Private Information, Limited Commitment, and Risk Sharing*

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Abstract

We consider an exchange economy with a continuum of agents, each of whom is subject to idiosyncratic endowment shocks. We study efficient allocations subject to two constraints: limited enforcement of financial contracts, and private information about the predictable component of the future endowment process. In our economy the immiseration result, common in this literature, does not hold, and a nontrivial steady state distribution exists. We calibrate our model to match the basic aggregate moments of the US economy, and find evidence that the efficient allocation implied by our model captures some of the very key features of observed partial risk sharing among households. Such crucial empirical features cannot be explained by existing models of endogenous incomplete markets.

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JEL Classification:D31, D63, D91, E21,G22

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1 Introduction

There is an existing literature that studies risk sharing among households assuming an exogenously incomplete market (see, for example, Deaton (1991), Carroll (1992, 1997), Huggett (1993), Aiyagari (1994), Storesletten, Telmer, Yaron (2004)). These models usually assumes a single risk free asset traded in the market and exogenous borrowing constraints. It leaves open the question why the market is incomplete and what determines the set of assets traded on the market.

We investigate the quantitative implications of private information and limited enforceability of private contracts on risk sharing among US households. We consider an exchange economy with a continuum of agents, each of whom is subject to idiosyncratic endowment shocks. Following Kehoe and Levine (1993) and Kocherlakota (1996), we assume a full set of state contingent contracts is available to all agents, but contracts cannot be legally enforced other than by exclusion from future intertemporal trade. The threat of exclusion from markets induces individuals to honor risk sharing contracts. Unlike the literature on private information optimal social insurance, which usually assumes private information on agents' income level, we introduced a different type of information asymmetry, which is more relevant as far as risk sharing among households is concerned. We consider the case in which, income is publicly obserbable and persistent, and agents privately observe a noisy signal of their future income.

We show that the component planning approach used in the optimal social insurance literature can be adapted to this environment to formulate the problem recursively. We develop a numerical algorithm to solve this model based on Fernandez and Phelan (2000). We also observe that unlike the private information optimal insurance literature, our model has endogenous upper and lower bounds on the steady state distribution of utility entitlements (that is, a nontrivial steady state of the economy does exist).

The model with private information and limited enforcement has the following qualitative feature. Agents' with a realization of high persistent shocks are rewarded higher current and future consumption to prevent default on the financial contract. This implies that they have an incentive to pretend to be high type. Therefore the optimal risk sharing would involve distorting the complete risk sharing plan of the high type in a way that prevent the low type from pretending to be of high type. This is done by "punishing the high type if his income is low". This punishment does not hurt the high type so much because high type's chance of getting a low income is very low; however, this provides incentives for low types to tell truth. For a low type agent, if he claims to be of high type, then the chance of getting a low income realization is high, therefore is more likely to be punished.

This mechanism has the following implications. First, it implies that consumption not only responds to income increases (as is true in models with only participation constraint), but also responds to income decreases. Next, elasticity of consumption with respect to income is increasing in income levels. The reason is the following: Consider an agent with low income realizations. Consumption's response to income only happens when the agent is of high type. However, among the income-poor people, the fraction of high type agent is small, consequently, consumption response to income is small when averaged out across agents. On the other hand, among the income-rich people, if the agent is of low type, consumption may or may not respond to income depending on whether the participation constraint is binding or not. If the agent is of high type, consumption respond to income since participation constraint is binding. Moreover, for high-income people, the fraction of high type agent is also high. Consequently, consumption respond more to income.

We decompose of income process into private information and public information. In the bench mark model we assume that the noisy signal is the same as the persistent component of the income process. The quantitative effect of private information depends crucially on the decomposition of the persistent component of income process between public information and private information.

The paper is organized as follows. In Section 2, we present the model and define the equilibrium. Section 3 provides some intuition for the lack of risk sharing. Section 4 describes the dual approach used to solve the model. The calibration of the model is presented in Section 5. In Section 6, I present the quantitative results of the benchmark model. Brief concluding remarks are provided in Section 7.

2 The Model

2.1 The Environment

We consider a pure exchange economy with a continuum of agents, each having expected utility of the form:

$$U(\{c_t\}_{t=0}^{\infty}) = (1-\beta)E\left\{\sum_{t=0}^{\infty}\beta^t u(c_t)\right\}.$$

Utility function $u : D \to W$, where D and W are intervals on R. Therefore, the range of U and u are the same. We index the consumers in the economy by their date 0 promised utility and type, $(w_0, \theta_{-1}) \in W \times \Theta$.

The endowment process of each individual is given by[1]

$$y_{t+1} = \exp\{\theta_t + \sigma\varepsilon_{t+1}\}, \tag{2}$$

where θ_t is a two state Markov chain with transition matrix

$$\begin{bmatrix} 1-\lambda & \lambda \\ \mu & 1-\mu \end{bmatrix}. \tag{3}$$

θ_t is i.i.d. across individuals. $\{\varepsilon_{t+1}\}$ is i.i.d. across people and time. This specification implies current income carries information about future income prospect only through θ_t. One can conveniently think of θ_t as a proxy of agent's ability, health status, or whatever private information that is relevant for the prospect of agent's future income. We assume that the realization of $\{y_t\}_{t=1}^{\infty}$ is public information. However, $\{\theta_t\}_{t=0}^{\infty}$ is agent's private information. Agent's type at date -1, i.e. θ_{-1} is also public information. Let Θ denote the space of possible realizations of θ. In our setting, $\Theta = \{\theta_H, \theta_L\}$. Let Y be the space of possible realizations of endowment y.

In this economy, an individual's history is an element in $\Theta^t \times Y^t$. Given the distribution of θ_{-1}, equation (2) and (3) determine the distribution of the infinite sequence $\{\theta_t, y_t\}_{t=0}^{\infty}$. Let μ^t denote the distribution of the sequence $(\{\theta_s\}_{s=-1}^{t}, \{y_s\}_{s=0}^{t})$, given that is the distribution of θ_{-1} is the steady state distribution of the Markov chain defined by the transition matrix (3). Therefore μ^t is a measure on $\Theta^t \times Y^t$.

Following Kehoe and Levine (1993) and Kocherlakota (1996), we assume a full

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[1] This formulation include the following formulation as a special case:

$$y_t = \exp\{\theta_t + \sigma\varepsilon_t\} \tag{1}$$

To see this, assume that θ_t is a AR(1) process and ε_t is normal i.i.d. distribution. To be more specific,

$$\theta_{t+1} = \rho\theta_t + \eta_{t+1}$$

Then we have:

$$\begin{aligned} y_{t+1} &= \exp\{\theta_{t+1} + \sigma\varepsilon_{t+1}\} \\ &= \exp\{\rho\theta_t + \eta_{t+1} + \sigma\varepsilon_{t+1}\} \end{aligned}$$

which is of the form in (2).

Since at date t, only θ_t is observed, θ_{t+1} is not observed. Specification (2) is a more convenient notation.

set of state contingent contracts is available to all agents, but contracts cannot be legally enforced other than by exclusion from future intertemporal trade. The threat of exclusion from markets induces individuals to honor risk sharing contracts.

We will first consider the social planner's problem and solve for the constrained Pareto optimal allocation of the economy. Decentralization of this allocation will follow Atkeson and Lucas (1995) and is omitted here. In solving for the planner's problem, it is convenient to think of the social planner as assigning utilities, in stead of consumptions. Therefore we also use $\{h_t(y^t, \theta^t)\}_{t=0}^{\infty}$ to denote a utility assignment plan, where $h_t(y^t, \theta^t) = u(c_t(y^t, \theta^t))$ is to be interpreted as the date t instantaneous utility assigned to an agent with past history (y^t, θ^t). We use $\{\{h_t(w_0, \theta_{-1}, y^t, \theta^t)\}_{t=0}^{\infty}\}_{(w_0,\theta_{-1})\in\Theta\times W}$ to denote an allocation. We use $C = u^{-1}$ to denote the inverse of agents' utility function.

An allocation of this economy is a sequence $\{\{h_t(w_0, \theta_{-1}, y^t, \theta^t)\}_{t=0}^{\infty}\}_{w_0\in W}$. In the following subsection, we state a notion of (constrained) efficiency in this environment.

2.2 Constrained Efficiency

An allocation $\{\{h_t(w_0, \theta_{-1}, \theta^t, y^t)\}_{t=0}^{\infty}\}_{(w_0,\theta_{-1})\in\Theta\times W}$ is feasible if it satisfies the following conditions:

1). Promise keeping: $\forall w_0 \in R$

$$w_0 = (1-\beta)E\left\{\sum_{t=0}^{\infty}\beta^t h_t(w_0, \theta_{-1}, \theta^t, y^t)|\theta_{-1}\right\} \tag{4}$$

2). Incentive Compatibility: for all reporting strategy $z = \{z_t\}_{t=0}^{\infty}$, $z_t : \Theta^t \times Y^t \to \Theta$,

$$E\left\{\sum_{t=0}^{\infty}\beta^t h_t(w_0, \theta_{-1}, \theta^t, y^t)|\theta_{-1}\right\} \geq E\left\{\sum_{t=0}^{\infty}\beta^t h_t(w_0, \theta_{-1}, z^t(\theta^t, y^t), y^t)|\theta_{-1}\right\} \tag{5}$$

3). Individual Rationality:

$$(1-\beta)E\left\{\sum_{j=0}^{\infty}\beta^j h_{t+j}(w_0, \theta_{-1}, \theta^{t+j}, y^{t+j})|\theta^t\right\} \geq U^{AUT}(y_t, \theta_t) \tag{6}$$

where

$$U^{AUT}(y,\theta) = (1-\beta)E\left\{\sum_{j=0}^{\infty}\beta^j u(y_{t+j})|\theta_t = \theta, y_t = y\right\} \tag{7}$$

is the Autarky utility.

4)

$$\lim_{t\to\infty}\beta^t \sup_{\theta^t,y^t}\sum_{j=0}^{\infty}\beta^j h_{t+j}(w_0,\theta_{-1},\theta^{t+j},y^{t+j}) = 0 \tag{8}$$

Let ψ_0 be a probability distribution over the space of $\Theta \times W$. Then an allocation is said to be efficient with respect to ψ_0 if it is feasible, and there does not exist another feasible allocation $\{\{\tilde{h}_t(w_0,\theta_{-1},\theta^t,y^t)\}_{t=0}^{\infty}\}_{w_0\in w}$ such that

$$\int_{\Theta\times W}\int_{\Theta^t\times Y^t} C(\tilde{h}_t(w_0,\theta_{-1},\theta^t,y^t))d\mu^t d\psi_0 \le \int_{\Theta\times W}\int_{\Theta^t\times Y^t} C(h_t(w_0,\theta_{-1},\theta^t,y^t))d\mu^t d\psi_0 \tag{9}$$

for all t and the inequality is strict for at least one t.

An allocation is said to satisfy the resource constraint if

$$\int_{\Theta\times W}\int_{\Theta^t\times Y^t} C(h_t(w_0,\theta_{-1},\theta^t,y^t))d\mu^t d\psi_0 \le \bar{y}\ \forall\ t \tag{10}$$

where $\bar{y}$ is the total resource in the economy and is given by:

$$\bar{y} = \int y_t d\mu^t \tag{11}$$

In the rest of this note, we will focus on constrained efficient allocations that satisfies the resource constraint.

3 Intuition for Lack of Risk Sharing

To understand the mechanism of incomplete risk sharing in our model, it is useful to compare this model with a model without the participation constraint (6). In the absence of the participation constraint, the optimal allocation is complete risk sharing among agents of the same type (since θ_{-1} is public information and there is continuum of agents of the same type, there is no need for risk sharing among agents of different types).

Adding the participation constraint may destroy the result of complete risk sharing among agents of the same type, if the participation constraint is violated along some history. This constraint is most likely to be violated for an agent of high type and with high income realization. Therefore, from a social planner's perspective, agents' of type are rewarded higher current and future consumption to prevent default on the financial contract. Since agent's type is private information, this implies incentive constraint is most likely to bind for agent of low type: they have an incentive to pretend to be high type. Therefore the optimal risk sharing would involve distorting the complete risk sharing plan of the high type in a way that prevent the low type from pretending to be of high type. This is done by "punishing the high type if his income is low". This punishment does not hurt the high type so much because high type's chance of getting a low income is very low; however, this provides incentives for low types to tell truth. For a low type agent, if he claims to be of high type, then the chance of getting a low income realization is high, therefore is more likely to be punished.

This mechanism has the following implications. First, it implies that consumption not only responds to income increases (as is true in models with only KL constraint), but also responds to income decreases. Next, elasticity of consumption with respect to income is increasing in income levels. The reason is the following: Consider an agent with low income realizations. Consumption's response to income only happens when the agent is of high type. However, among the income-poor people, the fraction of high type agent is small, consequently, consumption response to income is small when averaged out across agents. On the other hand, among the income-rich people, if the agent is of low type, consumption may or may not respond to income depending on whether the participation constraint is binding or not. If the agent is of high type, consumption respond to income since participation constraint is binding. Moreover, for high-income people, the fraction of high type agent is also high. Consequently, consumption respond more to income.

4 Solving the constrained efficient allocation: Dual approach.

Following Atkeson and Lucas (1992,1995), we formulate the constrained Pareto Optimality problem as a component planning problem. The insight of this approach is that the constrained Pareto optimal problem considered in section 3 can be decomposed into components indexed by (w_0, θ_{-1}). The original problem can be solved by considering a social planner who solve a cost minimization problem for each component, and who can transfer resources among the different component at some intertemporal price. This intuition is formalized in the following theorem, essentially a version of the first welfare theorem.

Theorem 1: *Suppose there exists a system of intertemporal prices* $\{p_t\}_{t=0}^{\infty}$ *and a consumption allocation rule* $\left\{\{\hat{h}_t(w_0, \theta_{-1}, \theta^t, y^t)\}_{t=0}^{\infty}\right\}_{(w_0,\theta_{-1})\in\Theta\times W}$ *such that for each* w_0, θ_{-1}, $\left\{\hat{h}_t(w_0, \theta_{-1}, \theta^t, y^t)\right\}_{t=0}^{\infty}$ *solves the the following cost minimization problem:*

$$\{\hat{h}_t(w_0, \theta_{-1}, \theta^t, y^t)\}_{t=0}^{\infty} \in \arg\min \sum p_t \int_{\Theta^t\times Y^t} \left[C(h_t(w_0, \theta_{-1}, \theta^t, y^t)\right] d\mu^t \tag{12}$$

$$s.t. \qquad w_0 = (1-\beta)E\left\{\sum_{t=0}^{\infty} \beta^t h_t(w_0, \theta_{-1}, \theta^t, y^t)|\theta_{-1}\right\} \tag{13}$$

$$(1-\beta)E[\sum_{j=0}^{\infty} \beta^j h_{t+j}(w_0, \theta_{-1}, \theta^{t+j}, y^{t+j}))|\theta^t, y^t] \geq u^{AUT}(y_t, \theta_t), \quad \textit{all } t, \theta^t, y^t \tag{14}$$

$$E[\sum_{t=0}^{\infty} \beta^t h_t(w_0, \theta_{-1}, \theta^t, y^t))] \geq E[\sum_{t=0}^{\infty} \beta^t h_t(w_0, \theta_{-1}, z^t(\theta^t, y^t), y^t))|\theta^t, y^t] \text{ for all } \hat{\theta} \text{ all } t, \theta^t, y^t \tag{15}$$

where the price system satisfies

$$\sum_{t=0}^{\infty} p_t < \infty$$

Then the consumption allocation is efficient.

In stead of considering an arbitrary prices vector $\{p_t\}_{t=0}^{\infty}$, we consider a planner's cost minimization problem with constant intertemporal price q (i.e. $p_t = q^t$). By doing so, we are considering the steady state of the component planning problem. Our general strategy is to solve the above problem for a particular q, and chose q such that the resource constraint is balanced.

One difficulty in solving the above problem, even with constant intertemporal price, is that the problem lacks a recursive structure. Following Fernandez and Phelan (2000), we consider the auxiliary planning problem. We introduce an additional constraint, called the threat keep constraint, so that the problem with this additional constraint is recursive. This is done via the following steps.

First, for each component (w_0, θ_{-1}), consider utility assignment plans $\{h_t(y^t, \theta^t)\}_{t=0}^{\infty}$ that satisfies the incentive compatibility constraint (5), participation constraint (6) and (8). Define:

$$w_0 = (1-\beta)E\left\{\sum_{t=0}^{\infty}\beta^t h_t(\theta^t, y^t)|\theta_{-1}\right\} \tag{16}$$

and

$$\hat{w}_0 = (1-\beta)E\left\{\sum_{t=0}^{\infty}\beta^t h_t(\theta^t, y^t)|\theta_{-1}^C\right\} \tag{17}$$

where we use θ^C to denote the complement of θ. Let $W^*(\theta_{-1})$ be the set of pairs $(w_0, \hat{w}_0)$ such that there exists an utility assignment $\{h_t(y^t, \theta^t)\}_{t=0}^{\infty}$ such that incentive compatibility and participation constraints are satisfied, and (16) and (17) are true. Note for an utility assignment to be a feasible allocation, it is enough that (5), (6), (8), and (16) are satisfied. By requiring (17) to be satisfied, we are putting an additional restriction of the utility assignment $\{h_t(y^t, \theta^t)\}_{t=0}^{\infty}$. Therefore we have the following theorem that characterize the set of feasible allocations.

Theorem 2: *Let $\{\{h_t(w_0, \theta_{-1}, \theta^t, y^t)\}_{t=0}^{\infty}\}_{(w_0,\theta_{-1})\in\Theta\times W}$ be a collection of utility assignments. It constitutes a feasible allocation if and only if $\forall(w_0, \theta_{-1})$, $(w_0, \hat{w}_0) \in W^*(\theta_{-1})$ for some $\hat{w}_0$.*

Therefore if we define the "auxiliary planning problem"as follows:

$$\begin{aligned} V_A(w_0, \hat{w}_0, \theta_{-1}) &= \min \sum p_t E\left[C(h_t(w_0, \theta_{-1}, \theta^t, y^t)\right] \\ s.t. \qquad w_0 &= (1-\beta)E\left\{\sum_{t=0}^{\infty}\beta^t h_t(w_0, \theta_{-1}, \theta^t, y^t)|\theta_{-1}\right\} \\ \hat{w}_0 &\geq (1-\beta)E\left\{\sum_{t=0}^{\infty}\beta^t h_t(w_0, \theta_{-1}, \theta^t, y^t)|\theta_{-1}^C\right\} \end{aligned}$$

$$(1-\beta)E[\sum_{j=0}^{\infty}\beta^j h_{t+j}(w_0, \theta_{-1}, \theta^{t+j}, y^{t+j}))|\theta^t, y^t] \geq u^{AUT}(y_t, \theta_t), \quad \textit{all } t, \theta^t, y^t$$

$$E[\sum_{t=0}^{\infty}\beta^t h_t(w_0, \theta_{-1}, \theta^t, y^t))] \geq E[\sum_{t=0}^{\infty}\beta^t h_t(w_0, \theta_{-1}, z^t(\theta^t, y^t), y^t))|\theta^t, y^t] \text{ for all } \hat{\theta} \text{ all } t, \theta^t, y^t$$

Then the original problem can be solved by:

$$\begin{aligned} &\min_{\hat{w}_0} V_A(w_0, \hat{w}_0, \theta_{-1}) \\ s.t.\ (w_0, \hat{w}_0) \ \in \ &W^*(\theta_{-1}) \end{aligned}$$

In stead of consider the original problem, we now work with the "auxiliary planning problem". First, we can show the constraint set of the auxiliary planning problem is recursive:

Theorem 3: $W^*(\theta_{-1})$ *is the largest fixed point of the following operator:*

$$
\begin{aligned}
BW(\theta_{-1}) = \{(w, \hat{w}) \in R^2 : \\
\forall\theta, \forall y \in R,\ \exists\ h(\theta, y),\ \textit{and}\ [w'(\theta, y), \hat{w}'(\theta, y)] \in W(\theta) \\
\textit{such that } 1).\ w = \sum_\theta \pi(\theta|\theta_{-1}) \int [(1-\beta)h(\theta, y)) + \beta w'(\theta, y)] dF(y|\theta_{-1}) \\
2).\ \hat{w} = \sum_\theta \pi(\theta|\theta^C_{-1}) \int [(1-\beta)h(\theta, y)) + \beta w'(\theta, y)] dF(y|\theta^C_{-1}) \\
3).\ (1-\beta)h(\theta, y) + \beta w'(\theta, y) \geq (1-\beta)h(\theta^C, y)) + \beta \hat{w}'(\theta^C, y) \quad \forall\theta, \forall y \\
4).\ (1-\beta)h(\theta, y) + \beta w'(\theta, y) \geq u^{AUT}(\theta, y)\} \forall\theta, \forall y
\end{aligned}
$$

and $B^n(W) \to W^*$ *if* $W \supseteq W^*$.

Now the value function of the auxiliary planning problem can be show to be the solution to some Bellman equation:

Theorem 4:

1) The value function of the auxiliary component planning problem is the unique fixed point of the following operator:

$$
Tf(w, \hat{w}, \theta_{-1}) = \min_{\substack{h(\theta,y) \\ [w'(\theta,y), \hat{w}'(\theta,y)] \\ \in W^*(\theta)}} \sum_\theta \pi(\theta|\theta_{-1}) \int [(1-q)C(h(\theta, y)) + qf(w'(\theta, y), \hat{w}'(\theta, y), \theta)] dF(y|\theta_{-1}) \tag{18}
$$

$$
\begin{aligned}
w = \sum_\theta \pi(\theta|\theta_{-1}) \int [(1-\beta)h(\theta, y)) + \beta w'(\theta, y)] dF(y|\theta_{-1}) \\
\hat{w} = \sum_\theta \pi(\theta|\theta^C_{-1}) \int [(1-\beta)h(\theta, y)) + \beta w'(\theta, y)] dF(y|\theta^C_{-1}) \\
(1-\beta)h(\theta, y) + \beta w'(\theta, y) \geq (1-\beta)h(\theta^C, y)) + \beta \hat{w}'(\theta^C, y) \quad \forall\theta, \forall y \\
(1-\beta)h(\theta, y) + \beta w'(\theta, y) \geq u^{AUT}(\theta, y) \forall\theta, \forall y
\end{aligned}
$$

2) The fixed point V_A *is strictly increasing in* w*, strictly convex in* w *and weakly convex in* $(w_0, \hat{w}_0)$*, and continuously differentiable.*

After we solved the problem for fixed q, we find the steady state allocation implied by the optimal solution associated with the q. We check if the resource constraint is satisfied. It can show that the excess demand function is a monotone function of q.

Therefore we can update q until the resource constraint is balanced.

5 Calibration

In this section, we document the calibrations and computations procedure of we did with the model outlined above. We use CRRA utility of the form

$$u(c) = \frac{c^{1-\eta}}{1-\eta}$$

with $\eta = 2$. We chose $\beta = 0.95$. In the calibration part, we discretize the space of possible realizations of endowment $Y = \{y_1, y_2, \cdots y_M\}$. To guarantee the boundedness of the utility function, we assume consumption of agents never exceeds the maximum amount of endowment y_M and never falls below the minimum amount of endowment y_1. We obtain parameters in the income process (2) from the estimation that is done in the literature. Storesletten, Telmer and Yaron (1998) used the following specification of the income process:

$$\begin{aligned} \ln y_t^i &= \alpha_i + \mu_{it} + \varepsilon_{it},\ \alpha_i \sim N(0, \sigma_\alpha^2), \varepsilon_{it} \sim N(0, \sigma_\varepsilon^2) \\ \mu_t &= \rho_y \mu_{t-1} + \nu_t,\ \nu_t \sim N(0, \sigma_\nu^2) \end{aligned}$$

where α_i is individual specific component, and ε_{it} is and i.i.d. shock. The parameters estimated by Storesletten, Telmer and Yaron (1998) are:

ρ_y	σ_ν^2	σ_α^2	σ_ε^2
0.98	0.019	0.32	0.005

We discretize the μ process into 2 stage Markov chain using the procedure in Tauchen and Hussy (1991), and obtain the persistent component θ. The parameters we got from this procedure are:

$$\theta^L = -0.1378 \quad \theta^H = 0.1378$$

$$\pi(\theta|\theta_{-1}) = \begin{bmatrix} 0.8765 & 0.1235 \\ 0.1235 & 0.8765 \end{bmatrix}$$

We set

$$\sigma = \sigma_\alpha + \sigma_\varepsilon \tag{19}$$

where σ is the parameter in equation (2).

By this procedure, we assume all the persistent component of income process is private information. This assumption is probably not a realistic one, but it gives us a bench mark case to understand the properties of the optimal contract. However, this exercise does provide important information on how large the quantitative effect of private information is likely to be.

We discretize the conditional distribution of $F(y|\theta^L)$ and the conditional distribution $F(y|\theta_H)$ by matching the first eight conditional moments of $F(y|\theta_H)$ and $F(y|\theta_L)$. The result of the discretization is:

$$Y = \left\{ \begin{array}{cccccc} y_1 & y_2 & y_3 & y_4 & y_5 & y_6 \\ 0.1487 & 0.3385 & 0.7021 & 1.4243 & 2.9545 & 6.7269 \end{array} \right\}$$

$$F(y|\theta^H) = \begin{array}{cccccc} 0.0053 & 0.1342 & 0.4630 & 0.3430 & 0.0535 & 0.0010 \end{array}$$

$$F(y|\theta^L) = \begin{array}{cccccc} 0.0010 & 0.0535 & 0.3430 & 0.4630 & 0.1342 & 0.0053 \end{array}$$

6 Quantitative Results

In the rest of this section, we show some computed result and discuss the intuition of the result.

Figure 1 is the set of implementable promised value-threat value pairs. The set on the upper graph is $W^*(\theta_H)$, and the set depicted on the lower graph is $W^*(\theta_L)$. By construction, $W^*(\theta_H)$ is the transpose of $W^*(\theta_L)$ by $\pi/2$. The southwest corner is the allocation on both graph associated with autarky. The w axis represents the possible life-time utilities that can be delivered to agents of each type. For each w, the line segment between the upper boundary and lower boundary of the set represents the set of possible utility levels that agent of the opposite type can achieve by mis-reporting to be of that type. Any point in the set represent a utility pair $(w, \hat{w})$ that can be implemented by some feasible consumption allocation as defined in section 2. The sets depicted on figure 1 are the domain of the value function $V(\cdot, \cdot, \theta_H)$ and $V(\cdot\cdot, \theta_L)$.

Figure 2 depicts the value function V. The upper-left panel is $V(\cdot, \hat{w}, \theta_L)$ for a fixed $\hat{w}$; the upper-right panel is $V(w, \cdot, \theta_H)$ for a fixed w. We see that the value function is convex and increasing in the w dimension. The value function $V(w, \hat{w}, \theta)$ is interpreted as the minimum cost to provide the agent of type θ life-time utility

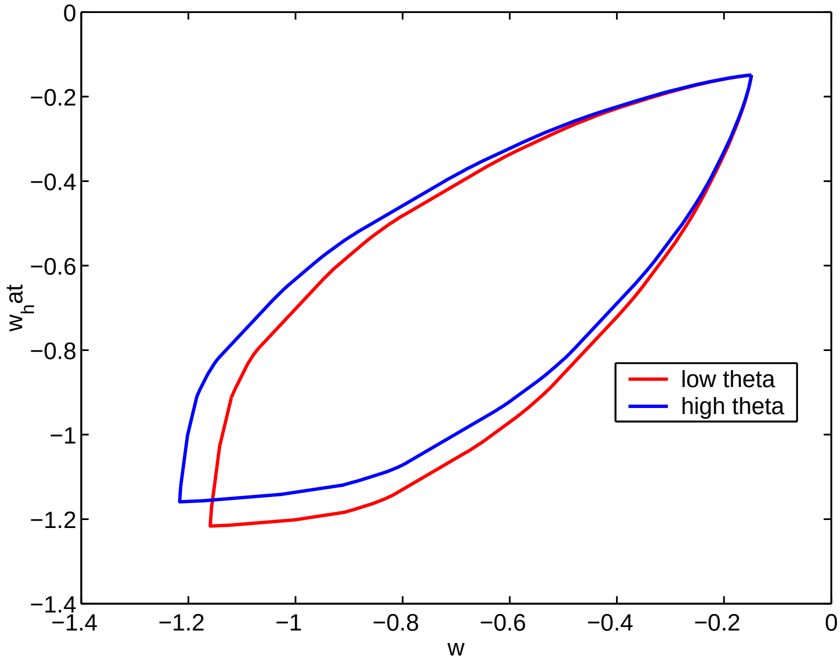

Figure 1: Feasible set

level w, and at same to guarantee agent not of type θ life-utility $\hat{w}$. The upper-right panel of figure 2 depict the typical shape of the value function along the $\hat{w}$ dimension. The fact that the slope of $V(w, \hat{w}, \theta)$ in $\hat{w}$ is very steep when it gets close to the boundary of the domain of the set $W^*(\theta_H)$ is because typically there is very few consumption allocation to support a utility pair when the later is close to the boundary. Consequently, for a fixed w, changing $\hat{w}$ is painful for the planner. As $\hat{w}$ moves closer to the center, there are many consumption allocations that can support a given utility pair, consequently, the minimum cost is lower. The fact that the the function $V(w, \cdot, \theta_H)$ is almost flat in the center reflects the fact that the likelihood of the two conditional distributions $F(y|\theta_H)$ and $F(y|\theta_L)$ is far from each other, therefore it is relatively easy for the planner to distinguish between the two types based the observation of realization of income levels. Consequently, for a fixed w, the planner can provide different levels of $\hat{w}$ to the mis-reporting agent almost without raising the cost. It indicates that in this region the problem of private information is less severe.

Figure 3 plot the typical consumption path under the optimal plan. Figure 3.1 plots the consumption path of a low type agent when he receive a sequence of low shocks, $[y_1, y_1, y_1, \cdots y_1]$. Figure 3.2 plots the consumption path of the same agent when he received a sequence of medium level income shocks $[y_3, y_3, y_3, \cdots y_3]$. We see the consumption path is identical, regardless of income realizations. This indicates perfect risk sharing for agent of low type and medium-low income level. The intuition for such result is that for agents of low type and relatively low income levels, neither the participation constraint bind, nor does the truth telling constraint bind. The fact that the consumption profile is downward sloping is because of the low interest rate, i.e. $q > \beta$. Figure 3.3 is the consumption path of an low type agent with income realizations $[y_1, y_6, y_1, \cdots y_1]$. Note upon receiving a high shock, the agent's consumption increases. This indicates a binding participation constraint. Figure 3.4 is the consumption path of a high type agent upon receiving a sequence of medium income shocks $[y_3, y_3, y_3, \cdots y_3]$. The consumption profile is downward sloping indicating neither the participation constraint nor the truth telling constraint binds. Figure 3.5 plots the consumption path of a high type agents upon receiving a sequence of high income shocks $[y_6, y_6, y_6, \cdots y_6]$. The upward sloping consumption profile is because the participation constraint is always binding. The consumption path asymptotes to a level that corresponds to the upper left corner of the set $W^*(\theta_H)$, that is, the set depicted in figure 1, lower panel. The last graph indicates one of the key differences of our model's prediction relative to that of a pure limited enforcement model. That is, consumption responds to negative income shocks for high type. Figure 3.6 plots the consumption path of a high type agent upon receiving the a sequence of income shocks $[y_6, y_1, y_1, y_6, y_6, y_6]$. It is clear that the consumption of a high type agent falls sharply when he receives a low income shock. This is because of the truth telling constraint. As we remarked before, it is the truth telling constraint of the low type agent that is most likely to be binding. Therefore the optimal consumption plan require distorting the high type's allocation in such way that low type has no incentive to claim to be of high type. This involves giving the high type a really low consumption level upon seeing a low income realization. This punishment does not affect high type's utility too much, since for high type agent the probability of a low income realization is low, such punishment only happens with small probability. However, this punishment is severe to low type agent if he claim to be of high type, since for low type agent, the probability of a low income realization is high and the low type is much more likely to be punished.

Our model also has the prediction that the consumption-wealth elasticity is increasing with respect to income level. The intuition for this result is that for high type agent, consumption changes is more likely to respond to income changes. If high type agents' income realization is high, then the participation constraint is more likely to bind, if the income realization is low, then the punishment described above is more likely to happen. As a consequence, consumption of high type agent is more likely to covary with income realizations.

6.1 Empirical Findings

Our empirical analysis draws on the CEX micro data. The CEX is carried out by the Bureau of Labor Statistics (BLS), and is a random sample rotating panel that contains information on demographic characteristics, major housing inventory and consumption expenditure. The survey consists of an Interview Survey in which each consumer unit in the sample is interviewed every three months over a 15-month period, and a Diary Survey which is completed by the sample consumer units for two consecutive one-week periods. The Diary Survey records consumer units' self-reported daily purchases over two consecutive one-week periods. The Interview Survey is designed to collect data on major items of expense, household characteristics, and income. In each quarter, BLS chooses randomly about 5000 households according to stratification criteria determined by the U.S. Census. The expenditures covered by the survey are those that respondents can recall fairly accurately for three months or longer. Consumer units are defined as members of a household related by blood, marriage, adoption, or other legal arrangement, single person living alone or sharing a household with others, or two or more persons living together who are financially dependent. The head of the household in the CEX is the person or one of the persons who owns or rents the unit. The CEX is the only micro-level data set reporting comprehensive measures of consumption expenditure for a large cross-section of households in the U.S. It has been widely used to study various issues such as inequality, consumption smoothing, and asset pricing).

We use 1986-2002 data from the CEX Interview Survey. Prior to 1980, the CEX was conducted about every 10 years and not on a regular basis. Data for years after 2002 are not yet released. We do not include the years 1980 and 1981 since the quality of the CEX consumption data is lower for this period (Attanasio and Weber (1995)), the years 1982 and 1983 since rural households are excluded (See Attanasio (1998) for details). The BLS changed its household identification numbering system in 1986, leaving no information about the correspondence between the household identification numbers in quarter 4 of 1985 and quarter 1 of 1986. Thus we use data starting from 1986. In the survey, expenditure is reported in each interview (after the first) and refers to the months of the previous quarter. For example, a household interviewed in April 1986 reports expenditure for January, February, and

March 1986. Income is reported in the second and fifth interview, and it refers to the previous twelve months. Thus we pick the corresponding consumption data at the second and the fifth interviews. We exclude households with non-positive income, non-positive consumption, or whose income is considered incomplete. We also exclude households who did not complete either the first or the last interview. We will focus on households whose head is between 25-65 years old. The final sample consists of about 34,000 consumption and income growth observations.

Consumption is defined as quarterly expenditures on nondurable consumption, which includes food, alcoholic beverages, tobacco, apparel and services, gasoline and auto oil, household operations, utilities, public transportation, personal care, entertainment, and miscellaneous expenditures[2]. It excludes expenditure on various durables, housing, vehicles, education and health[3]. Income consists of after-tax labor earnings, plus transfers minus mandatory deductions. This income definition includes all sources of income due to public insurance programs, but excludes all income that results from private insurance contracts or self-insurance via asset accumulation, such as borrowing and lending on the credit market, unrealized gain in securities, and other formal or informal insurance arrangements. Nominal consumption is deflated by the corresponding three month average CPI for nondurables for urban households. Income values are deflated by the corresponding annualized monthly CPI for urban households.

The regression we run is

$$\triangle \log c_{it} = \alpha_1 + \alpha_2 \triangle \log y_{it} + \alpha_3 \sum \triangle \log c_{it} + \varepsilon_{it}$$

Table 3 shows the regression results of coefficients and corresponding standard errors. The parameter α is positive and significantly different from 0 at 1%. The parameter β is not significantly different from 1. We interpret our results as that, conditioning on aggregate consumption, an individual consumption depends on the individual's own income, thus rejecting full risk-sharing assumption. This confirms the earlier results in the literature rejecting full-insurance. We also notices that the coefficient on individual income is quite small, with one percent change of income changes consumption by 16 percent. This indicates that the degree of consumption

[2]The definition of total non durable consumption is similar to Attanasio and Weber (1995).

[3]The main reason that we exclude durables, health, vehicles, education and housing is to aviod treating issues related with duarability.

smoothing is quite large.

CEX	model
0.1685	0.2689

Next, we study the response of changes in consumption to changes in income for negative and positive income shocks. Altonji and Siow (1987) and Dynarski et. al. (1997) find that the coefficients on negative and positive income change are not significant different from each other and are subject to large standard errors. However, Krueger and Perri (2004) find the asymmetry to be statistically significant.

Since the U.S. economy is constantly growing, we use the median growth rate of income as the trend. We divide the whole sample into two subsamples, one contains households whose growth rate of income is lower than the trend, the other contains households whose growth rate of income is higher than the trend. We run separate regressions for each sample. Table 4 shows the regression results. The sample displays a considerable amount of asymmetry, where the quantities in the parenthesis are the standard deviations. When income decreases (to be more precise, increases less than the trend), the response of consumption change to income change is larger and significant at the 1% level. When income increases (to be more precise, increases more than the trend), the response of consumption change to income change is smaller. However, these coefficients are not statistically distinguishable.

	CEX	model
Negative	0.1310	0.0370
Positive	0.0819	0.2689

Furthermore, we see how households with different lifetime income respond to income shocks. Ideally we should differentiate households by their lifetime income but CEX only contains households' income data within 2 years. So we use the mean of two annual income observations as a proxy for lifetime income and define those with a mean income above the median of the mean in the sample to be rich and those with a mean income below the median of the mean in the sample to be poor. Table 5 shows the regression results. Here the conclusion is that rich people's consumption change is more sensitive to their income change than poor people. Table 6 shows the regression results if we classify income into four groups. The same conclusion remains true. Figure 1 shows the elasticity of consumption w.r.t. income for each income decile. We see clearly that the elasticity increases dramatically with income, with the exception of the top two deciles. The low elasticity of top two deciles may be caused by the topcoding in CEX which may bias the OLS regression downwards.

Table 5 shows the regression results of α_2 of the model and CEX data. Here the conclusion is that rich people's consumption change is more sensitive to their income change than poor people.

	CEX	model
poor	0.1231	0.0370
rich	0.2577	0.2689

Table 6 shows the regression results if we classify income into four groups. The same conclusion remains true.

Quartile	CEX	Model
1	0.0512	0.0211
2	0.2240	0.0452
3	0.2841	0.2229
4	0.2325	0.3434

We find the consumption-income elasticity is increasing in income levels except for the very top income levels. The current calibration is qualitatively consistent with this pattern. We run the same regression using the CEX data and the data generated by the simulation of our model. Our model predicts the similar pattern.

7 Conclusions

We formulated a model in which, agents have private information about their future income, and contingent contracts can only be enforced by the threat of exclusion from future intertemporal trade. We applied techniques developed by Atkeson and Lucas (1992, 1995) and Fernandez and Phelan (2000) to characterize the stationary efficient allocation in this environment. We calibrate the model to the US data and compare its quantitative predictions with the US data.

In the calibration exercise we do now, we obtain the parameters of the income process from the estimation that is done for a similar but different model. We need to estimate the parameters of our model directly from PSID.

Right now, we are assuming all predictability of future income is private information. In the future, we plan to parameterize the private information component in predicting agents' future income and calibrate this parameter.

8 Appendix

8.1 The Computation Procedure

In this section, we outline the basic procedure of computing the numerical solutions to the model.

Step 1: Pick a intertemporal price $q \in (\beta, 1)$. Compute the fixed point of the B operator, i.e. the set W^* defined in theorem 3. In this step we use the computation algorithm of Judd, Yeltken and Conklin (2003).

Step 2: Given W^*, compute the fixed point of the T operator in (18). In this step we approximate the upper boundary and lower boundary of the set W^* using the shape preserving method developed by Schumaker (1983) and further elaborated by Judd and Solnick (1994). This procedure ensure that we have a convex programming problem and thus improve the accuracy and efficiency of the computation.

Step 3: The optimal policy function of the dynamic programming problem in (18) define a Markov operator on the space $D \times D \times \Theta$ through the following procedure:

$$\begin{aligned} & Q(w, \hat{w}, \theta_{-1}; A) \\ = & \int I_{[w'(w,\hat{w},\theta_{-1},\theta,y),\hat{w}'(w,\hat{w},\theta_{-1},\theta,y),\theta]\in A}(d)F(y|\theta^L) \cdot \pi(\theta|\theta_{-1}) \end{aligned}$$

We compute the steady state distribution of the Markov operator Q. This distribution is the steady state distribution of $(w, \hat{w}, \theta)$ under the optimal policy function of the dynamic programming programming problem (18).

Step 4: Compute the excess demand of the steady state:

$$d(q) = \int_{D\times D\times\Theta} \int C[h(w, \hat{w}, \theta; \theta', y')]dF(y|\theta^L) \cdot \pi(\theta|\theta_{-1})d\phi_q - \int y d\phi_q \tag{20}$$

If $d(q) > 0$, then increase q, come back to step 1; If $d(q) < 0$ then decrease q and come back to step 1. Continue this procedure until $||d(q)||$ is small enough.

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