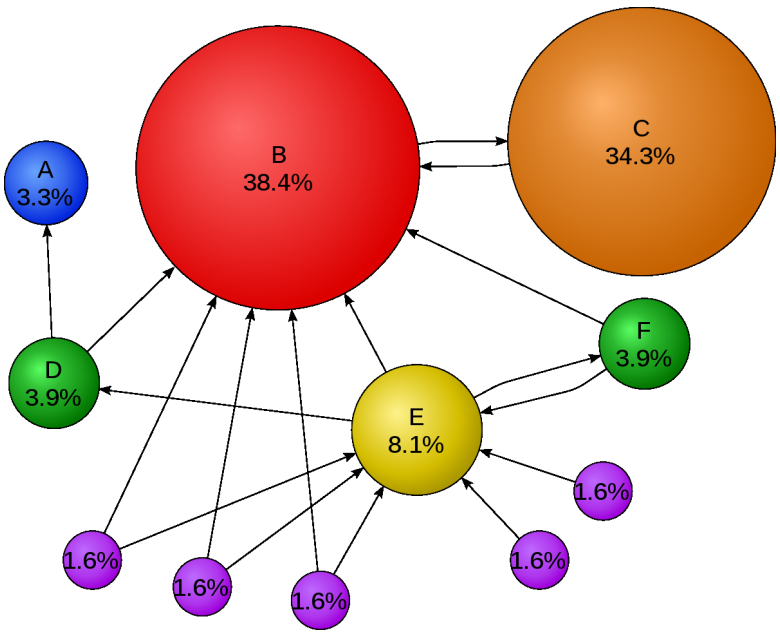
PageRank

PageRank is a link analysis algorithm, named after Larry Page[1] and used by the Google Internet search engine, that assigns a numerical weighting to each element of a hyperlinked set of documents, such as the World Wide Web, with the purpose of "measuring" its relative importance within the set. The algorithm may be applied to any collection of entities with reciprocal quotations and references [related to webpages]. The numerical weight that it assigns to any given element E is referred to as the *PageRank of E* and denoted by $PR(E)$.



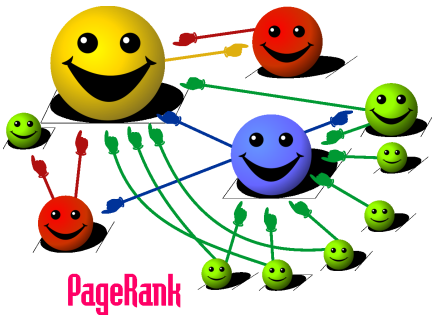
Mathematical **PageRanks** for a simple network, expressed as percentages. (Google uses a logarithmic scale.) Page C has a higher PageRank than Page E, even though there are fewer links to C; the one link to C comes from an important page and hence is of high value. If web surfers who start on a random page have an 85% likelihood of choosing a random link from the page they are currently visiting, and a 15% likelihood of jumping to a page chosen at random from the entire web, they will reach Page E 8.1% of the time. (The 15% likelihood of jumping to an arbitrary page corresponds to a damping factor of 85%.) Without damping, all web surfers would eventually end up on Pages A, B, or C, and all other pages would have PageRank zero. In the presence of damping, Page A effectively links to all pages in the web, even though it has no outgoing links of its own.

The name "PageRank" is a trademark of Google, and the PageRank process has been patented (U.S. Patent 6,285,999 [2]). However, the patent is assigned to Stanford University and not to Google. Google has exclusive license rights on the patent from Stanford University. The university received 1.8 million shares of Google in exchange for use of the patent; the shares were sold in 2005 for $336 million.[3][4]

The value of incoming links is colloquially referred to as "Google juice".

Description

A PageRank results from a mathematical algorithm based on the webgraph, created by all World Wide Web pages as nodes and hyperlinks as edges, taking into consideration authority hubs such as cnn.com or usa.gov. The rank value indicates an importance of a particular page. A hyperlink to a page counts as a vote of support. The PageRank of a page is defined recursively and depends on the number and PageRank metric of all pages that link to it ("incoming links"). A page that is linked to by many pages with high PageRank receives a high rank itself. If there are no links to a web page, then there is no support for that page.



Cartoon illustrating basic principle of PageRank. The size of each face is proportional to the total size of the other faces which are pointing to it.

Numerous academic papers concerning PageRank have been published since Page and Brin's original paper.[] In practice, the PageRank concept may be vulnerable to manipulation. Research has been conducted into identifying falsely influenced PageRank rankings. The goal is to find an effective means of ignoring links from documents with falsely influenced PageRank.Wikipedia:No original research

Other link-based ranking algorithms for Web pages include the HITS algorithm invented by Jon Kleinberg (used by Teoma and now Ask.com)[*citation needed*], the IBM CLEVER project, and the TrustRank algorithm.

History

PageRank was developed at Stanford University by Larry Page (hence the name *Page*-Rank[5]) and Sergey Brin in 1996[6] as part of a research project about a new kind of search engine.[7] Sergey Brin had the idea that information on the web could be ordered in a hierarchy by "link popularity": a page is ranked higher as there are more links to it.[8] It was co-authored by Rajeev Motwani and Terry Winograd. The first paper about the project, describing PageRank and the initial prototype of the Google search engine, was published in 1998:[] shortly after, Page and Brin founded Google Inc., the company behind the Google search engine. While just one of many factors that determine the ranking of Google search results, PageRank continues to provide the basis for all of Google's web search tools.[]

PageRank has been influenced by citation analysis, early developed by Eugene Garfield in the 1950s at the University of Pennsylvania, and by Hyper Search, developed by Massimo Marchiori at the University of Padua. In the same year PageRank was introduced (1998), Jon Kleinberg published his important work on HITS. Google's founders cite Garfield, Marchiori, and Kleinberg in their original paper.[]

A small search engine called "RankDex" from IDD Information Services designed by Robin Li was, since 1996, already exploring a similar strategy for site-scoring and page ranking.[9] The technology in RankDex would be patented by 1999[10] and used later when Li founded Baidu in China.[11][12] Li's work would be referenced by some of Larry Page's U.S. patents for his Google search methods.[13]

Algorithm

PageRank is a probability distribution used to represent the likelihood that a person randomly clicking on links will arrive at any particular page. PageRank can be calculated for collections of documents of any size. It is assumed in several research papers that the distribution is evenly divided among all documents in the collection at the beginning of the computational process. The PageRank computations require several passes, called "iterations", through the collection to adjust approximate PageRank values to more closely reflect the theoretical true value.

A probability is expressed as a numeric value between 0 and 1. A 0.5 probability is commonly expressed as a "50% chance" of something happening. Hence, a PageRank of 0.5 means there is a 50% chance that a person clicking on a random link will be directed to the document with the 0.5 PageRank.

Simplified algorithm

Assume a small universe of four web pages: **A**, **B**, **C** and **D**. Links from a page to itself, or multiple outbound links from one single page to another single page, are ignored. PageRank is initialized to the same value for all pages. In the original form of PageRank, the sum of PageRank over all pages was the total number of pages on the web at that time, so each page in this example would have an initial PageRank of 1. However, later versions of PageRank, and the remainder of this section, assume a probability distribution between 0 and 1. Hence the initial value for each page is 0.25.

The PageRank transferred from a given page to the targets of its outbound links upon the next iteration is divided equally among all outbound links.

If the only links in the system were from pages **B**, **C**, and **D** to **A**, each link would transfer 0.25 PageRank to **A** upon the next iteration, for a total of 0.75.

$$PR(A) = PR(B) + PR(C) + PR(D).$$

Suppose instead that page **B** had a link to pages **C** and **A**, while page **D** had links to all three pages. Thus, upon the next iteration, page **B** would transfer half of its existing value, or 0.125, to page **A** and the other half, or 0.125, to page **C**. Since **D** had three outbound links, it would transfer one third of its existing value, or approximately 0.083, to A.

$$PR(A) = \frac{PR(B)}{2} + \frac{PR(C)}{1} + \frac{PR(D)}{3}.$$

In other words, the PageRank conferred by an outbound link is equal to the document's own PageRank score divided by the number of outbound links **L( )**.

$$PR(A) = \frac{PR(B)}{L(B)} + \frac{PR(C)}{L(C)} + \frac{PR(D)}{L(D)}.$$

In the general case, the PageRank value for any page **u** can be expressed as:

$$PR(u) = \sum_{v \in B_u} \frac{PR(v)}{L(v)},$$

i.e. the PageRank value for a page **u** is dependent on the PageRank values for each page **v** contained in the set $\mathbf{B_u}$ (the set containing all pages linking to page **u**), divided by the number *L*(*v*) of links from page **v**.

Damping factor

The PageRank theory holds that even an imaginary surfer who is randomly clicking on links will eventually stop clicking. The probability, at any step, that the person will continue is a damping factor *d*. Various studies have tested different damping factors, but it is generally assumed that the damping factor will be set around 0.85.[]

The damping factor is subtracted from 1 (and in some variations of the algorithm, the result is divided by the number of documents (*N*) in the collection) and this term is then added to the product of the damping factor and the sum of the incoming PageRank scores. That is,

$$PR(A) = \frac{1-d}{N} + d\left(\frac{PR(B)}{L(B)} + \frac{PR(C)}{L(C)} + \frac{PR(D)}{L(D)} + \cdots\right).$$

So any page's PageRank is derived in large part from the PageRanks of other pages. The damping factor adjusts the derived value downward. The original paper, however, gave the following formula, which has led to some confusion:

$$PR(A) = 1 - d + d\left(\frac{PR(B)}{L(B)} + \frac{PR(C)}{L(C)} + \frac{PR(D)}{L(D)} + \cdots\right).$$

The difference between them is that the PageRank values in the first formula sum to one, while in the second formula each PageRank is multiplied by *N* and the sum becomes *N*. A statement in Page and Brin's paper that "the sum of all PageRanks is one"[] and claims by other Google employees[14] support the first variant of the formula above.

Page and Brin confused the two formulas in their most popular paper "The Anatomy of a Large-Scale Hypertextual Web Search Engine", where they mistakenly claimed that the latter formula formed a probability distribution over web pages.[]

Google recalculates PageRank scores each time it crawls the Web and rebuilds its index. As Google increases the number of documents in its collection, the initial approximation of PageRank decreases for all documents.

The formula uses a model of a *random surfer* who gets bored after several clicks and switches to a random page. The PageRank value of a page reflects the chance that the random surfer will land on that page by clicking on a link. It can be understood as a Markov chain in which the states are pages, and the transitions, which are all equally probable, are the links between pages.

If a page has no links to other pages, it becomes a sink and therefore terminates the random surfing process. If the random surfer arrives at a sink page, it picks another URL at random and continues surfing again.

When calculating PageRank, pages with no outbound links are assumed to link out to all other pages in the collection. Their PageRank scores are therefore divided evenly among all other pages. In other words, to be fair with pages that are not sinks, these random transitions are added to all nodes in the Web, with a residual probability usually set to d = 0.85, estimated from the frequency that an average surfer uses his or her browser's bookmark feature.

So, the equation is as follows:

$$PR(p_i) = \frac{1-d}{N} + d \sum_{p_j \in M(p_i)} \frac{PR(p_j)}{L(p_j)}$$

where $p_1, p_2, \ldots, p_N$ are the pages under consideration, $M(p_i)$ is the set of pages that link to p_i, $L(p_j)$ is the number of outbound links on page p_j, and N is the total number of pages.

The PageRank values are the entries of the dominant eigenvector of the modified adjacency matrix. This makes PageRank a particularly elegant metric: the eigenvector is

$$\mathbf{R} = \begin{bmatrix} PR(p_1) \\ PR(p_2) \\ \vdots \\ PR(p_N) \end{bmatrix}$$

where **R** is the solution of the equation

$$\mathbf{R} = \begin{bmatrix} (1-d)/N \\ (1-d)/N \\ \vdots \\ (1-d)/N \end{bmatrix} + d \begin{bmatrix} \ell(p_1,p_1) & \ell(p_1,p_2) & \cdots & \ell(p_1,p_N) \\ \ell(p_2,p_1) & \ddots & & \vdots \\ \vdots & & \ell(p_i,p_j) & \\ \ell(p_N,p_1) & \cdots & & \ell(p_N,p_N) \end{bmatrix} \mathbf{R}$$

where the adjacency function $\ell(p_i,p_j)$ is 0 if page p_j does not link to p_i, and normalized such that, for each j

$$\sum_{i=1}^{N} \ell(p_i,p_j) = 1,$$

i.e. the elements of each column sum up to 1, so the matrix is a stochastic matrix (for more details see the computation section below). Thus this is a variant of the eigenvector centrality measure used commonly in network analysis.

Because of the large eigengap of the modified adjacency matrix above,[15] the values of the PageRank eigenvector can be approximated to within a high degree of accuracy within only a few iterations.

As a result of Markov theory, it can be shown that the PageRank of a page is the probability of arriving at that page after a large number of clicks. This happens to equal t^{-1} where t is the expectation of the number of clicks (or random jumps) required to get from the page back to itself.

One main disadvantage of PageRank is that it favors older pages. A new page, even a very good one, will not have many links unless it is part of an existing site (a site being a densely connected set of pages, such as Wikipedia).

Several strategies have been proposed to accelerate the computation of PageRank.[16]

Various strategies to manipulate PageRank have been employed in concerted efforts to improve search results rankings and monetize advertising links. These strategies have severely impacted the reliability of the PageRank concept, which purports to determine which documents are actually highly valued by the Web community.

Since December 2007, when it started *actively* penalizing sites selling paid text links, Google has combatted link farms and other schemes designed to artificially inflate PageRank. How Google identifies link farms and other PageRank manipulation tools is among Google's trade secrets.

Computation

PageRank can be computed either iteratively or algebraically. The iterative method can be viewed as the power iteration method[17][18] or the power method. The basic mathematical operations performed are identical.

Iterative

At $t = 0$, an initial probability distribution is assumed, usually

$$PR(p_i; 0) = \frac{1}{N}.$$

At each time step, the computation, as detailed above, yields

$$PR(p_i; t+1) = \frac{1-d}{N} + d \sum_{p_j \in M(p_i)} \frac{PR(p_j; t)}{L(p_j)},$$

or in matrix notation

$$\mathbf{R}(t+1) = d\mathcal{M}\mathbf{R}(t) + \frac{1-d}{N}\mathbf{1}, \qquad (*)$$

where $\mathbf{R}_i(t) = PR(p_i; t)$ and $\mathbf{1}$ is the column vector of length N containing only ones.

The matrix $\mathcal{M}$ is defined as

$$\mathcal{M}_{ij} = \begin{cases} 1/L(p_j), & \text{if } j \text{ links to } i \\ 0, & \text{otherwise} \end{cases}$$

i.e.,

$$\mathcal{M} := (K^{-1}A)^T,$$

where A denotes the adjacency matrix of the graph and K is the diagonal matrix with the outdegrees in the diagonal.

The computation ends when for some small ϵ

$$|\mathbf{R}(t+1) - \mathbf{R}(t)| < \epsilon\,,$$

i.e., when convergence is assumed.

Algebraic

For $t \to \infty$ (i.e., in the steady state), the above equation (*) reads

$$\mathbf{R} = d\mathcal{M}\mathbf{R} + \frac{1-d}{N}\mathbf{1}. \qquad (**)$$

The solution is given by

$$\mathbf{R} = (\mathbf{I} - d\mathcal{M})^{-1}\frac{1-d}{N}\mathbf{1},$$

with the identity matrix $\mathbf{I}$.

The solution exists and is unique for $0 < d < 1$. This can be seen by noting that $\mathcal{M}$ is by construction a stochastic matrix and hence has an eigenvalue equal to one as a consequence of the Perron–Frobenius theorem.

Power Method

If the matrix $\mathcal{M}$ is a transition probability, i.e., column-stochastic with no columns consisting of just zeros and $\mathbf{R}$ is a probability distribution (i.e., $|\mathbf{R}| = 1, \mathbf{E}\mathbf{R} = 1$ where $\mathbf{E}$ is matrix of all ones), Eq. (**) is equivalent to

$$\mathbf{R} = \left(d\mathcal{M} + \frac{1-d}{N}\mathbf{E} \right) \mathbf{R} =: \widehat{\mathcal{M}}\mathbf{R}. \qquad (***)$$

Hence PageRank $\mathbf{R}$ is the principal eigenvector of $\widehat{\mathcal{M}}$. A fast and easy way to compute this is using the power method: starting with an arbitrary vector $x(0)$, the operator $\widehat{\mathcal{M}}$ is applied in succession, i.e.,

$$x(t+1) = \widehat{\mathcal{M}}x(t),$$

until

$$|x(t+1) - x(t)| < \epsilon.$$

Note that in Eq. (***) the matrix on the right-hand side in the parenthesis can be interpreted as

$$\frac{1-d}{N}\mathbf{I} = (1-d)\mathbf{P}\mathbf{1}^t,$$

where $\mathbf{P}$ is an initial probability distribution. In the current case

$$\mathbf{P} := \frac{1}{N}\mathbf{1}.$$

Finally, if $\mathcal{M}$ has columns with only zero values, they should be replaced with the initial probability vector $\mathbf{P}$. In other words

$$\mathcal{M}' := \mathcal{M} + \mathcal{D},$$

where the matrix $\mathcal{D}$ is defined as

$$\mathcal{D} := \mathbf{P}\mathbf{D}^t,$$

with

$$\mathbf{D}_i = \begin{cases} 1, & \text{if } L(p_i) = 0 \\ 0, & \text{otherwise} \end{cases}$$

In this case, the above two computations using $\mathcal{M}$ only give the same PageRank if their results are normalized:

$$\mathbf{R}_{\text{power}} = \frac{\mathbf{R}_{\text{iterative}}}{|\mathbf{R}_{\text{iterative}}|} = \frac{\mathbf{R}_{\text{algebraic}}}{|\mathbf{R}_{\text{algebraic}}|}.$$

PageRank MATLAB/Octave implementation

```
% Parameter M adjacency matrix where M_i,j represents the link from 'j'
 to 'i', such that for all 'j' sum(i, M_i,j) = 1
% Parameter d damping factor
% Parameter v_quadratic_error quadratic error for v
% Return v, a vector of ranks such that v_i is the i-th rank from [0,
1]

function [v] = rank(M, d, v_quadratic_error)

N = size(M, 2); % N is equal to half the size of M
v = rand(N, 1);
v = v ./ norm(v, 2);
last_v = ones(N, 1) * inf;
M_hat = (d .* M) + (((1 - d) / N) .* ones(N, N));
```

```
while(norm(v - last_v, 2) > v_quadratic_error)
      last_v = v;
      v = M_hat * v;
      v = v ./ norm(v, 2);
end

endfunction
```

Example of code calling the rank function defined above:

```
M = [0 0 0 0 1 ; 0.5 0 0 0 0 ; 0.5 0 0 0 0 ; 0 1 0.5 0 0 ; 0 0 0.5 1 0];
rank(M, 0.80, 0.001)
```

This example takes 13 iterations to converge.

Efficiency

Depending on the framework used to perform the computation, the exact implementation of the methods, and the required accuracy of the result, the computation time of the these methods can vary greatly.

Variations

PageRank of an undirected graph

The PageRank of an undirected graph G is statistically close to the degree distribution of the graph G,[19] but they are generally not identical: If R is the PageRank vector defined above, and D is the degree distribution vector

$$D = \frac{1}{2|E|}\begin{bmatrix} deg(p_1) \\ deg(p_2) \\ \vdots \\ deg(p_N) \end{bmatrix}$$

where $deg(p_i)$ denotes the degree of vertex p_i, and E is the edge-set of the graph, then, with $Y = \frac{1}{N}\mathbf{1}$, by:[20]

$$\frac{1-d}{1+d}\|Y - D\|_1 \leq \|R - D\|_1 \leq \|Y - D\|_1,$$

that is, the PageRank of an undirected graph equals to the degree distribution vector if and only if the graph is regular, i.e., every vertex has the same degree.

Distributed Algorithm for PageRank Computation

There are simple and fast random walk-based distributed algorithms for computing PageRank of nodes in a network.[21] They present a simple algorithm that takes $O(\log n/\epsilon)$ rounds with high probability on any graph (directed or undirected), where n is the network size and ϵ is the reset probability ($1 - \epsilon$ is also called as damping factor) used in the PageRank computation. They also present a faster algorithm that takes $O(\sqrt{\log n}/\epsilon)$ rounds in undirected graphs. Both of the above algorithms are scalable, as each node processes and sends only small (polylogarithmic in n, the network size) number of bits per round. For directed graphs, they present an algorithm that has a running time of $O(\sqrt{\log n/\epsilon})$, but it requires a polynomial number of bits to processed and sent per node in a round.

Google Toolbar

The Google Toolbar's PageRank feature displays a visited page's PageRank as a whole number between 0 and 10. The most popular websites have a PageRank of 10. The least have a PageRank of 0. Google has not disclosed the specific method for determining a Toolbar PageRank value, which is to be considered only a rough indication of the value of a website.

PageRank measures the number of sites that link to a particular page.[22] The PageRank of a particular page is roughly based upon the quantity of inbound links as well as the PageRank of the pages providing the links. The algorithm also includes other factors, such as the size of a page, the number of changes, the time since the page was updated, the text in headlines and the text in hyperlinked anchor texts.[8]

The Google Toolbar's PageRank is updated infrequently, so the values it shows are often out of date.

SERP Rank

The search engine results page (SERP) is the actual result returned by a search engine in response to a keyword query. The SERP consists of a list of links to web pages with associated text snippets. The SERP rank of a web page refers to the placement of the corresponding link on the SERP, where higher placement means higher SERP rank. The SERP rank of a web page is a function not only of its PageRank, but of a relatively large and continuously adjusted set of factors (over 200),.[23] Search engine optimization (SEO) is aimed at influencing the SERP rank for a website or a set of web pages.

After the introduction of Google Places into the mainstream organic SERP, PageRank played little to no role in ranking a business in the Local Business Results.[24] While the theory of citations still plays a role in the algorithm, PageRank is not a factor since business listings, rather than web pages, are ranked.

Google directory PageRank

The Google Directory PageRank is an 8-unit measurement. Unlike the Google Toolbar, which shows a numeric PageRank value upon mouseover of the green bar, the Google Directory only displays the bar, never the numeric values.

False or spoofed PageRank

In the past, the PageRank shown in the Toolbar was easily manipulated. Redirection from one page to another, either via a HTTP 302 response or a "Refresh" meta tag, caused the source page to acquire the PageRank of the destination page. Hence, a new page with PR 0 and no incoming links could have acquired PR 10 by redirecting to the Google home page. This spoofing technique, also known as 302 Google Jacking, was a known vulnerability. Spoofing can generally be detected by performing a Google search for a source URL; if the URL of an entirely different site is displayed in the results, the latter URL may represent the destination of a redirection.

Manipulating PageRank

For search engine optimization purposes, some companies offer to sell high PageRank links to webmasters.[] As links from higher-PR pages are believed to be more valuable, they tend to be more expensive. It can be an effective and viable marketing strategy to buy link advertisements on content pages of quality and relevant sites to drive traffic and increase a webmaster's link popularity. However, Google has publicly warned webmasters that if they are or were discovered to be selling links for the purpose of conferring PageRank and reputation, their links will be devalued (ignored in the calculation of other pages' PageRanks). The practice of buying and selling links is intensely debated across the Webmaster community. Google advises webmasters to use the nofollow HTML attribute value on sponsored links. According to Matt Cutts, Google is concerned about webmasters who try to game the system, and thereby reduce the quality and relevance of Google search results.[]

The intentional surfer model

The original PageRank algorithm reflects the so-called random surfer model, meaning that the PageRank of a particular page is derived from the theoretical probability of visiting that page when clicking on links at random. A page ranking model that reflects the importance of a particular page as a function of how many actual visits it receives by real users is called the *intentional surfer model*.[] The Google toolbar sends information to Google for every page visited, and thereby provides a basis for computing PageRank based on the intentional surfer model. The introduction of the nofollow attribute by Google to combat Spamdexing has the side effect that webmasters commonly use it on outgoing links to increase their own PageRank. This causes a loss of actual links for the Web crawlers to follow, thereby making the original PageRank algorithm based on the random surfer model potentially unreliable. Using information about users' browsing habits provided by the Google toolbar partly compensates for the loss of information caused by the nofollow attribute. The SERP rank of a page, which determines a page's actual placement in the search results, is based on a combination of the random surfer model (PageRank) and the intentional surfer model (browsing habits) in addition to other factors.[]

Other uses

A version of PageRank has recently been proposed as a replacement for the traditional Institute for Scientific Information (ISI) impact factor,[25] and implemented at eigenfactor.org [26]. Instead of merely counting total citation to a journal, the "importance" of each citation is determined in a PageRank fashion.

A similar new use of PageRank is to rank academic doctoral programs based on their records of placing their graduates in faculty positions. In PageRank terms, academic departments link to each other by hiring their faculty from each other (and from themselves).[27]

PageRank has been used to rank spaces or streets to predict how many people (pedestrians or vehicles) come to the individual spaces or streets.[28][29] In lexical semantics it has been used to perform Word Sense Disambiguation[30] and to automatically rank WordNet synsets according to how strongly they possess a given semantic property, such as positivity or negativity.[31]

A dynamic weighting method similar to PageRank has been used to generate customized reading lists based on the link structure of Wikipedia.[32]

A Web crawler may use PageRank as one of a number of importance metrics it uses to determine which URL to visit during a crawl of the web. One of the early working papers [33] that were used in the creation of Google is *Efficient crawling through URL ordering*,[34] which discusses the use of a number of different importance metrics to determine how deeply, and how much of a site Google will crawl. PageRank is presented as one of a number of these importance metrics, though there are others listed such as the number of inbound and outbound links for a URL, and the distance from the root directory on a site to the URL.

The PageRank may also be used as a methodology [35] to measure the apparent impact of a community like the Blogosphere on the overall Web itself. This approach uses therefore the PageRank to measure the distribution of attention in reflection of the Scale-free network paradigm.

In any ecosystem, a modified version of PageRank may be used to determine species that are essential to the continuing health of the environment.[36]

An application of PageRank to the analysis of protein networks in biology is reported recently.[37]

How to increase page rank

Page Rank is one of the important factors in Google's SERP rankings. To increase the PageRank of a website, get back links from popular, relevant niche sites. Where links from the relevant niche sites helps in getting good rank in SERP, there the back links from the high PR sites help in attaining high PageRank.

nofollow

In early 2005, Google implemented a new value, "nofollow",[38] for the rel attribute of HTML link and anchor elements, so that website developers and bloggers can make links that Google will not consider for the purposes of PageRank—they are links that no longer constitute a "vote" in the PageRank system. The nofollow relationship was added in an attempt to help combat spamdexing.

As an example, people could previously create many message-board posts with links to their website to artificially inflate their PageRank. With the nofollow value, message-board administrators can modify their code to automatically insert "rel='nofollow'" to all hyperlinks in posts, thus preventing PageRank from being affected by those particular posts. This method of avoidance, however, also has various drawbacks, such as reducing the link value of legitimate comments. (See: Spam in blogs#nofollow)

In an effort to manually control the flow of PageRank among pages within a website, many webmasters practice what is known as PageRank Sculpting[39]—which is the act of strategically placing the nofollow attribute on certain internal links of a website in order to funnel PageRank towards those pages the webmaster deemed most important. This tactic has been used since the inception of the nofollow attribute, but may no longer be effective since Google announced that blocking PageRank transfer with nofollow does not redirect that PageRank to other links.[40]

Deprecation

PageRank was once available for the verified site maintainers through the Google Webmaster Tools interface. However on October 15, 2009, a Google employee confirmed[] that the company had removed PageRank from its *Webmaster Tools* section, explaining that "We've been telling people for a long time that they shouldn't focus on PageRank so much. Many site owners seem to think it's the most important metric for them to track, which is simply not true."[] In addition, The PageRank indicator is not available in Google's own Chrome browser.

The visible page rank is updated very infrequently.[41]

On 6 October 2011, many users mistakenly thought Google PageRank was gone. As it turns out, it was simply an update to the URL used to query the PageRank from Google.[]

Google now also relies on other strategies as well as PageRank, such as Google Panda.[42]

Notes

[2] http://www.google.com/patents?vid=6285999

[7] Page, Larry, "PageRank: Bringing Order to the Web" (http://web.archive.org/web/20020506051802/www-diglib.stanford.edu/cgi-bin/WP/get/SIDL-WP-1997-0072?1), Stanford Digital Library Project, talk. August 18, 1997 (archived 2002)

[8] 187-page study from Graz University, Austria (http://www.google-watch.org/gpower.pdf), includes the note that also human brains are used when determining the page rank in Google

[10] USPTO, "Hypertext Document Retrieval System and Method" (http://www.google.com/patents?hl=en&lr=&vid=USPAT5920859&id=x04ZAAAAEBAJ&oi=fnd&dq=yanhong+li&printsec=abstract#v=onepage&q=yanhong li&f=false), U.S. Patent number: 5920859, Inventor: Yanhong Li, Filing date: Feb 5, 1997, Issue date: Jul 6, 1999

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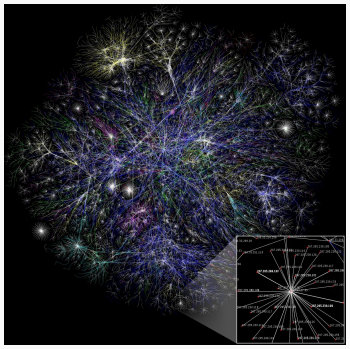
Random graph

Network science

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In mathematics, a **random graph** is a graph that is generated by some random process.[1] The theory of random graphs lies at the intersection between graph theory and probability theory, and studies the properties of typical random graphs.

Random graph models

A random graph is obtained by starting with a set of *n* vertices and adding edges between them at random. Different **random graph models** produce different probability distributions on graphs. Most commonly studied is the one proposed by Edgar Gilbert, denoted *G(n,p)*, in which every possible edge occurs independently with probability *p*. A closely related model, the Erdős–Rényi model denoted *G(n,M)*, assigns equal probability to all graphs with exactly *M* edges. The fastest known algorithm for generating the former model is proposed by Nobari *et al.* in.[2] The latter model can be viewed as a snapshot at a particular time (*M*) of the **random graph process** $\tilde{G}_n$, which is a stochastic process that starts with *n* vertices and no edges, and at each step adds one new edge chosen uniformly from the set of missing edges.

If instead we start with an infinite set of vertices, and again let every possible edge occur independently with probability *p*, then we get an object *G* called an **infinite random graph**. Except in the trivial cases when *p* is 0 or 1, such a *G* almost surely has the following property:

> Given any $n + m$ elements $a_1, \ldots, a_n, b_1, \ldots, b_m \in V$, there is a vertex $c \in V$ that is adjacent to each of $a_1, \ldots, a_n$ and is not adjacent to any of $b_1, \ldots, b_m$.

It turns out that if the vertex set is countable then there is, up to isomorphism, only a single graph with this property, namely the Rado graph. Thus any countably infinite random graph is almost surely the Rado graph, which for this reason is sometimes called simply the **random graph**. However, the analogous result is not true for uncountable graphs, of which there are many (nonisomorphic) graphs satisfying the above property.

Another model, which generalizes Gilbert's random graph model, is the **random dot-product model**. A random dot-product graph associates with each vertex a real vector. The probability of an edge *uv* between any vertices *u* and *v* is some function of the dot product **u** • **v** of their respective vectors.

The network probability matrix models random graphs through edge probabilities, which represent the probability $p_{i,j}$ that a given edge $e_{i,j}$ exists for a specified time period. This model is extensible to directed and undirected; weighted and unweighted; and static or dynamic graphs.

For $M \simeq pn$ the two most widely used models, *G(n,M)* and *G(n,p)*, are almost interchangeable.[3]

Random regular graphs form a special case, with properties that may differ from random graphs in general.

Properties of random graphs

The theory of random graphs studies typical properties of random graphs, those that hold with high probability for graphs drawn from a particular distribution. For example, we might ask for a given value of n and p what the probability is that $G(n,p)$ is connected. In studying such questions, researchers often concentrate on the asymptotic behavior of random graphs—the values that various probabilities converge to as n grows very large. Percolation theory characterizes the connectedness of random graphs, especially infinitely large ones.

Percolation is related to the robustness of the graph (called also network). Given a random graph of n nodes and an average degree $\langle k \rangle$. Next we remove randomly a fraction $1-p$ of nodes and leave only a fraction p. There exists a critical percolation threshold $p_c = 1/\langle k \rangle$ below which the network becomes fragmented while above p_c a giant connected component exists [1] [3] [4] [5] [6] [7].

(threshold functions, evolution of G~)

Random graphs are widely used in the probabilistic method, where one tries to prove the existence of graphs with certain properties. The existence of a property on a random graph can often imply, via the famous Szemerédi regularity lemma, the existence of that property on almost all graphs.

Random trees

A random tree is a tree or arborescence that is formed by a stochastic process. Types of random trees include uniform spanning tree, random minimal spanning tree, random binary tree, treap, rapidly exploring random tree, Brownian tree, and random forest.

Interdependent Graphs

Interdependent graphs or networks is a system of coupled networks where nodes of one or more networks depend on nodes in other networks. This dependency is enhanced by the developments in modern technology. Such dependencies may lead to cascading failures between the networks and a relatively small damage can lead to a catastrophic breakdown of the system. Blackouts are a fascinating demonstration of the important role played by the dependencies between networks. A recent study developed a framework to study the cascading failures in an interdependent networks system[8]

History

Random graphs were first defined by Paul Erdős and Alfréd Rényi in their 1959 paper "On Random Graphs"[6] and independently by Gilbert in his paper "Random graphs".[7]

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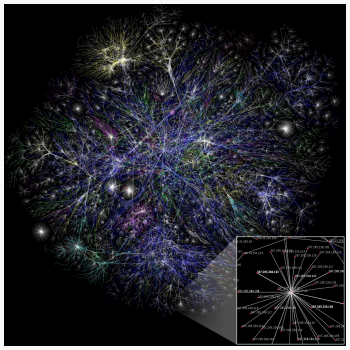
[7] .

Exponential random graph models

Network science

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Exponential random graph models (ERGMs) are a family of statistical models for analyzing data about social and other networks.

Background

Many metrics exist to describe the structural features of an observed network such as the density, centrality, or assortativity.[1][2] However, these metrics describe the observed network which is only one instance of a large number of possible alternative networks. This set of alternative networks may have similar or dissimilar structural features. To support statistical inference on the processes influencing the formation of network structure, a statistical model should consider the set of all possible alternative networks weighted on their similarity to an observed network. However because network data is inherently relational, it violates the assumptions of independence and identical distribution of standard statistical models like linear regression.[] Alternative statistical models should reflect the uncertainty associated with a given observation, permit inference about the relative frequency about network substructures of theoretical interest, disambiguating the influence of confounding processes, efficiently representing complex structures, and linking local-level processes to global-level properties.[]

Definition

Formally a random graph Y consists of a set of n nodes and m dyads $\{Y_{ij} : i = 1, \ldots, n; j = 1, \ldots, n\}$ where $Y_{ij} = 1$ if the nodes (i, j) are connected and $Y_{ij} = 0$ otherwise.
The basic assumption of these models is that the structure in an observed graph y can be explained by any statistics $s(y)$ depending on the observed network and nodal attributes. This way, it is possible to describe any kind of dependence between the dyadic variables:

$$P(Y = y|\theta) = \frac{\exp(\theta^T s(y))}{c(\theta)}$$

where θ is a vector of model parameters associated with $s(y)$ and $c(\theta)$ is a normalising constant.
These models represent a probability distribution on each possible network on n nodes. However, the size of the set of possible networks for an undirected network (simple graph) of size n is $2^{\frac{n(n-1)}{2}}$. Because the number of possible networks in the set vastly outnumbers the number of parameters which can constrain the model, the ideal probability distribution is the one which maximizes the Gibbs entropy.[]

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Modularity (networks)

Modularity is one measure of the structure of networks or graphs. It was designed to measure the strength of division of a network into modules (also called groups, clusters or communities). Networks with high modularity have dense connections between the nodes within modules but sparse connections between nodes in different modules. Modularity is often used in optimisation methods for detecting community structure in networks. However, it has been shown that modularity suffers a resolution limit and, therefore, it is unable to detect small communities.

Motivation

Many scientifically important problems can be represented and empirically studied using networks. For example, biological and social patterns, the World Wide Web, metabolic networks, food webs, neural networks and pathological networks are a few examples of real world problems that can be mathematically represented and topologically studied to reveal some unexpected structural features.[] Most of these networks possess a certain community structure that has substantial importance in building an understanding regarding the dynamics of the network. For instance, a closely connected social community will imply a faster rate of transmission of information or rumor among them than a loosely connected community. Thus, if a network is represented by a number of individual nodes connected by links which signify a certain degree of interaction between the nodes, communities are defined as groups of densely interconnected nodes that are only sparsely connected with the rest of the network. Hence, it may be imperative to identify the communities in networks since the communities may have quite different properties such as node degree, clustering coefficient, betweenness, centrality.[] etc., from that of the average network. Modularity is one such measure, which when maximized; it leads to the appearance of communities in a given network.

Definition

Modularity is the fraction of the edges that fall within the given groups minus the expected such fraction if edges were distributed at random. The value of the modularity lies in the range [−1/2,1). It is positive if the number of edges within groups exceeds the number expected on the basis of chance. For a given division of the network's vertices into some modules, modularity reflects the concentration of nodes within modules compared with random distribution of links between all nodes regardless of modules.

There are different methods for calculating modularity.[] In the most common version of the concept, the randomization of the edges is done so as to preserve the degree of each vertex. Let us consider a graph with n nodes and m links (edges) such that the graph can be partitioned into 2 communities using a membership variable s. If a node i belongs to community 1, $s_i = 1$, or if i belongs to community 2, $s_i = -1$. Let the adjacency matrix for the network be represented by A, where $A_{ij} = 0$ means there's no edge (no interaction) between nodes i and j and $A_{ij} = 1$ means there is an edge between the two. Also for simplicity we consider an undirected network. Thus $A_{ij} = A_{ji}$. (It is important to note that multiple edges may exist between 2 nodes and here we assess the simplest case).
Modularity Q is then defined as the fraction of edges that fall within group 1 or 2, minus the expected number of edges within group 1 and 2 for a random graph with same node degree distribution as the given network.

The expected number of edges shall be computed using the concept of Configuration Models.[] The configuration model is a randomized realization of a particular network. Given a network with *n* nodes, where each node *i* has a node degree k_i, the configuration model cuts each edge into 2 halves, and then each half edge, called a stub, is rewired randomly with any other stub in the network even allowing self loops. Thus, even though the node degree distribution of the graph remains intact, the configuration model results in a completely random network.

Let the total number of stubs be l_n

$$l_n = \sum_i^n k_i = 2m$$

Now, if we randomly select two nodes *i* and *j* with node degrees k_i and k_j respectively and rewire the stubs for these 2 nodes, then,

Expected [Full edges between *i* and *j*] = (Full edges between *i* and *j*)/(Total number of rewirings possible) *(2)*

Total number of rewirings possible = number of stubs remaining after choosing a particular stub = $l_{n-1} = l_n$ for large *n*

Thus, Expected [Number of full edges between *i* and *j*]=$(k_i * k_j)/l_n = (k_i k_j)/2m$.

Hence, the actual number of edges between *i* and *j* minus expected number of edges between them is $A_{ij}-(k_i k_j)/2m$. Thus using []

$$Q = \frac{1}{2m}\sum_{ij}\left[A_{ij} - \frac{k_i * k_j}{2m}\right]\frac{s_i s_j + 1}{2}(3)$$

It is important to note that (3) holds good for partitioning into 2 communities only. Hierarchical partitioning (i.e. partitioning into 2 communities, then the 2 sub-communities further partitioned into 2 smaller sub communities only to maximize *Q*) is a possible approach to identify multiple communities in a network. Additionally, (3) can be generalized for partitioning a network into *c* communities.[]

$$Q = \sum_{ij}\left[\frac{A_{ij}}{2m} - \frac{k_i * k_j}{(2m)(2m)}\right]\delta((c_i, c_j) = \sum_{i=1}^{c}(e_{ii} - a_i^2)(4)$$

where e_{ii} is the fraction of edges with both end vertices in the same community *i*.

$$e_{ii} = \sum_j \frac{A_{ij}}{2m}\delta(c_i, c_j)$$

and a_i is the fraction of edges with at least one end vertex in community *i*.

$$a_i = \frac{k_i}{2m} = \sum_j a_{ij}$$

Example of multiple community detection

We consider an undirected network with 10 nodes and 12 edges and the following adjacency matrix.

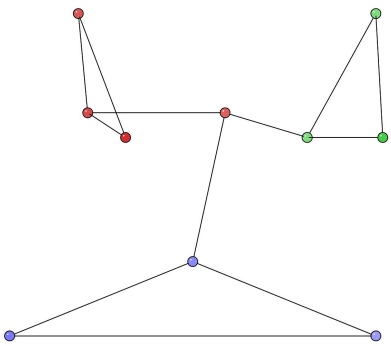

Fig 1. Sample Network corresponding to the Adjacency matrix with 10 nodes, 12 edges.

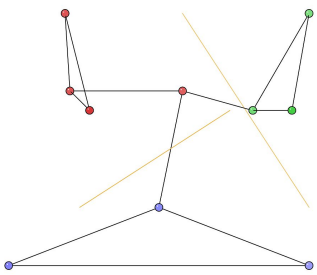

Fig 2. Network partitions that maximize Q. Maximum Q=0.4985

Node ID	1	2	3	4	5	6	7	8	9	10

1	0	1	1	0	0	0	0	0	0	1
2	1	0	1	0	0	0	0	0	0	0
3	1	1	0	0	0	0	0	0	0	0
4	0	0	0	0	1	1	0	0	0	1
5	0	0	0	1	0	1	0	0	0	0
6	0	0	0	1	1	0	0	0	0	0
7	0	0	0	0	0	0	0	1	1	1
8	0	0	0	0	0	0	1	0	1	0
9	0	0	0	0	0	0	1	1	0	0
10	1	0	0	1	0	0	1	0	0	0

The communities in the graph are represented by the red, green and blue node clusters in Fig 1. The optimal community partitions are depicted in Fig 2.

Matrix formulation

An alternative formulation of the modularity, useful particularly in spectral optimization algorithms, is as follows.[] Define S_{ir} to be 1 if vertex i belongs to group r and zero otherwise. Then

$$\delta(c_i, c_j) = \sum_r S_{ir} S_{jr}$$

and hence

$$Q = \frac{1}{2m} \sum_{ij} \sum_r \left[A_{ij} - \frac{k_i k_j}{2m} \right] S_{ir} S_{jr} = \frac{1}{2m} \mathrm{Tr}(\mathbf{S}^{\mathrm{T}} \mathbf{B} \mathbf{S}),$$

where **S** is the (non-square) matrix having elements S_{ir} and **B** is the so-called modularity matrix, which has elements

$$B_{ij} = A_{ij} - \frac{k_i k_j}{2m}.$$

All rows and columns of the modularity matrix sum to zero, which means that the modularity of an undivided network is also always zero.

For networks divided into just two communities, one can alternatively define $s_i = \pm 1$ to indicate the community to which node i belongs, which then leads to

$$Q = \frac{1}{2m} \sum_{ij} B_{ij} s_i s_j = \frac{1}{2m} \mathbf{s}^{\mathrm{T}} \mathbf{B} \mathbf{s},$$

where **s** is the column vector with elements s_i.[]

This function has the same form as the Hamiltonian of an Ising spin glass, a connection that has been exploited to create simple computer algorithms, for instance using simulated annealing, to maximize the modularity. The general form of the modularity for arbitrary numbers of communities is equivalent to a Potts spin glass and similar algorithms can be developed for this case also.[]

Resolution limit

Modularity compares the number of edges inside a cluster with the expected number of edges that one would find in the cluster if the network were a random network with the same number of nodes and where each node keeps its degree, but edges are otherwise randomly attached. This random null model implicitly assumes that each node can get attached to any other node of the network. Such assumption is however unreasonable if the network is very large, as the horizon of a node includes a small part of the network, ignoring most of it. Moreover, this implies that the

expected number of edges between two groups of nodes decreases if the size of the network increases. So, if a network is large enough, the expected number of edges between two groups of nodes in modularity's null model may be smaller than one. If this happens, a single edge between the two clusters would be interpreted by modularity as a sign of a strong correlation between the two clusters, and optimizing modularity would lead to the merge of the two clusters, independently of the clusters' features. So, even weakly interconnected complete graphs, which have the highest possible density of internal edges, and represent the best identifiable communities, would be merged by modularity optimization if the network is sufficiently large.[1] For this reason, optimizing modularity in large networks would fail to resolve small communities, even when they are well defined. This bias is inevitable for methods like modularity optimization, which rely on a global null model.[2]

Multiresolution methods

There are two main approaches which try to solve the resolution limit within the modularity context: the addition of a resistance r to every node, in the form of a self-loop, which increases ($r>0$) or decreases ($r<0$) the aversion of nodes to form communities;[3] or the addition of a parameter $\gamma>0$ in front of the null-case term in the definition of modularity, which controls the relative importance between internal links of the communities and the null model.[] Optimizing modularity for values of these parameters in their respective appropriate ranges, it is possible to recover the whole mesoscale of the network, from the macroscale in which all nodes belong to the same community, to the microscale in which every node forms its own community, thus the name *multiresolution methods*. However, it has been recently demonstrated that these methods are intrinsically deficient and their use will not produce reliable solutions.[4]

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External links

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Erdős–Rényi model

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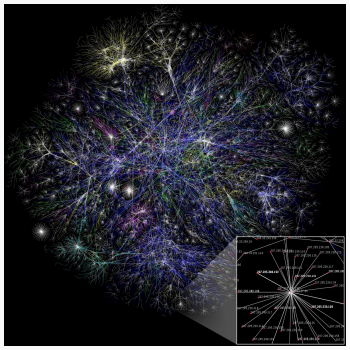
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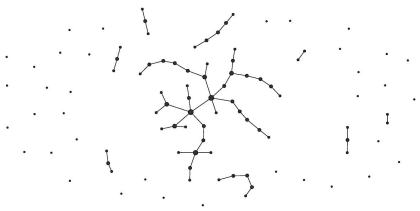
In graph theory, the **Erdős–Rényi model**, named for Paul Erdős and Alfréd Rényi, is either of two models for generating random graphs, including one that sets an edge between each pair of nodes with equal probability, independently of the other edges. It can be used in the probabilistic method to prove the existence of graphs

satisfying various properties, or to provide a rigorous definition of what it means for a property to hold for almost all graphs.

Definition

There are two closely related variants of the Erdős–Rényi (ER) random graph model.

- In the $G(n, M)$ model, a graph is chosen uniformly at random from the collection of all graphs which have n nodes and M edges. For example, in the $G(3, 2)$ model, each of the three possible graphs on three vertices and two edges are included with probability 1/3.
- In the $G(n, p)$ model, a graph is constructed by connecting nodes randomly. Each edge is included in the graph with probability p independent from every other edge. Equivalently, all graphs with n nodes and M edges have equal probability of



A graph generated by the binomial model of Erdős and Rényi ($p = 0.01$)

$$p^M(1-p)^{\binom{n}{2}-M}.$$

The parameter p in this model can be thought of as a weighting function; as p increases from 0 to 1, the model becomes more and more likely to include graphs with more edges and less and less likely to include graphs with fewer edges. In particular, the case $p = 0.5$ corresponds to the case where all $2^{\binom{n}{2}}$ graphs on n vertices are chosen with equal probability.

The behavior of random graphs are often studied in the case where n, the number of vertices, tends to infinity. Although p and M can be fixed in this case, they can also be functions depending on n. For example, the statement

> Almost every graph in $G(n, 2\ln(n)/n)$ is connected.

means

> As n tends to infinity, the probability that a graph on n vertices with edge probability $2\ln(n)/n$ is connected, tends to 1.

Comparison between the two models

The expected number of edges in $G(n, p)$ is $\binom{n}{2}p$, and by the law of large numbers any graph in $G(n, p)$ will almost surely have approximately this many edges (provided the expected number of edges tends to infinity). Therefore a rough heuristic is that if $pn^2 \to \infty$, then $G(n,p)$ should behave similarly to $G(n, M)$ with $M = \binom{n}{2}p$ as n increases. For many graph properties, this is the case. If P is any graph property which is monotone with respect to the subgraph ordering (meaning that if A is a subgraph of B and A satisfies P, then B will satisfy P as well), then the statements "P holds for almost all graphs in $G(n, p)$" and "P holds for almost all graphs in $G\left(n, \binom{n}{2}p\right)$" are equivalent (provided $pn^2 \to \infty$). For example, this holds if P is the property of being connected, or if P is the property of containing a Hamiltonian cycle. However, this will not necessarily hold for non-monotone properties (e.g. the property of having an even number of edges).

In practice, the $G(n, p)$ model is the one more commonly used today, in part due to the ease of analysis allowed by the independence of the edges.

Properties of $G(n, p)$

With the notation above, a graph in $G(n, p)$ has on average $\binom{n}{2}p$ edges. The distribution of the degree of any particular vertex is binomial:[1]

$$P(\deg(v) = k) = \binom{n-1}{k} p^k (1-p)^{n-1-k},$$

where n is the total number of vertices in the graph. Since

$$P(\deg(v) = k) \to \frac{(np)^k \mathrm{e}^{-np}}{k!} \quad \text{as } n \to \infty \text{ and } np = \text{const},$$

this distribution is Poisson for large n and np = const.

In a 1960 paper, Erdős and Rényi[2] described the behavior of $G(n, p)$ very precisely for various values of p. Their results included that:

- If $np < 1$, then a graph in $G(n, p)$ will almost surely have no connected components of size larger than $O(\log(n))$.
- If $np = 1$, then a graph in $G(n, p)$ will almost surely have a largest component whose size is of order $n^{2/3}$.
- If $np \to c > 1$, where c is a constant, then a graph in $G(n, p)$ will almost surely have a unique giant component containing a positive fraction of the vertices. No other component will contain more than $O(\log(n))$ vertices.
- If $p < \frac{(1-\epsilon)\ln n}{n}$, then a graph in $G(n, p)$ will almost surely contain isolated vertices, and thus be disconnected.
- If $p > \frac{(1+\epsilon)\ln n}{n}$, then a graph in $G(n, p)$ will almost surely be connected.

Thus $\frac{\ln n}{n}$ is a sharp threshold for the connectedness of $G(n, p)$.

Further properties of the graph can be described almost precisely as n tends to infinity. For example, there is a $k(n)$ (approximately equal to $2\log_2(n)$) such that the largest clique in $G(n, 0.5)$ has almost surely either size $k(n)$ or $k(n) + 1$.

Thus, even though finding the size of the largest clique in a graph is NP-complete, the size of the largest clique in a "typical" graph (according to this model) is very well understood.

Relation to percolation

In percolation theory one examines a finite or infinite graph and removes edges (or links) randomly. Thus the Erdős–Rényi process is in fact unweighted link percolation on the complete graph. (One refers to percolation in which nodes and/or links are removed with heterogeneous weights as weighted percolation). As percolation theory has much of its roots in physics, much of the research done was on the lattices in Euclidean spaces. The transition at $np = 1$ from giant component to small component has analogs for these graphs, but for lattices the transition point is difficult to determine. Physicists often refer to study of the complete graph as a mean field theory. Thus the Erdős–Rényi process is the mean-field case of percolation.

Some significant work was also done on percolation on random graphs. From a physicist's point of view this would still be a mean-field model, so the justification of the research is often formulated in terms of the robustness of the graph, viewed as a communication network. Given a random graph of $n \gg 1$ nodes with an average degree $\langle k \rangle$. Remove randomly a fraction $1 - p'$ of nodes and leave only a fraction p' from the network. There exists a critical percolation threshold $p'_c = \frac{1}{\langle k \rangle}$ below which the network becomes fragmented while above p'_c a giant connected component of order n exists. The relative size of the giant component, P_∞, is given by[3][4][5][6]

$$P_\infty = p'[1 - \exp(-\langle k \rangle P_\infty)].$$

Caveats

Both of the two major assumptions of the $G(n, p)$ model (that edges are independent and that each edge is equally likely) may be inappropriate for modeling real-life phenomena. In particular, an Erdős–Rényi graph does not have heavy tails, as is the case in many real networks. Moreover, it has low clustering, unlike many social networks. For popular modeling alternatives, see Barabási–Albert model and Watts and Strogatz model. One should note that these alternative models are not percolation processes, but instead represent a growth and rewiring model, respectively. A model for interacting ER networks was developed recently by Buldyrev *et al.*.[7]

History

The $G(n, p)$ model was first introduced by Edgar Gilbert in a 1959 paper which studied the connectivity threshold mentioned above. The $G(n, M)$ model was introduced by Erdős and Rényi in their 1959 paper. As with Gilbert, their first investigations were as to the connectivity of $G(n, M)$, with the more detailed analysis following in 1960.

References

[1] , Eq. (1)

[2] The probability p used here refers there to $p^M(1-p)^{\binom{n}{2}-M}$.

Further reading

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Watts and Strogatz model

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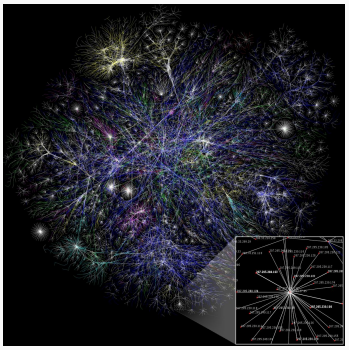
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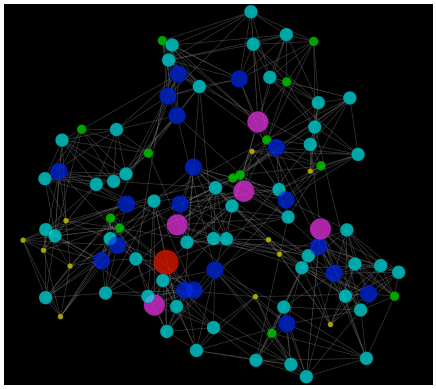
The **Watts-Strogatz model** is a random graph generation model that produces graphs with small-world properties, including short average path lengths and high clustering. It was proposed by Duncan J. Watts and Steven Strogatz in their joint 1998 Nature paper.[] The model also became known as the (Watts) *beta* model after Watts used β to formulate it in his popular science book *Six Degrees*.



Watts-Strogatz small-world model generated by igraph and visualized by Cytoscape 2.5. 100 nodes.

Rationale for the model

The formal study of random graphs dates back to the work of Paul Erdős and Alfréd Rényi.[] The graphs they considered, now known as the classical or Erdős–Rényi (ER) graphs, offer a simple and powerful model with many applications.

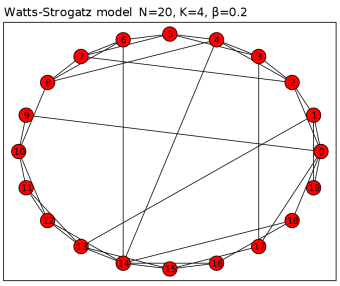
However the ER graphs do not have two important properties observed in many real-world networks:

1. They do not generate local clustering and triadic closures. Instead because they have a constant, random, and independent probability of two nodes being connected, ER graphs have a low clustering coefficient.
2. They do not account for the formation of hubs. Formally, the degree distribution of ER graphs converges to a Poisson distribution, rather than a power law observed in many real-world, scale-free networks.

The Watts and Strogatz model was designed as the simplest possible model that addresses the first of the two limitations. It accounts for clustering while retaining the short average path lengths of the ER model. It does so by interpolating between an ER graph and a regular ring lattice. Consequently, the model is able to at least partially explain the "small-world" phenomena in a variety of networks, such as the power grid, neural network of C. elegans, and a network of movie actors.

Algorithm

Given the desired number of nodes N, the mean degree K (assumed to be an even integer), and a special parameter β, satisfying $0 \leq \beta \leq 1$ and $N \gg K \gg \ln(N) \gg 1$, the model constructs an undirected graph with N nodes and $\frac{NK}{2}$ edges in the following way:



Watts-Strogatz graph

1. Construct a regular ring lattice, a graph with N nodes each connected to K neighbors, $K/2$ on each side. That is, if the nodes are labeled $n_0 \ldots n_{N-1}$, there is an edge (n_i, n_j) if and only if $0 < |i-j| \leq \frac{K}{2}$.
2. For every node $n_i = n_0, \ldots, n_{N-1}$ take every edge (n_i, n_j) with $i < j$, and rewire it with probability β. Rewiring is done by replacing (n_i, n_j) with (n_i, n_k) where k is chosen with uniform probability from all possible values that avoid loops ($k \neq i$) and link duplication (there is no edge $(n_i, n_{k'})$ with $k' = k$ at this point in the algorithm).

Properties

The underlying lattice structure of the model produces a locally clustered network, and the random links dramatically reduce the average path lengths. The algorithm introduces about $\beta\frac{NK}{2}$ non-lattice edges. Varying β makes it possible to interpolate between a regular lattice ($\beta = 0$) and a random graph ($\beta = 1$) approaching the Erdős–Rényi random graph $G(n, p)$ with $n = N$ and $p = \frac{NK}{2\binom{N}{2}}$.

The three properties of interest are the average path length, the clustering coefficient, and the degree distribution.

Average path length

For a ring lattice the average path length is $l(0) = N/2K \gg 1$ and scales linearly with the system size. In the limiting case of $\beta \to 1$ the graph converges to a classical random graph with $l(1) = \frac{\ln N}{\ln K}$. However, in the intermediate region $0 < \beta < 1$ the average path length falls very rapidly with increasing β, quickly approaching its limiting value.

Clustering coefficient

For the ring lattice the clustering coefficient is $C(0) = 3/4$ which is independent of the system size. In the limiting case of $\beta \to 1$ the clustering coefficient attains the value for classical random graphs, $C(1) = K/N$ and is thus inversely proportional to the system size. In the intermediate region the clustering coefficient remains quite close to its value for the regular lattice, and only falls at relatively high β. This results in a region where the average path length falls rapidly, but the clustering coefficient does not, explaining the "small-world" phenomenon.

If we use the Barrat and Weigt[] measure for clustering $C'(\beta)$ defined as the fraction between the average number of edges between the neighbors of a node and the average number of possible edges between these neighbors, or, alternatively,

$$C'(\beta) \equiv \frac{3 \times \text{number of triangles}}{\text{number of connected triples}}$$

then we get $C'(\beta) \sim C(0)\,(1-\beta)^3$.

Degree distribution

The degree distribution in the case of the ring lattice is just a Dirac delta function centered at K. In the limiting case of $\beta \to 1$ it is Poisson distribution, as with classical graphs. The degree distribution for $0 < \beta < 1$ can be written as,[]

$$P(k) = \sum_{n=0}^{f(k,K)} C_{K/2}^{n}\,(1-\beta)^n\,\beta^{K/2-n}\frac{(\beta K/2)^{k-K/2-n}}{(k-K/2-n)!}e^{-\beta K/2}$$

where k_i is the number of edges that the i^{th} node has or its degree. Here $k \geq K/2$, and $f(k, K) = \min(k - K/2, K/2)$. The shape of the degree distribution is similar to that of a random graph and has a pronounced peak at $k = K$ and decays exponentially for large $|k - K|$. The topology of the network is relatively homogeneous, and all nodes have more or less the same degree.

Limitations

The major limitation of the model is that it produces an unrealistic degree distribution. In contrast, real networks are often scale-free networks inhomogeneous in degree, having hubs and a scale-free degree distribution. Such networks are better described in that respect by the preferential attachment family of models, such as the Barabási–Albert (BA) model. (On the other hand, the Barabási–Albert model fails to produce the high levels of clustering seen in real networks, a shortcoming not shared by the Watts and Strogatz model. Thus, neither the Watts and Strogatz model nor the Barabási–Albert model should be viewed as fully realistic.)

The Watts and Strogatz model also implies a fixed number of nodes and thus cannot be used to model network growth.

References

Epidemic model

An **Epidemic model** is a simplified means of describing the transmission of communicable disease through individuals.

Introduction

The outbreak and spread of disease has been questioned and studied for many years. The ability to make predictions about diseases could enable scientists to evaluate inoculation or isolation plans and may have a significant effect on the mortality rate of a particular epidemic. The modeling of infectious diseases is a tool which has been used to study the mechanisms by which diseases spread, to predict the future course of an outbreak and to evaluate strategies to control an epidemic (Daley & Gani, 2005).

The first scientist who systematically tried to quantify causes of death was John Graunt in his book *Natural and Political Observations made upon the Bills of Mortality*, in 1662. The bills he studied were listings of numbers and causes of deaths published weekly. Graunt's analysis of causes of death is considered the beginning of the "theory of competing risks" which according to Daley and Gani (Daley & Gani, 2005, p. 2) is "a theory that is now well established among modern epidemiologists".

The earliest account of mathematical modeling of spread of disease was carried out in 1766 by Daniel Bernoulli. Trained as a physician, Bernoulli created a mathematical model to defend the practice of inoculating against smallpox (Hethcote, 2000). The calculations from this model showed that universal inoculation against smallpox would increase the life expectancy from 26 years 7 months to 29 years 9 months (Bernoulli & Blower, 2004).

Daniel Bernoulli's work, of course, preceded our modern understanding of germ theory, and it was not util the research of Ronald Ross into the spread of malaria, that modern theoretical epidemiology began. This was soon followed by the acclaimed work of A. G. McKendrick and W. O. Kermack, whose paper *A Contribution to the Mathematical Theory of Epidemics* was published in 1927. A simple deterministic (compartmental) model was formulated in this paper. The model was successful in predicting the behavior of outbreaks very similar to that observed in many recorded epidemics (Brauer & Castillo-Chavez, 2001).

Types of Epidemic Models

Stochastic

"Stochastic" means being or having a random variable. A stochastic model is a tool for estimating probability distributions of potential outcomes by allowing for random variation in one or more inputs over time. Stochastic models depend on the chance variations in risk of exposure, disease and other illness dynamics. They are used when these fluctuations are important, as in small populations (Trottier & Philippe, 2001).

Deterministic

When dealing with large populations, as in the case of tuberculosis, deterministic or compartmental mathematical models are used. In the deterministic model, individuals in the population are assigned to different subgroups or compartments, each representing a specific stage of the epidemic. Letters such as M, S, E, I, and R are often used to represent different stages.

The transition rates from one class to another are mathematically expressed as derivatives, hence the model is formulated using differential equations. While building such models, it must be assumed that the population size in a compartment is differentiable with respect to time and that the epidemic process is deterministic. In other words, the changes in population of a compartment can be calculated using only the history used to develop the model (Brauer & Castillo-Chavez, 2001).

Another approach is through discrete analysis on a lattice (such as a two-dimensional square grid), where the updating is done through asynchronous single-site updates (Kinetic Monte Carlo) or synchronous updating (Cellular Automata). The lattice approach enables inhomogeneities and clustering to be taken into account. Lattice systems are usually studied through computer simulation, and are discussed in the Wikipedia page Epidemic models on lattices.

Terminology

The following is a summary of the notation used in this and the next sections.

- M : Passively Immune Infants
- S : Susceptibles
- E : Exposed Individuals in the Latent Period
- I : Infectives
- R : Removed with Immunity
- β : Contact Rate
- μ : Average Death Rate
- B : Average Birth Rate
- $1/\epsilon$: Average Latent Period
- $1/\gamma$: Average Infectious Period
- R_0: Basic Reproduction Number
- N : Total Population
- f : Average Loss of Immunity Rate of Recovered Individuals
- δ : Average Temporary Immunity Period

Deterministic Compartmental Models

The SIR Model

In 1927, W. O. Kermack and A. G. McKendrick created a model in which they considered a fixed population with only three compartments: susceptible, $S(t)$; infected, $I(t)$; and recovered, $R(t)$. The compartments used for this model consist of three classes:

- $S(t)$ is used to represent the number of individuals not yet infected with the disease at time t, or those susceptible to the disease.
- $I(t)$ denotes the number of individuals who have been infected with the disease and are capable of spreading the disease to those in the susceptible category.
- $R(t)$ is the compartment used for those individuals who have been infected and then recovered from the disease. Those in this category are not able to be infected again or to transmit the infection to others.

The flow of this model may be considered as follows:

$$\mathcal{S} \to \mathcal{I} \to \mathcal{R}$$

Using a fixed population, $N = S(t) + I(t) + R(t)$, Kermack and McKendrick derived the following equations:

$$\frac{dS}{dt} = -\beta SI$$
$$\frac{dI}{dt} = \beta SI - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

Several assumptions were made in the formulation of these equations: First, an individual in the population must be considered as having an equal probability as every other individual of contracting the disease with a rate of β, which is considered the contact or infection rate of the disease. Therefore, an infected individual makes contact and is able to transmit the disease with βN others per unit time and the fraction of contacts by an infected with a susceptible is S/N. The number of new infections in unit time per infective then is $\beta N(S/N)$, giving the rate of new infections (or those leaving the susceptible category) as $\beta N(S/N)I = \beta SI$ (Brauer & Castillo-Chavez, 2001). For the second and third equations, consider the population leaving the susceptible class as equal to the number entering the infected class. However, a number equal to the fraction (γ which represents the mean recovery rate, or $1/\gamma$ the mean infective period) of infectives are leaving this class per unit time to enter the removed class. These processes which occur simultaneously are referred to as the Law of Mass Action, a widely accepted idea that the rate of contact between two groups in a population is proportional to the size of each of the groups concerned (Daley & Gani, 2005). Finally, it is assumed that the rate of infection and recovery is much faster than the time scale of births and deaths and therefore, these factors are ignored in this model.

The SIR Model with Births and Deaths

Using the case of measles, for example, there is an arrival of new susceptible individuals into the population. For this type of situation births and deaths must be included in the model. The following differential equations represent this model:

$$\frac{dS}{dt} = -\beta SI + \mu(N - S)$$
$$\frac{dI}{dt} = \beta SI - \gamma I - \mu I$$
$$\frac{dR}{dt} = \gamma I - \mu R$$

The SIS Model with Births and Deaths

The SIS model can be easily derived from the SIR model by simply considering that the individuals recover with no immunity to the disease, that is, individuals are immediately susceptible once they have recovered.

$$\mathcal{S} \to \mathcal{I} \to \mathcal{S}$$

Removing the equation representing the recovered population from the SIR model and adding those removed from the infected population into the susceptible population gives the following differential equations:

$$\frac{dS}{dt} = -\beta SI + \mu(N - S) + \gamma I$$
$$\frac{dI}{dt} = \beta SI - \gamma I - \mu I$$

The SIRS Model

This model is simply an extension of the SIR model as we will see from its construction.

$$\mathcal{S} \to \mathcal{I} \to \mathcal{R} \to \mathcal{S}$$

The only difference is that it allows members of the recovered class to be free of infection and rejoin the susceptible class.

$$\frac{dS}{dt} = -\beta SI + \mu(N - S) + fR$$
$$\frac{dI}{dt} = \beta SI - \gamma I - \mu I$$
$$\frac{dR}{dt} = \gamma I - \mu R - fR$$

Models with More Compartments

The SEIS Model

The SEIS model takes into consideration the exposed or latent period of the disease, giving an additional compartment, E(t).

$$\mathcal{S} \to \mathcal{E} \to \mathcal{I} \to \mathcal{S}$$

In this model an infection does not leave a long lasting immunity thus individuals that have recovered return to being susceptible again, moving back into the S(t) compartment. The following differential equations describe this model:

$\frac{dS}{dT}$ = B - βSI - μS + γI

$\frac{dE}{dT}$ = βSI - (ε + μ)E

$\frac{dI}{dT}$ = εE - (γ + μ)I

The SEIR Model

The SIR model discussed above takes into account only those diseases which cause an individual to be able to infect others immediately upon their infection. Many diseases have what is termed a latent or exposed phase, during which the individual is said to be infected but not infectious.

$$\mathcal{S} \rightarrow \mathcal{E} \rightarrow \mathcal{I} \rightarrow \mathcal{R}$$

In this model the host population (N) is broken into four compartments: susceptible, exposed, infectious, and recovered, with the numbers of individuals in a compartment, or their densities denoted respectively by S(t), E(t), I(t), R(t), that is N = S(t) + E(t) + I(t) + R(t)

$$\frac{dS}{dT} = B - \beta SI - \mu S$$
$$\frac{dE}{dT} = \beta SI - (\varepsilon + \mu)E$$
$$\frac{dI}{dT} = \varepsilon E - (\gamma + \mu)I$$
$$\frac{dR}{dT} = \gamma I - \mu R$$

The MSIR Model

There are several diseases where an individual is born with a passive immunity from its mother.

$$\mathcal{M} \rightarrow \mathcal{S} \rightarrow \mathcal{I} \rightarrow \mathcal{R}$$

To indicate this mathematically, an additional compartment is added, M(t), which results in the following differential equations:

$$\frac{dM}{dT} = B - \delta MS - \mu M$$
$$\frac{dS}{dT} = \delta MS - \beta SI - \mu S$$
$$\frac{dI}{dT} = \beta SI - \gamma I - \mu I$$
$$\frac{dR}{dT} = \gamma I - \mu R$$

The MSEIR Model

For the case of a disease, with the factors of passive immunity, and a latency period there is the MSEIR model.

$$\mathcal{M} \rightarrow \mathcal{S} \rightarrow \mathcal{E} \rightarrow \mathcal{I} \rightarrow \mathcal{R}$$

$$\frac{dM}{dT} = B - \delta MS - \mu M$$
$$\frac{dS}{dT} = \delta MS - \beta SI - \mu S$$
$$\frac{dE}{dT} = \beta SI - (\varepsilon + \mu)E$$
$$\frac{dI}{dT} = \varepsilon E - (\gamma + \mu)I$$
$$\frac{dR}{dT} = \gamma I - \mu R$$

The MSEIRS Model

An MSEIRS model is similar to the MSEIR, but the immunity in the R class would be temporary, so that individuals would regain their susceptibility when the temporary immunity ended.

$$\mathcal{M} \to \mathcal{S} \to \mathcal{E} \to \mathcal{I} \to \mathcal{R} \to \mathcal{S}$$

Reproduction Number

There is a threshold quantity which determines whether an epidemic occurs or the disease simply dies out. This quantity is called the basic reproduction number, denoted by R_0, which can be defined as the number of secondary infections caused by a single infective introduced into a population made up entirely of susceptible individuals (S(0) ≈ N) over the course of the infection of this single infective. This infective individual makes βN contacts per unit time producing new infections with a mean infectious period of 1/γ. Therefore, the basic reproduction number is

$$R_0 = (\beta N)/\gamma$$

This value quantifies the transmission potential of a disease. If the basic reproduction number falls below one ($R_0 < 1$), i.e. the infective may not pass the infection on during the infectious period, the infection dies out. If $R_0 > 1$ there is an epidemic in the population. In cases where $R_0 = 1$, the disease becomes endemic, meaning the disease remains in the population at a consistent rate, as one infected individual transmits the disease to one susceptible (Trottier & Philippe, 2001).

In cases of diseases with varying latent periods, the basic reproduction number can be calculated as the sum of the reproduction number for each transition time into the disease. An example of this is tuberculosis. Blower et al. (1995) calculated from a simple model of TB the following reproduction number:

$$R_0 = R_0^{\text{FAST}} + R_0^{\text{SLOW}}$$

In their model, it is assumed that the infected individuals can develop active TB by either direct progression (the disease develops immediately after infection) considered above as FAST tuberculosis or endogenous reactivation (the disease develops years after the infection) considered above as SLOW tuberculosis.

Other Considerations within Compartmental Epidemic Models

Vertical Transmission

In the case of some diseases such as AIDS and Hepatitis B, it is possible for the offspring of infected parents to be born infected. This transmission of the disease down from the mother is called Vertical Transmission. The influx of additional members into the infected category can be considered within the model by including a fraction of the newborn members in the infected compartment (Brauer & Castillo-Chavez, 2001).

Vector Transmission

Diseases transmitted from human to human indirectly, i.e. malaria spread by way of mosquitoes, are transmitted through a vector. In these cases, the infection transfers from human to insect and an epidemic model must include both species, generally requiring many more compartments than a model for direct transmission. For more information on this type of model see the reference *Population Dynamics of Infectious Diseases: Theory and Applications*, by R. M. Anderson (Brauer & Castillo-Chavez, 2001).

Others

Other occurrences (taken from *Mathematical Models in Population Biology and Epidemiology* by Fred Brauer and Carlos Castillo-Chávez) which may need to be considered when modeling an epidemic include things such as the following:

- Nonhomogeneous mixing
- Age-Structured populations
- Variable infectivity
- Distributions that are spatially non-uniform
- Diseases caused by macroparasites
- Acquired immunity through vaccinations

References

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- Anderson, R. M. & May, R. M. (1991). *Infectious Diseases of Humans*. Oxford: Oxford University Press.
- Bernoulli, D. & Blower, S. (2004). "An attempt at a new analysis of the mortality caused by smallpox and of the advantages of inoculation to prevent it." *Reviews in Medical Virology*, 14, 275 – 288.
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- Daley, D. J. & Gani, J. (2005). *Epidemic Modeling: An Introduction.* NY: Cambridge University Press.
- Hethcote, H. W. (2000). "The mathematics of infectious diseases." *Society for Industrial and Applied Mathematics,* 42, 599 – 653.
- Trottier, H., & Philippe, P. (2001). "Deterministic modeling of infectious diseases: theory and methods." [1] *The Internet Journal of Infectious Diseases.*

Further reading

- An Introduction to Infectious Disease Modelling [2] by Emilia Vynnycky and Richard G White. An introductory book on infectious disease modelling and its applications.

External links

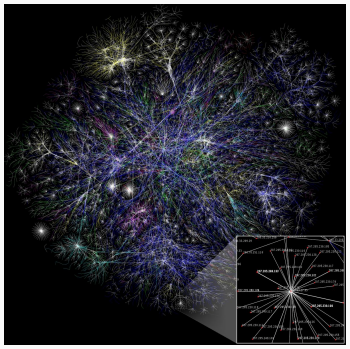
Software

- AnyLogic [3]
- Epigrass [4]
- Model-Builder
- NetLogo [5]
- STEM [6]
- World Health Organization Public Health Mapping and GIS Program [7]

References

[1] http://www.ispub.com/ostia/index.php?xmlFilePath=journals/ijid/volln2/model.xml
[2] http://anintroductiontoinfectiousdiseasemodelling.com/
[3] http://www.xjtek.com/
[4] http://sourceforge.net/projects/epigrass/
[5] http://ccl.northwestern.edu/netlogo/
[6] http://www.eclipse.org/stem/
[7] http://www.who.int/health_mapping/en/

Hierarchical network model

Network science

Theory · History
Graph · Complex network · Contagion Small-world · Scale-free · Community structure · Percolation · Evolution · Controllability · Topology · Graph drawing · Social capital · Link analysis · Optimization Reciprocity · Closure · Homophily Transitivity · Preferential attachment Balance · Network effect · Influence
Types of Networks
Information · Telecommunication Social · Biological · Neural · Semantic Random · Dependency · Flow
Graphs
Vertex · Edge · Component Directed · Multigraph · Bipartite Weighted · Hypergraph · Random Cycle · Loop · Path Neighborhood · Clique · Complete · Cut Data structure · Adjacency list & matrix Incidence list & matrix
Metrics and Algorithms
Centrality · Degree · Betweenness Closeness · PageRank · Motif Clustering · Degree distribution · Assortativity · Distance · Modularity
Models

Random · Erdős–Rényi
Barabási–Albert · Watts–Strogatz
ERGM · Epidemic · Hierarchical

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Hierarchical network models are iterative algorithms for creating networks which are able to reproduce the unique properties of the scale-free topology and the high clustering of the nodes at the same time. These characteristics are widely observed in nature, from biology to language to some social networks.

Concept

The hierarchical network model is part of the scale-free model family sharing their main property of having proportionally more hubs among the nodes than by random generation; however, it significantly differs from the other similar models (Barabási–Albert, Watts–Strogatz) in the distribution of the nodes' clustering coefficients: as other models would predict a constant clustering coefficient as the function of the degree of the node, in hierarchical models nodes with more links are expected to have a lower clustering coefficient. Moreover, while the Barabási-Albert model predicts a decreasing average clustering coefficient as the number of nodes increases, in the case of the hierarchical models there is no relationship between the size of the network and its average clustering coefficient.

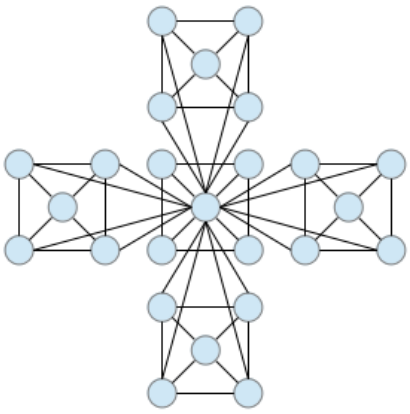
The development of hierarchical network models was mainly motivated by the failure of the other scale-free models in incorporating the scale-free topology and high clustering into one single model. Since several real-life networks (metabolic networks, the protein interaction network, the world wide web or some social networks) exhibit such properties, different hierarchical topologies were introduced in order to account for these various characteristics.

Algorithm

Hierarchical network models are usually derived in an iterative way by replicating the initial cluster of the network according to a certain rule. For instance, consider an initial network of five fully interconnected nodes (N=5). As a next step, create four replicas of this cluster and connect the peripherial nodes of each replica to the central node of the original cluster (N=25). This step can be repeated indefinitely, thereby for any k steps the number of nodes in the system can be derived by $N=5^{k+1}$.[]

Of course there have been several different ways for creating hierarchical systems proposed in the literature. These systems generally differ in the structure of the initial cluster as well as in the degree of expansion which is often referred to as the *replication factor* of the model.[][]

Properties



Example of a hierarchical network structure.

Degree distribution

Being part of the scale-free model family, the degree distribution of the hierarchical network model follows the power law meaning that a randomly selected node in the network has k edges with a probability

$$P(k) \sim ck^{-\gamma}$$

where c is a constant and γ is the degree exponent. In most real world networks exhibiting scale-free properties γ lies in the interval [2,3].[]

As a specific result for hierarchical models it has been shown that the degree exponent of the distribution function can be calculated as

$$\gamma = 1 + \frac{lnM}{ln(M-1)}$$

where M represents the replication factor of the model.[]

Clustering coefficient

In contrast to the other scale-free models (Erdős–Rényi, Barabási–Albert, Watts–Strogatz) where the clustering coefficient is independent of the degree of a specific node, in hierarchical networks the clustering coefficient can be expressed as a function of the degree in the following way:

$$C(k) \sim k^{-\beta}$$

It has been analytically shown that in deterministic scale-free networks the exponent β takes the value of 1.[]

Examples

Actor network

Based on the actor database available at www.IMDB.com the network is defined by Hollywood actors who are connected to each other if they both appeared in the same movie, resulting in a data set of 392,340 nodes and 15,347,957 edges. As earlier studies have shown, this network exhibits scale-free properties at least for high values of k. Moreover, the clustering coefficients seem to follow the required scaling law with the parameter -1 providing evidence for the hierarchical topology of the network. Intuitively, one-performance actors have by definition a clustering coefficient of one while actors starring in several movies are highly unlikely to work with the same crew which in general results in a decreasing clustering coefficient as the number of co-stars grows.[]

Language network

Words can be regarded a network if one specifies the linkage criteria between them. Defining links as appearance as a synonym in the Merriam-Webster dictionary a semantic web of 182,853 nodes with 317,658 edges was constructed. As it turned out, the obtained network of words indeed follows a power law in its degree distribution while the distribution of the clustering coefficient indicates that the underlying web follows a hierarchical structure with γ=3.25 and β=1.[]

Network of webpages

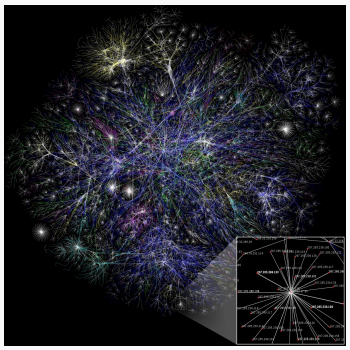
By mapping the www.nd.edu domain a network of 325,729 nodes and 1,497,135 edges was obtained whose degree distribution followed a power law with γ_{out}=2.45 and γ_{in}=2.1 for the out- and in-degrees, respectively. The evidence for the scaling law distribution of the clustering coefficients is significantly weaker than in the previous cases although there is a clearly visible declining pattern in the distribution of *C(k)* indicating that the more links a domain has the less interconnected the linked/linking web pages are.[][]

Domain network

The domain network, i.e. the internet at the autonomuous system (AS) level where the administrative domains are said to be connected in case there is a router which connects them, was found to comprise 65,520 nodes and 24,412 links between them and exhibit the properties of a scale-free network. The sample distribution of the clustering coefficients was fitted by the scaling function $C(k) \sim k^{-0.75}$ whose exponent is (in absolute terms) somewhat smaller than the theoretical parameter for deterministic scale-free networks.[][]

References

Barabási–Albert model

Network science

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The **Barabási–Albert (BA) model** is an algorithm for generating random scale-free networks using a preferential attachment mechanism. Scale-free networks are widely observed in natural and human-made systems, including the Internet, the world wide web, citation networks, and some social networks.

Concepts

Many observed networks fall into the class of scale-free networks, meaning that they have power-law (or scale-free) degree distributions, while random graph models such as the Erdős–Rényi (ER) model and the Watts–Strogatz (WS) model do not exhibit power laws. The Barabási–Albert model is one of several proposed models that generates scale-free networks. It incorporates two important general concepts: growth and preferential attachment. Both growth and preferential attachment exist widely in real networks.

Growth means that the number of nodes in the network increases over time.

Preferential attachment means that the more connected a node is, the more likely it is to receive new links. Nodes with higher degree have stronger ability to grab links added to the network. Intuitively, the preferential attachment can be understood if we think in terms of social networks connecting people. Here a link from A to B means that person A "knows" or "is acquainted with" person B. Heavily linked nodes represent well-known people with lots of relations. When a newcomer enters the community, s/he is more likely to become acquainted with one of those more visible people rather than with a relative unknown. Similarly, on the web, new pages link preferentially to hubs, i.e. very well known sites such as Google or Wikipedia, rather than to pages that hardly anyone knows. If someone selects a new page to link to by randomly choosing an existing link, the probability of selecting a particular page would be proportional to its degree. This explains the preferential attachment probability rule.

Preferential attachment is an example of a positive feedback cycle where initially random variations (one node initially having more links or having started accumulating links earlier than another) are automatically reinforced, thus greatly magnifying differences. This is also sometimes called the Matthew effect, "the rich get richer", and in chemistry autocatalysis.

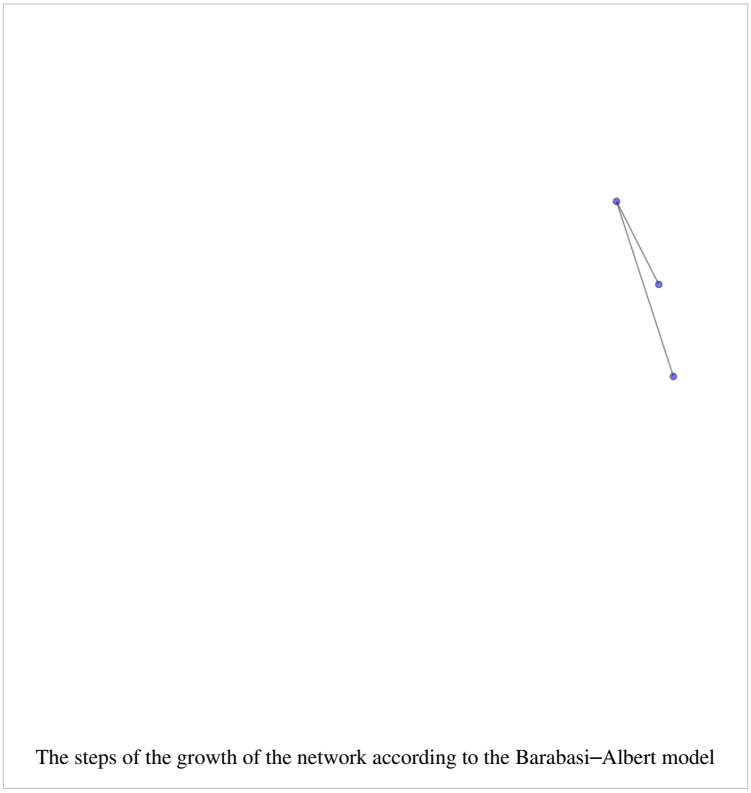
Algorithm

The network begins with an initial network of m_0 nodes. $m_0 \geq 2$ and the degree of each node in the initial network should be at least 1, otherwise it will always remain disconnected from the rest of the network.

New nodes are added to the network one at a time. Each new node is connected to m existing nodes with a probability that is proportional to the number of links that the existing nodes already have. Formally, the probability p_i that the new node is connected to node i is[]

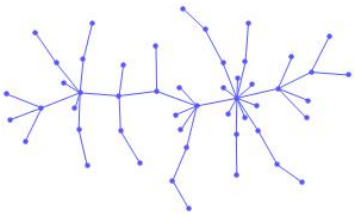
$$p_i = \frac{k_i}{\sum_j k_j},$$

where k_i is the degree of node i and the sum is made over all preexisting nodes j. Heavily linked nodes ("hubs") tend to quickly accumulate even more links, while nodes with only a few links are unlikely to be chosen as the destination for a new link. The new nodes have a "preference" to attach themselves to the already heavily linked nodes.



The steps of the growth of the network according to the Barabasi–Albert model

Properties

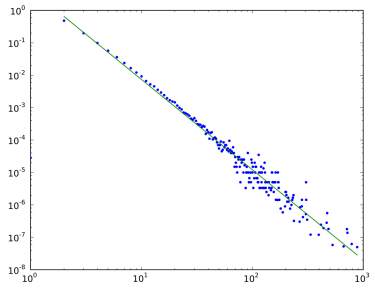


A network generated according to the Barabasi Albert model. The network is made of 50 vertices with initial degrees m=1.

Degree distribution

The degree distribution resulting from the BA model is scale free, in particular, it is a power law of the form

$$P(k) \sim k^{-3}$$



The degree distribution of the BA Model, which follows a power law. In loglog scale the power law function is a straight line.[]

Average path length

The average path length of the BA model increases approximately logarithmically with the size of the network. The actual form has a double logarithmic correction[] and goes as

$$\ell \sim \frac{\ln N}{\ln \ln N}.$$

The BA model has a systematically shorter average path length than a random graph.

Node degree correlations

Correlations between the degrees of connected nodes develop spontaneously in the BA model because of the way the network evolves. The probability, $n_{k\ell}$, of finding a link between nodes of degrees k and ℓ in the BA model when $m = 1$ is given by

$$n_{k\ell} = \frac{4(\ell-1)}{k(k+1)(k+\ell)(k+\ell+1)(k+\ell+2)} + \frac{12(\ell-1)}{k(k+\ell-1)(k+\ell)(k+\ell+1)(k+\ell+2)}.$$

This is certainly not the result expected if the distributions were uncorrelated, $n_{k\ell} = k^{-3}\ell^{-3}$[]

Clustering coefficient

While there is no analytical result for the clustering coefficient of the BA model, the empirically determined clustering coefficients are generally significantly higher for the BA model than for random networks. The clustering coefficient also scales with network size following approximately a power law

$$C \sim N^{-0.75}.$$

This behavior is still distinct from the behavior of small-world networks where clustering is independent of system size. In the case of hierarchical networks, clustering as a function of node degree also follows a power-law,

$$C(k) = k^{-1}.$$

This result was obtained analytically by Dorogovtsev, Goltsev and Mendes.[1]

Spectral properties

The spectral density of BA model has a different shape from the semicircular spectral density of random graph. It has a triangle-like shape with the top lying well above the semicircle and edges decaying as a power law.

Limiting cases

Model A

Model A retains growth but does not include preferential attachment. The probability of a new node connecting to any pre-existing node is equal. The resulting degree distribution in this limit is geometric,[2] indicating that growth alone is not sufficient to produce a scale-free structure.

Model B

Model B retains preferential attachment but eliminates growth. The model begins with a fixed number of disconnected nodes and adds links, preferentially choosing high degree nodes as link destinations. Though the degree distribution early in the simulation looks scale-free, the distribution is not stable, and it eventually becomes nearly Gaussian as the network nears saturation. So preferential attachment alone is not sufficient to produce a scale-free structure.

The failure of models A and B to lead to a scale-free distribution indicates that growth and preferential attachment are needed simultaneously to reproduce the stationary power-law distribution observed in real networks.[]

History

The first use of a preferential attachment mechanism to explain power-law distributions appears to have been by Yule in 1925,[3] although Yule's mathematical treatment is opaque by modern standards because of the lack of appropriate tools for analyzing stochastic processes. The modern master equation method, which yields a more transparent derivation, was applied to the problem by Herbert A. Simon in 1955[4] in the course of studies of the sizes of cities and other phenomena. It was first applied to the growth of networks by Derek de Solla Price in 1976[5] who was interested in the networks of citation between scientific papers. The name "preferential attachment" and the present popularity of scale-free network models is due to the work of Albert-László Barabási and Réka Albert, who rediscovered the process independently in 1999 and applied it to degree distributions on the web.[]

References

[1] S.N. Dorogovtsev, A.V. Goltsev, and J.F.F. Mendes, e-print cond-mat/0112143.

External links

- "This Man Could Rule the World" (http://www.popsci.com/science/article/2011-10/man-could-rule-world)

List of social networking websites

This is a **list of** major active **social networking websites** and excludes dating websites (see Comparison of online dating websites). For defunct social networking websites, see List of defunct social networking websites.

This list is not exhaustive, and is limited to notable, well-known sites.

Contents

Name	Description/Focus	Date launched	Registered users	Registration	Global Alexa[1] Page ranking
43 Things	Goal setting and achievement	1 January 2005	3,000,000[2]	Open	13,574[3]
Academia.edu	Social networking site for academics/researchers	September 2008	211,000[4]	Open	3,872[5]
Advogato	Free and open source software developers	1999	13,575[6]	Open	318,165[7]
aNobii	Books	2006		Open	13,131[8]
AsianAvenue	A social network for the Asian American community	1997		Open	133,043[9]

aSmallWorld	European jet set and social elite world-wide	March 2004	550,000[10]	Invite-only	8,780[11]
Athlinks	Running, Swimming	2001	139,458[12]	Open	60,677[13]
Audimated.com	Independent music	2010		Open	656,507[14]
Bebo	General	July 2005	117,000,000[15]	Open to people 13 and older	4,169[16]
BIGADDA	Indian Social Networking Site	August 2007	3,000,000[17]	Open to people 16 and older	42,765[18]
Biip.no	Norwegian community	1 June 2005	430,000[19]	Requires Norwegian phone number	81,916[20]
BlackPlanet	Black Americans	1 September 1999	20,000,000[21]	Open	7,193[22]
Blauk	Anyone who wants to tell something about a stranger or acquaintance.	2010		Open to people 13 and older.	1,081,215[23]
Blogster	Blogging community	24 November 2005	85,579[24]	Open	12,048[25]
Bolt.com	General	1996		Open	589,595[26]
Busuu	Language learning community (headquartered in Madrid, Spain)	16 May 2008	12,000,000[27]	Open	4,976[28]
Buzznet	Music and pop-culture	2005	10,000,000[29]	Open	6,955[30]
CafeMom	Mothers	December 2006	1,250,000[31]	Open to moms and moms-to-be	1,293[32]
Care2	Green living and social activism	1998	9,961,947[33]	Open	1,930[34]
CaringBridge	Not for profit providing free websites that connect family and friends during a serious health event, care and recovery.[35]		9,500,000[36]	Open to people 18 and older[37]	4,279[38]
Classmates.com	School, college, work and the military	1995	50,000,000[39]	Open to people 18 and older[40]	3,284[41]
Cloob	General. Popular in Iran	2004		Open	749[42]
CouchSurfing	Worldwide network for making connections between travelers and the communities they visit.	2003	2,967,421[43]	Open	2,231[44]
CozyCot	East Asian and Southeast Asian women	2001	150,000[45]	Open	36,959[46]
Cross.tv	Faith Based social network for Christian believers from around the world	2008	450,000[47]	Open	53,394[48]
Crunchyroll	Anime and forums.	2006		Open	5,721[49]

Cyworld	General. Popular in South Korea.	1999	24,000,000[50]	Open	1,616[51]
DailyBooth	Photo-blogging site where users upload a photo every day	13 February 2009		Closed	5,963[52]
DailyStrength	Medical & emotional support community - Physical health, Mental health, Support groups	4 November 2007		Open	10,511[53]
delicious	Social bookmarking allowing users to locate and save websites that match their own interests	September 2003	8,822,921[54]	Open	436[55]
deviantART	Art community	7 August 2000	22,000,000[56]	Open to people 13 and older	131[57]
Diaspora*	Decentralized, privacy aware, general (open source)	November 2010	370,000+[58][59]	Open	36,792[60][61]
Disaboom	People with disabilities (Amputee, cerebral palsy, MS, and other disabilities)			Open	218,135[62]
Dol2day	Politic community, Social network, Internet radio (German-speaking countries)		40,200[63]	Open	385,237[64]
DontStayIn	Clubbing (primarily UK)			Open	86,535[65]
Draugiem.lv	General (primarily LV, LT, HU)		2,600,466[66]	Invitation only	2,698[67]
douban	Chinese Web 2.0 website providing user review and recommendation services for movies, books, and music.	2005	46,850,000[68]	Open	106[69]
DXY.cn	Chinese online community for physicians, health care professionals, pharmacies and facilities	2000[70]	2,000,000[71]	Open	8,367[72]
Elftown	Community and wiki around Fantasy and sci-fi.		185,000[73]	Open, approval needed	43,277[74]
Elixio	Business executives jet set and global elite.	July 2007	80,000[75]	Invite-only	545,268[76]
English, baby!	Students and teachers of English as a second language	2000	1,600,000[77]	Open	29,585[78]
Epernicus	For research scientists			Open	798,086[79]
Eons.com	For baby boomers and mature internet users age 40 and beyond.			Open to people age 40 years and older	73,767[80]
eToro	Social Investing, Finance	May 2007	2,500,000	Open	1,591[81]
Experience Project	Life experiences			Open	2,658[82]
Exploroo	Travel Social Networking.			Open	163,545[83]

Facebook	General: Photos, Videos, Blogs, Apps.	February 2004	1,000,000,000[84]	Open to people 13 and older	2[85]
Faceparty	General. Popular UK.		200,000[86]	Invitation only to people 18 and older	163,639[87]
Faces.com	Adult Social Network, Mainly UK & USA	2011	700,000[88]	Open to people 18 and older worldwide	149,839[89]
Fetlife	People who are into BDSM	January 2008	1,200,000[90]	Open to people "of legal age to see adult content"	4,131[91]
FilmAffinity	Movies and TV Series	2002	250,000[92]	Open	3,151[93]
Filmow	Movies and TV Series	2009		Open	44,630[94]
FledgeWing	Entrepreneural community targeted towards worldwide university students			Open to university students	696,513[95]
Flixster	Movies	2007	63,000,000[96]	Open to people 13 and older	4,012[97]
Flickr	Photo sharing, commenting, photography related networking, worldwide	February 2004	32,000,000[98]	Open to people 13 and older (Yahoo! Login)	48[99]
Focus.com	Business to Business, worldwide	2005	850,000[100]	Open to people 13 and older	12,114[101]
Fotki	Photo sharing, video hosting, photo contests, journals, forums, flexible privacy protection, friend's feed, audio comments and unlimited custom design integration.	October 1998	1,632,565	Open	8,011[102]
Fotolog	Photoblogging. Popular in South America and Spain	2002	20,000,000[103]	Open	3,168[104]
Foursquare	Location based mobile social network	2009	20,000,000[105]	Open	817[106]
Friendica	Distributed, federated, privacy aware, open source, general			Open	354,946[107][60]
Friends Reunited	UK based. School, college, work, sport and streets	July 2000	19,000,000[108]	Open to people 13 and older	29,239[109]
Friendster	General. Popular in Southeast Asia. No longer popular in the western world	2002	90,000,000[110]	Open to people 16 and older.	12,167[111]
Frühstückstreff	General	July 2001	14,800[*citation needed*]	Open	1,827,621[112]
Fubar	dating, an "online bar" for 18 and older	October 2007	1,200,000[113]	Open to people 18+	10,453[114]
Fuelmyblog	Blogging community	July 2007	50,000[115]	Open	85,547[116]
FullCircle	Geosocial networking and location-based services portal for mobile devices	April 2009		Open	1,167,425[]

Gaia Online	Anime and games. Popular in USA, Canada and Europe. Moderately popular around Asia.		23,523,663[117]	Open to people 13 and older	6,277[118]
GamerDNA	Computer and video games	21 September 2006	310,000[119]	Open	59,730[120]
Gapyear.com	Travel social network	July 1998		Open	74,763[121]
Gather.com	Article, picture, and video sharing, as well as group discussions		465,000[122]	Open	3,062[123]
Gays.com	Social network for LGBT community, Guide for LGBT bars, restaurants, clubs, shopping	16 May 2008	100,000[124]	Open, Global	224,302[125]
Geni.com	Families, genealogy	16 January 2007	15,000,000[126]	Open	5,958[127]
GetGlue	Social network for entertainment	28 October 2008	2,000,000[128]	Open	3,735[129]
Gogoyoko	Fair play in Music - Social networking site for musicians and music lovers			Invite only while in beta	145,552[130]
Goodreads	Library cataloging, book lovers	December 2006	10,000,000[131]	Open	544[132]
Goodwizz	Social network with matchmaking and personality games to find new contacts. Global, based in France.	October 2010	110,000[133][134]	Open	597,008[135]
Google+	General	28 June 2011	400,000,000[136]	Open to people 13 and older [137]	NA[138]
GovLoop	For people in and around government				138,576[139]
Grono.net	Poland		2,000,000[*citation needed*]	Open	395,036[140]
Habbo	General for teens. Over 31 communities worldwide. Chat room and user profiles.	August 2000	268,000,000[141][142][143]	Open to people 13 and older	15,255[144]
hi5	General. Popular in Nepal, Mongolia, Thailand, Romania, Jamaica, Central Africa, Portugal and Latin America. Not very popular in the USA.	2003	80,000,000[]	Open to people 13 and older.	902[145]
Hospitality Club	Hospitality		328,629[146]	Open	99,562[147]
Hotlist	Geo-Social Aggregator rooted in the concept of knowing where users' friends are, were, and will be.		80,000[148]	Open	155,811[149]

HR.com	Social networking site for Human Resources professionals	1999	194,000[150]	Open	79,591[151]
Hub Culture	Global influencers focused on worth creation	November 2002	20,000[152]	Invite-only	152,756[153]
Hyves	General, mostly popular in the Netherlands.	September 2004	10,097,000[154]	Open	2,243[155]
Ibibo	Talent based social networking site that allows to promote one's self and also discover new talent. Most popular in India.		3,500,000[156]	Open	1,143[157]
Identi.ca	Twitter-like service popular with hackers and software freedom advocates.		395,695[*citation needed*]	Open	7,290[158]
Indaba Music	Online collaboration for musicians, remix contests, and networking.		350,000[159]	Open, Global	45,039[160]
IRC-Galleria	Finland		505,000[161]	Open to Finnish speaking people 12 and older	36,982[162]
italki.com	Language learning social network. 100+ languages.		500,000[163]	Open, Global	18,956[164]
Itsmy	Mobile community worldwide, blogging, friends, personal TV-shows		2,500,000[165]		770,100[166]
iWiW	Hungary	14 April 2002	4,000,000[167]	Invite-only	3,831[168]
Jaiku	General. Microblogging. Owned by Google	February 2006		Open to people 13 and older	62,910[169]
Jiepang	Location based mobile social network. In Chinese language	2010	3,000,000[170]	Open	14,790[171]
Kaixin001	General. In Simplified Chinese; caters for mainland China users			Open to the General Public	284[172]
Kiwibox	General.	1999	2,400,000[173]	Open to people 13 and older	104,563[174]
Lafango	Talent-Focused media sharing site			Open, Global	161,778[175]

LAGbook	African-based Social Networking Site. Popular in Germany, Denmark, France, China, Hong Kong, Pakistan, Turkey, United Kingdom, United States, Indonesia, Bangladesh, Malaysia, Philippines, Canada, India, Romania, Mexico, Ukraine, Bulgaria, Japan, Chile, Spain, South Africa, Israel, Australia and especially in Nigeria.[176][]	17 April 2010	350,000+[][177][178]	Specifically targeted at the youth demographic (18-30), but permits registration of people 13 and older.	24,429[179]
LaiBhaari	Marathi social networking	2010	250,000	Open	952,782[180]
Last.fm	Music	2002	30,000,000[181]	Open	772[182]
LibraryThing	Book lovers	29 August 2005	1,300,000[183]	Open to people 13 and older	10,800[184]
Lifeknot	Shared interests, hobbies			Open to people 18 and older	1,071,686[185]
LinkedIn	Business and professional networking	May 2003	160,000,000[186]	Open to people 18 and older	12[187]
LinkExpats	Social networking website for expatriates. 100+ countries.			Open, Global	800,451[188]
Listography	Lists. Autobiography			Open	92,525[189]
LiveJournal	Blogging. Popular in Russia and among the Russian-speaking diaspora abroad.	15 April 1999	17,564,977[190]	Open (OpenID)	115[191]
Livemocha	Online language learning		5,000,000[192]	Open	4,100[193]
Makeoutclub	General	9 August 1999		Open	756,300[194]
MEETin	General			Open	318,979[195]
Meetup (website)	General. Used to plan offline meetings for people interested in various activities			Open to people 18 and older	481[196]
Meettheboss	Business and Finance community, worldwide.			Open	315,513[197]
MillatFacebook	General, Created in Response to Facebook [198]	May 2010	461,200[199]	Open	236,663[200]
mixi	Japan	25 October 2000	24,323,160[201]	Open	232[202]
MocoSpace	mobile community, worldwide	2005	3,000,000[203]	Open to people 14 and older	9,882[204]
MOG	Music			Open to people 14 and older	13,955[205]

MouthShut.com	Social Network, social media, consumer reviews			Open	3,340[206]
Mubi	Auteur cinema		200,000	Open	18,225[207]
MyHeritage	family-oriented social network service		30,000,000 [208]	Open	3,756[209]
MyLife	Locating friends and family, keeping in touch (formerly Reunion.com)		51,000,000[210]	Open	1,765[211]
My Opera	Blogging, mobile blogging, photo sharing, connecting with friends, Opera Link and Opera Unite. Global		7,300,000[212]	Open	NA[138]
Myspace	General	August 2003	30,000,000+[213][214]	Open to ages 13 and older.	161[215]
Nasza-klasa.pl	School, college and friends. Popular in Poland		11,000,000[216]	Open	75,145[217]
Netlog	General. Popular in Europe, Turkey, the Arab World and Canada's Québec province. Formerly known as Facebox and Redbox.[218]		95,000,000[219]	Open to people 13 and older	545[220]
Nexopia	Canada		1,400,000[221]	Open to people 13 and older [222]	25,241[223]
NGO Post	Non-Profit news sharing and networking, mainly in India		15,000[224]	Open	42,076[225]
Ning	Users create their own social websites and social networks	2005 (Networks 2007)		Open to people 13 and older	378[226]
Odnoklassniki	Connect with old classmates. Popular in Russia and former Soviet republics		45,000,000[227]	Open	65[228]
Open Diary	First online blogging community, founded in 1998	1998	5,000,000[229]	Open to people 13 and older	28,134[230]
Orkut	General. Owned by Google Inc. Popular in India and Brazil.[231]	22 January 2004	100,000,000[232]	Open to people 18 and older, (Google login)	319[233]
OUTeverywhere	Gay/LGBTQ Community			Open	445,440[234]
PatientsLikeMe	Online community for patients with life-changing illnesses to find other patients like them, share their data with others, and learn more about their condition to improve their outcome.	2006	109,587[235]	Open to people 13 years and up	69,514[236]
Partyflock	Dutch virtual community for people interested in house music and other electronic dance music	10 November 2001	321,125	Open to people 18 years and up	16,828[237]

Pingsta	Collaborative platform for the world's Internetwork Experts			Invite-only, only Internet Experts	2,533,640[238]
Pinterest	Online pinboard for organizing and sharing things you love	2011		Open	38[239]
Plaxo	Aggregator		50,000,000[240]	Open	5,310[241]
Playfire	Computer and video games			Open to people 13 and older	38,259[242]
Playlist.com	General, Music			Open to people over 13	8,026[243]
Plurk	Micro-blogging, RSS, updates. Very popular in Taiwan			Open	1,991[244]
Qapacity	A a business-oriented social networking site and a business directory			Closed	20,640[245]
Quechup	General, friendship, dating	2007		Open to those over 16	82,914[246]
Qzone	General. In Simplified Chinese; caters for mainland China users		480,000,000[][247]	Open to the general public	NA[138]
Raptr	Video games			Open	25,664[248]
Ravelry	Knitting and crochet		1,814,460[249]	Open	3,498[250]
Renren	Significant site in China. Was known as 校内 (Xiaonei) until August 2009.		160,000,000[251]	Open	95[252]
ReverbNation.com	Social network for musician and bands		500,000[253]	Open to people 16 and older	1,806[254]
Ryze	Business		500,000[255]	Closed	71,324[256]
ScienceStage	Science-oriented multimedia platform and network for scientists			Open	61,668[257]
ShareTheMusic	Music Community. Sharing and listening to music for free and legally			Open	612,429[258]
Shelfari	Books	11 October 2006		Open	18,152[259]
Sina Weibo	Social microblogging site in Mainland China.	14 August 2009	300,000,000[260]	Open	28[261]
Skoob	Collaborative social network for Brazilian readers	2009	420,000[262]	Open	28,449[263]
Skyrock	Social Network in French-speaking world		22,000,000[264]	Open	710[265]
SocialVibe	Social Network for Charity		435,000[*citation needed*]	Open	101,364[266]
Sonico.com	General. Popular in Latin America and Spanish and Portuguese speaking regions.		50,000,000[267][268]	Open to people 13 and older	2,676[269]

SoundCloud	Repository of original music pieces and networking.		10,000,000[270]	Open	299[271]
Spaces	Russian Social Network targeted to mobile phone users			Open	9,953[272]
Stickam	Live video streaming and chat.		9,000,000[273]	Open	9,201[274]
StudiVZ	University students, mostly in the German-speaking countries. School students and those out of education sign up via its partner sites schülerVZ and meinVZ.		17,000,000[275]	Open	7,080[276]
Students Circle Network	A Social Network connecting students, teachers and institutions to course resources, study groups and learning spaces.	December 2010		Open	345,110[277]
StumbleUpon	Stumble through websites that match users' selected interests		20,000,000[278]	Open	146[279]
Tagged	General.	October 2004	100,000,000[280]	Open	288[281]
Talkbiznow	Business networking			Open	136,346[282]
Taltopia	Online artistic community			Open	281,501[283]
Taringa!	General (primarily Argentina)		11,000,000[284]	Open to people 13 and older	214[285]
TeachStreet	Education / Learning / Teaching - More than 400 subjects			Open	422,704[286]
TermWiki	Learning / Languages / Translation - 1.2m terms in more than 1300 subjects	May 2010		Open	22,997[287]
The Sphere	A Private Online Social Luxury Network with Exclusive Personalized Services	December 2008	1,300[288]	Invite-only	1,115,305[289]
TravBuddy.com	Travel	2005	1,588,000[290]	Open to people 18 and older	21,523[291]
Travellerspoint	Travel	2002	310,000[292]	Open	15,275[293]
tribe.net	General			Open	8,081[294]
Trombi.com	French subsidiary of Classmates.com		4,400,000[295]		16,227[296]
Tuenti	Spanish-based university and High School social network.		12,000,000[297]	Invite-only	1,083[298]
Twitter	General. Micro-blogging, RSS, updates	15 July 2006	500,000,000[299]	Open to all ages [300]	8[301]
Tylted	Mobile social game network[302]	2007	3,000,000[303]	Open to people 14 and older	737,508[304]

Vkontakte	General, including music upload, listening and search. Popular in Russia and former Soviet republics.	September 2006	123,612,100[305]	Open	38[306]
Vampirefreaks.com	Gothic and industrial subculture	1999	1,931,049[307]	Open to users 13 and over	21,052[308]
Viadeo	Global Social Networking and Campus Networking available in English, French, German, Spanish, Italian and Portuguese		35,000,000[309]	Open	435[310]
Virb	Social network that focuses heavily on artists, including musicians and photographers	2007		Open	33,371[311]
Vox	Blogging			Open	56,558[312]
Wakoopa	For computer fans that want to discover new software and games		100,000[*citation needed*]	Open	33,350[313]
Wattpad	For readers and authors to interact & e-book sharing			Open	7,036[314]
WAYN	Travel and lifestyle	May 2003	10,000,000[315]	Open to people 18 and older	1,367[316]
WeeWorld	Teenagers - 10 to 17		30,000,000[317]	Open to ages 13 and older.	18,317[318]
Wellwer	Community without borders, where sharing is everything.	September 2011		Open to people 13 years and older	370,755[319]
WeOurFamily	General with emphasis on privacy and security			Open, subscription-based	3,359,241[320]
Wepolls.com	Social polling network			Open	1,204,376[321]
Wer-kennt-wen	General			General	2,147[322]
weRead	Books	June 2007	4,000,000[*citation needed*]	Open	305,720[323]
Wiser.org	Online community space for the social justice and environmental movement[324]	April 2007	71,600[325]	Open to people 16 and older	239,901[326]
Wooxie	Blogging and micro-blogging			Open	111,538[327]
WriteAPrisoner.com	Site networking inmates, friends, family	September 2000	66,000[328]	Open to people 18 years and older	228,915[329]
Xanga	Blogs and "metro" areas	Unknown	27,000,000[330]	Open	3,943[331]
XING	Business (primarily Europe (Germany, Austria, Switzerland))		11,100,000[332]	Open	270[333]
Xt3	Catholic social networking, created for World Youth Day 2008			Open	712,292[334]
Yammer	Social networking for office colleagues	2008		Must have company email	3,622[335]

Yelp, Inc.	Local Business Review and Talk			Open	186[336]
Zoo.gr	Greek Web Meeting point	2004	890,000[337]	Open	34,577[338]
Zooppa	Online Community for Creative Talent (host of brand sponsored advertising contests)		60,000[339]	Open to people 14 and older[340]	36,373[341]

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References

Comparison of social networking software

Social networking software provide the basis for community driven content and social/business networking.

Social networking software comparison

Category	ONEsite	phpBB Social Network	Tiki Wiki CMS Groupware	BoonExDolphin	BuddyPress	EngineY	PeopleAggregator	ELGG	Mugshot (Discontinued)	Drupal	Communifire	Telligent Community	iScripts Socialware	ImpressCMS	Joomla!	PHPizabi	ODS	Oxwall	Jcow	phpFox	XOOPS	JomSocial	webNetwork	Noosfero	SocialEngine	Kune
Last Access	2013-02-05	2012-10-29	2011-02-21	2010-09-9	2010-03-25	2010-02-26	2007-03-05	2012-03-07	2008-10-31	2008-11-02	2012-1-04	2009-06-23	2012-5-30	2012-04-22	2007-03-05	2007-03-05	2008-03-06	2012-12-21	2010-03-29	2011-03-10	2010-06-06	2011-07-14	2011-05-02	2012-09-05	2012-02-24	2012-04-22
License	Custom	GPL 2.0	LGPL [] 2.1	CC-BY	GPL 2.0	Apache 2.0	[] Custom [5]	Open source under [] the GPL 2.0 [6]	GPL 2.0	[] GPL 2.0 [7]	Custom	Custom	Custom	GPL 2.0	GPL 2.0	Custom	[] Custom (based on GPL 2.0)	CPAL 1.0	Creative Commons	[] Custom	GPL 2.0	GPL 2.0	Custom	AGPL	Custom	AGPLv3
Cost	$100–$2500	Free	Free	Free, $99	Free	Free	Free (not free, $2700–$20,000) Wikipedia:Please clarify	Free	Free	Free	Per License	Per License	Per License, $147.00	Free	Free	Free	Free	Free	30 Day Trial, $199, $219, $299	$99, $199, $299	Free	$99, $149, $499	$199	Free	from $299, to $669 with all options	Free
Source Code	No	Yes	Yes	Yes	Yes	Yes	Yes	Freely available via stable releases and development SVN	[] Terminated	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Version	N/A	0.7.2	9.0	7.0.x	1.2	N/A	1.2pre6	1.8.3	N/A	5.12, 6.16	3.3	5.0	3.0	1.3.2	1.5.20	0.848b C1 HFP1	5.0.5	1.5.0	1.3.5	3.3.0	2.4.4	2.2.3	?	0.38.2	4.7	0.2.0 (beta/production)
Installation	Hosted	Out of the box	Out of the box	Out of the box	Out of the box	Out of the box	Out of the box	Out of the box	?	Out of the box	Out of the box	Out of the box	Out of the box	Out of the box	Out of the box	Out of the box	Out of the box	Out of the box/hosted	?	Out of the box	Out of the box	Out of the box	?	Out of the box on Debian or manual install	Out of the box	Out of the box (deb package)
Codebase	PHP, MySQL	PHP, MySQL	PHP, MySQL	PHP, MySQL	PHP, MySQL	Ruby, Rails	PHP, MySQL	PHP, MySQL/PostgreSQL	?	PHP, MySQL	ASP.NET 4.0 / Microsoft SQL Server	ASP.NET 3.5 / Microsoft SQL Server	PHP, MySQL	PHP, MySQL	PHP, MySQL	PHP, MySQL	VSP/VSPX	PHP, MySQL	PHP, MySQL	PHP, MySQL	PHP, MySQL	PHP, MySQL	?	Ruby, Rails	PHP, MySQL	Java-based GWT, auto-generated AJAX

SSO support	Facebook Connect Twitter Connect OpenID Yahoo Steam GitHub Flickr XboxLive OAuth2	Under development	OpenID + LDAP + Active Directory built in	Facebook Connect OpenID Janrain	Facebook Connect Twitter Connect LDAP OpenID (via plugins)	Facebook Connect OpenID	Yahoo Flickr ID, OpenID , VardSpace (under dev)	LDAP built in, OpenID by plugin	No	OpenID, module	OpenID LDAP Active Directory Facebook Cookie Forms	OpenID LDAP Active Directory Facebook Cookie Forms	No	OpenID + LDAP + Yubikey + Active Directory built in	OpenID, module	?	OpenID and Yadis compliant	Facebook Connect	OpenID, module	Facebook Connect	LDAP + Active Directory built in	Facebook Connect	?	Under development, scheduled for 0.32 [1]	Facebook Connect, module	Under development
RSS	Yes	Yes	Yes	Yes	Yes	Yes	Blog	Blog, Files, Groups, Users, Tags	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	?	Plugin	Yes	Yes	Yes	Yes	?	Yes	?	?
RSS Reader/Aggregator	Module	No	Yes	Yes	No	Yes	No	Yes	Simple aggregator from many sites	Module	Yes	Yes	Yes	Yes	?	Yes	No	Yes	Yes	Yes	Yes	Yes	?	Yes	?	?
Access control	Users, Groups, Pages, Modules, User Tiers within Control Panel	Users, Groups	Yes	User levels, Groups, Page blocks, Pages	User levels, Groups	Yes	Group-based	Users, groups (extensible via plugins)	Group-based	Yes	Yes	Yes	Users, Groups	Users, Groups	Yes	?	Users, Groups	Users roles	Users roles	Users, Groups	Users, Groups	Yes	?	Users roles	Users levels, Groups, Pages	Users (with roles), Groups
Wiki	Yes	No	Yes	Plugin	Plugin	Under development	No	Plugin	No	Module	Yes	Yes	No	Module	No	No	Yes	No	No	Plugin	Module	Component	?	No, but collective articles	No	Yes
Forum	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Plugin	No	Yes	Yes	Yes	Yes	Module	Component	Yes	Yes	Plugin	Yes	Yes	Module	Component	?	Yes	Plugin, $40	Yes
Blog	Yes	Under development	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Module	Yes	Yes	Yes	Plugin	Yes	Yes	Module	Component	?	Yes	Plugin, $40	Yes
Media sharing	Photos, Videos, Blogs, Comments, Forums, Podcasts, Articles	Yes	File Galleries (Image, Video, Audio) and Kaltura [] integration	Image, Video, Audio, Boards, Files	via choice of plugins	Yes	Image, Video, Audio	Image, video, audio, documents, any filetype, automatic podcast support (Kaltura Elgg [] Plugin)	?	Module (Kaltura All-in-One Video [] Module)	Yes	Yes	Image, Video, Audio	Yes	?	Yes	Yes	via plugins	Image, Video, Audio	Yes	Yes	Yes	?	Media on Articles, Media directory, Image Gallery	plugins : Photo $40, Music $30, Video $40	Image, Video, Audio, Maps, etc
Messaging	Yes	Yes	Yes	Text, Video	Yes	Yes	Yes	Yes	Yes	Module	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Plugin	Yes	Yes	Yes	Yes	?	XMPP over web. Community talk and friends talk	Yes	XMPP compliant & chat-rooms
Event Calendar	Yes	Under development	Yes	Yes	Plugin	Yes	Personal & Group	Plugin	?	Module	Yes	Module	Yes	Module	Plugin	Yes	Yes	Plugin	Application	Yes	Module	Yes	?	Yes	Plugin, $40	Yes
Social Grouping	Yes	Under development	Yes	Yes	Yes	Yes	Yes	Yes	?	Module	Yes	Yes	Yes	Module	Plugin	Commercial module	Yes	Plugin	Yes	Yes	Module	Yes	?	Yes	Plugin, $40	Yes
Tagging	Yes	Yes	Yes	Yes	Forum tagging and Group tagging via plugin	Yes	Yes	Yes	?	Module	Yes	Yes	Yes	Module	?	No	SKOS, SCOT and MOAT	Yes	Yes	Yes	Module	Yes	?	Yes	Plugin, $40	Under development

Connectivity	Chat system	Under development	Webservices	Facebook (more via plugins)	Plugin (Facebook, YouTube, Twitter, Flickr, YouTube)	Facebook	MySpace, Facebook, Google Service, YouTube, AIM, Flickr	Plugin (Facebook, YouTube, Twitter, VEOH)	?	via third party integration like Octazen scripts	Webservices	Facebook, Google, Content Mirroring, RSS importer, External Widgets	YouTube	@all by Module	?	Facebook, Flickr, Amazon, Ebay, Google, del.icio.us, Web importer, Mash-ups	Open ID		Yes	Yes	Module	Facebook, Various video providers	?	RSS importer	Yes	Depends
Bookmark Sharing / Management	Yes	Yes	Yes	Yes	?	Yes	?	via plugin	?	?	Module	Module	?	@all by Module	?	Yes	Yes	Plugin	Yes	Yes	Yes	Yes	?	No	Facebook, Twitter	?
Poll Generation/Tally	Polls, Quizzes, Surveys and Petitions	Yes	Yes	Yes, with sharing	?	?	?	Plugin	?	Yes	Yes	?	Module	Yes	Yes	Yes	?	?	?	Yes	Module	via plugins	?	?	Yes	Yes
Contact Management	Integration through API	Yes	Yes	Yes	?	?	?	Yes	?	?	Yes	Yes	?	Module	?	Yes	?	Yes	Yes	Yes	Module	Component	?	Yes	Yes	Yes
Supports the Semantic Web	Yes	Yes	?	Yes	?	?	?	?	?	Yes	Yes	?	?	Yes	?	Yes, and Linked Data	?	?	Yes	?	?	?	?	?	?	Under development
Supports DataPortability	?	?	?	?	?	?	?	?	?	?	?	?	?	?	?	Yes	Yes	Yes	Yes	?	?	?	?	?	?	?
Customizable	Page templates, layout, CSS skinnable, custom HTML, smarty templates, REST API	Open source, Plugins, Themes, Widgets	Yes	Templates, page builders, database builders, open-source, extensions, plugins	Yes, based on WordPress theme and plugin architecture	Yes	Under development	Extensible via plugins with a flexible API; skinnable; available in many languages	?	Module	Extensible via plugins, widgets, tasks, events, REST; skinnable; available in many languages	Extensible via plugins, widgets, tasks, events, REST; skinnable; available in many languages	Skinnable, Module Manager	Module, Plugins, ImpressCMS Persitable Framework (IPF)	?	?	Open Source, Skinnable, and more	Open source, Plugins, Themes, Events, Widgets	Yes	Yes	Module	Yes	?	Site configurations, Templates, Plugins, CMS (for each profile), and open-source	Yes	Open source; extensible via gadgets (wave extensions) & via modules; skinnable; available in many languages
Comment	Modular, drag and drop layout, content management and social login.	Based on phpBB, should be familiar to anyone who has used or developed with phpBB before.		Comes with iPhone App, Android App, Desktop App, Flash Players and Converter. Backed by large community. Hundreds of extensions.	Based on WordPress, should be familiar to anyone who has used or developed with WordPress before.		Integration of many services	"Clean, simple"	More like RSS aggrators	Equipped with a powerful blend of features, Drupal supports a variety of websites ranging from personal weblogs to large community-driven websites.	Community and collaboration platform with many integrated core applications	Community and collaboration platform with many integrated core applications	Supports Free and paid membership plans	Install only if you like with modules. ImpressCMS = Make a Lasting Impression	Content management		All component applications fully integrated, and all data accessible by multiple means including SPARQL/RDF, ODBC, JDBC, ADO.NET, OLE DB, XMLA, and other protocols.	Simple, plugin-based, user-friendly, SEO-friendly, customizable Facebook-style newsfeed	Ajax comment		Modular approach, with hundreds of free modules and thousands of free [] themes.	A Joomla component, can be installed alongside other Joomla components		Focus on a content productizer user, has professional support by Colivre [2], has enterprise specific profile type with products showroom and global search.	plugin based, white label, well documented, 30 days free trial	Federated, focused on real-time group collaboration and social networking, wave-based, check Kune
Category	**ONEsite**	**phpBB Social Network**	**Tiki Wiki CMS Groupware**	**BoonExDolphin**	**BuddyPress**	**EngineY**	**PeopleAggregator**	**ELGG**	**Mugshot (Discontinued)**	**Drupal**	**Communifire**	**Telligent Community**	**iScripts Socialware**	**ImpressCMS**	**Joomla!**	**PHPizabi**	**ODS**	**Oxwall**	**Jcow**	**phpFox**	**XOOPS**	**JomSocial**	**webNetwork**	**Noosfero**	**SocialEngine**	**Kune**

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