

Social Security Reform with Impure Intergenerational Altruism*

Fang Yang[†]

SUNY-Albany

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Abstract

This paper studies the long-run aggregate and welfare effects of eliminating Social Security in a quantitative dynamic general equilibrium life-cycle model where parents and their children are linked by voluntary and accidental bequests. Social security in this model with impure altruism has a smaller effect on capital accumulation than in a pure life-cycle model, a bigger effect than in a model with two-sided altruism. The welfare gain of eliminating Social Security system under impure altruism is smaller than in a pure life-cycle model, and bigger than in a model with two-sided altruism.

Keywords: Social Security; Altruism; Heterogeneous agents; Welfare

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[†]Mailing Address: Department of Economics, BA 123E, University at Albany, State University of New York, Albany, NY 12222. Email: fyang@albany.edu. URL: www.albany.edu/~fy554862.

1 Introduction

The effect of Social Security depends crucially on whether and how generations are altruistically linked. Many studies of Social Security reform find that imposing an unfunded Social Security system crowds out a large quantity of household saving and eliminating it has important welfare and distributional effects. Those studies assume that households do not have bequest motives.¹ On the other extreme, Fuster, Imrohoroglu and Imrohoroglu (2003, 2007) find that the crowding-out effects and the welfare loss of unfunded Social Security is very small in a two-sided altruistic framework.²

The importance of transmission of physical capital across generations in aggregate saving has been well established. Kotlikoff and Summers (1981) report that 80 percent of the U.S. household wealth is inherited. This finding implies that the simple life-cycle model without bequest motive is an inadequate description of saving behavior in the U.S. De Nardi (2004) shows that voluntary bequests and transmission of human capital are important mechanisms for generating a highly concentrated wealth distribution. Yang (2008) adopts a similar framework and shows that the model with voluntary bequests can generate the empirical observation that households with similar lifetime earnings hold very different amounts of wealth at retirement.

This paper quantitatively assesses whether the existence of impure warm-glow altruism and intergenerational links in a life-cycle model is important for understanding the long-run effects of Social Security on aggregate allocations and on the distribution of welfare across households. The model that I am using mainly incor-

¹Contributors to this literature include, among others, Feldstein (1985), Auerbach and Kotlikoff (1987), Hubbard and Judd (1987), Imrohoroglu, Imrohoroglu and Joines (1995, 1999), Huggett and Ventura (1999), Storesletten, Telmer and Yaron (1999), Conesa and Krueger (1999), Krueger and Kubler (2006), Rojas and Urrutia (2008), and Chen (2008).

²Other papers that examine the effect of Social Security under two-sided altruism include, among others, Laitner (1988), Altig and Davis (1993), and Fuster (1999).

porates two basic forces, bequests and human capital transmission, into an otherwise standard life-cycle model. Agents care about the total bequests left to their children, but not about consumption of their children. In this model, households face uninsurable labor income risk, uncertain lifetimes and a borrowing constraint. Households save to self-insure against labor earning shocks and life-span risk, for retirement, and possibly to leave bequests to their children. Pay-as-you-go Social Security provides insurance against income risk and lifespan risk, since retirement benefits are financed by a payroll tax and partially linked to lifetime contributions. The model under Social Security system is calibrated to U.S. data and it generates household asset composition over the life cycle and wealth inequality similar to the data.

I compare the steady state in the benchmark model with Social Security to one in which the replacement rate is zero. The main findings can be summarized as follows. Consistent with the existing literature that uses a life-cycle environment, the steady-state values of aggregate consumption, output and capital are higher after elimination of Social Security system. An unfunded Social Security system crowds out only 35% of the capital stock, less than in pure life-cycle models, regardless of whether accidental bequests are inherited by the descendent of the deceased, or are evenly distributed among agents of the same age group. This difference is partially due to the existence of a bequest motive. In my model, old individuals might save for bequest motives and respond less to the elimination of Social Security system. As a result, Social Security in the benchmark model has a smaller effect on aggregate saving and capital accumulation than in a pure life-cycle model.

I then compare the results from the benchmark model with those from a model with two-sided altruism. In a model with two-sided altruism, the transfer induced by Social Security is partially undone by altruistic transfers from parents to children, thus Social Security has a small impact on saving. In the benchmark model with

one-sided impure altruism, households do not receive much transfer and there is less insurance among generations. Therefore Social Security has a bigger crowding-out effect on the capital stock.

The model with intergenerational links of bequest and human capital can generate a more skewed wealth distribution than models without a bequest motive and the model with two-sided altruism. However, a redistributive Social Security system leads to higher wealth dispersion in all models. The current Social Security system gives households with lower social security earnings a higher replacement rate, which reduces wealth holding at the bottom of the distribution relatively more.

I also look at the overall welfare effects of eliminating Social Security for the economy. The welfare gain of eliminating Social Security system under impure altruism is positive, although smaller than that in a pure life-cycle model. Middle-aged households in a model with intended altruism receive larger bequests than in pure life-cycle models, which relax borrowing constraints. Thus the benefit of removing Social Security tax from relaxing borrowing-constraints is smaller. In addition, aggregate consumption increases less after elimination than in pure life-cycle models. The welfare gain in the benchmark model is bigger than in a model with two-sided altruism. In the benchmark model, households have one-sided impure altruism which does not provide much insurance among generations. Besides, the crowding-out effect of Social Security in the model with one-sided altruism is bigger. Therefore, as in the vast majority of the life-cycle models, the large negative effect of Social Security on capital accumulation and on consumption smoothing leads to an important reduction in steady-state welfare.

There is a literature that studies the effect of Social Security in a life-cycle framework with intergenerational transfer of bequest or bequest motive. Abel (1985) shows that, in an environment with intergenerational transfer of accidental bequests, the

introduction of fully funded Social Security crowds out steady-state private wealth. Abel (1986) analyzes theoretically the introduction of a fully funded Social Security system in an environment with a voluntary bequest motive and a private annuity market that is subject to adverse selection. De Nardi, Imrohorglu and Sargent (1999) study the effect of demographics on Social Security in an environment with voluntary bequest motive and without intergenerational transfer of bequest.

The paper is organized as follows. In Section 2, I present the benchmark model and define the equilibrium. The calibration of the model is presented in Section 3. In Section 4, I compare the quantitative implications of eliminating Social Security in the benchmark model with pure life-cycle models without altruism and with a two-sided altruism model. Brief concluding remarks are provided in Section 5.

2 The Model

The economy is a discrete-time overlapping generations world with an infinitely-lived government. There are idiosyncratic earnings shocks. These shocks are uninsurable: The only financial instrument is a one-period bond. Households cannot engage in unsecured borrowing; net assets must be non-negative. There is mortality risk but private annuity markets do not exist.³ Members of successive generations are linked by bequests and the children's inheritance of part of their parent's productivity. At age 20, each agent enters the model and starts consuming, working, and paying labor and capital income taxes.⁴ At age 35, the agent procreates. After retirement, the agent no longer works but receives interest from accumulated assets and benefits from Social Security. The government taxes labor earnings, capital income and estates,

³Although private annuity market do exist in the U.S., due to reasons such as the lack of actuarially fair price, and the existence of bequest motives, the demand for annuities is quite low.

⁴In order to focus on the effect of Social Security on capital accumulation, I assume inelastic labor supply, omitting the distortion caused by Social Security payroll tax.

pays Social Security benefits to retirees, and provides government consumption.

2.1 Demographics

During each model period, which is 5 years long, a continuum of people is born. Since there are no inter-vivos transfers, all agents start their working life with no assets.⁵ I denote age $t = 1$ as 20 years old, age $t = 2$ as 25 years old, and so on. At the beginning of period 4, the agent's children are born, and four periods later (when the agent is 55 years old) the children are 20 years old and start working. The agents retire at $t = 10$ (when they are 65 years old) and die by the end of age $T = 14$ (before turning 90 years old). From $t = 10$ (when they are 65 years old), each person faces a positive probability of dying, given by $(1 - p_t)$. The probability of dying is exogenous and independent of other household characteristics. The population grows at rate n . Since the demographic patterns are stable, agents at age t make up a constant fraction of the population at any point in time. Figure 1 illustrates the demographics in the model.

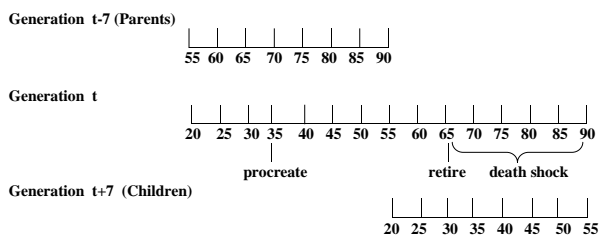


Figure 1: Demographics

⁵Data from the HRS suggests that observed inter-vivos monetary transfers are fairly small (Cardia and Ng (2000)). Given the small size of observed inter-vivos monetary transfers, I doubt that this simplification would affect the quantitative predictions of my model much.

2.2 Government

The government taxes labor earnings, capital income and estates to finance the exogenous public expenditure, G . Income from labor is taxed at a flat rate τ_l .⁶ Income from capital is taxed at a flat rate τ_a . Government taxes bequests at the rate τ_b for the proportion above the exemption level ex_b .

The structure of the Social Security system is the following: Government taxes labor earnings below e_{max} , at a flat rate of τ_{ss} . Retired households receive Social Security benefits from the government each period until they die. The Social Security benefits that individuals receive are linked to their average lifetime earnings according to a piecewise linear function, as in the US Social Security program.

2.3 Technology

There is one type of good produced according to the aggregate production function $F(K; L) = K^\alpha L^{1-\alpha}$, where K is the aggregate capital stock and L is the aggregate labor input. The final goods can be either consumed or invested into physical capital. Physical capital depreciates at rate δ . Households rent capital and efficient labor units to the representative firm each period, and receive rental income at the interest rate r and wage income at the wage rate w .

2.4 Labor Productivity

In this economy, all agents of the same birth cohort face the same exogenous age-efficiency profile, ϵ_t . Each worker i also faces stochastic productivity shocks y_t^i , which

⁶In the model, labor earnings are estimated using after-tax earnings. Thus all the progressive features of the tax system are already reflected in the calibrated after-tax earnings distribution. I introduce a constant tax rate τ_l to balance the government budget.

follows a Markov process Q_y

$$\ln y_t^i = \rho_y \ln y_{t-1}^i + \mu_t^i, \quad \mu_t^i \sim N(0, \sigma_y^2).$$

This Markov process is the same for all households, so that there is no uncertainty over the aggregate labor endowment. The total productivity of a worker at age t is given by the product of the worker's age- t productivity shock and age- t deterministic efficiency index: $y_t^i \epsilon_t$.

To capture the intergenerational correlation of earnings, I assume the parent's productivity shock at age 55 is transmitted to children at age 20 according to the following transition function Q_{yh}

$$\ln y_1^i = \rho_{yh} \ln y_{h,8}^i + \nu_1^i, \quad \nu_1^i \sim N(0, \sigma_{yh}^2).$$

What the children inherit is only their first draw; from age 20 on, their productivity y_t evolves stochastically according to Q_y .

2.5 Consumer's Maximization Problem

2.5.1 Preferences

Individuals derive utility from consumption and from bequests transferred to their children upon death. Preferences are assumed to be time separable, with a constant discount factor β . The momentary utility function from consumption is of the constant relative-risk aversion class given by

$$(1) \quad U(c) = \frac{c^{1-\eta}}{1-\eta}.$$

Following De Nardi (2004), the utility from bequest b is denoted by

$$(2) \quad \phi(b) = \phi_1(1 + b/\phi_2)^{1-\eta}.$$

The term ϕ_1 reflects the parent's concern about leaving bequests to his/her children,

while ϕ_2 measures the extent to which bequests are luxury goods.⁷

2.5.2 The Household's Recursive Problem

In a stationary equilibrium, the interest rate is constant at r and the wage rate is at w . I assume that children have full information about their parents' state and children solve their decision problems after observing their parents' decisions. Children infer the size of the bequests they are likely to receive based on this information. The household's state variables are given by $(t, a, y, \tilde{y}, S_p)$. The first four variables denote the agent's age, financial assets carried from the previous period, the agent's productivity, and cumulated productivity, respectively. The last term S_p denotes the agent's parent's state variables and differs in each of the following four cases.

(i) From $t = 1$ to $t = 2$ (from 20 to 25 years of age), the agent survives for sure until next period and does not expect to receive a bequest because his/her parent is younger than 65.

$$(3) \quad V(t, a, y, \tilde{y}, a_p, y_p, \tilde{y}_p) = \max_{c, a'} U(c) + \beta E(V(t+1, a', y', \tilde{y}', a'_p, y'_p, \tilde{y}'_p))$$

subject to

$$(4) \quad c + a' = (1 - \tau_l)w\epsilon_t y - \tau_{ss} \min(w\epsilon_t y, e_{max}) + (1 + r(1 - \tau_a))a$$

$$(5) \quad a' \geq 0, c \geq 0,$$

$$(6) \quad \tilde{y}' = ((t-1)\tilde{y} + \min(w\epsilon_t y, e_{max}))/t,$$

$$(7) \quad \tilde{y}'_p = ((t+6)\tilde{y}_p + \min(w\epsilon_{t+7}y_p, e_{max}))/t + 7.$$

At any subperiod, the agent's resources depend on asset holdings, a , and labor endowment, $\epsilon_t y$. Asset holdings pay a risk-free rate r and labor receives a real wage

⁷With the specifications, bequests are luxury goods. The scalar 1 inside the parameters ensures that the marginal utility of small bequests is bounded, while the marginal utility of large bequests declines more slowly than the marginal utility of consumption.

w . Average Social Security earnings for children and parents accumulate according to equations (6) and (7), respectively.

(ii) From $t = 3$ to $t = 7$ (from 30 to 50 years of age), the worker survives for sure until the next period. However, the agent's parent is at least 65 years old and faces a positive probability of dying at any period; hence, a bequest might be received at the beginning of the next period. Let $V^I(t, a, y, \tilde{y})$ and $V(t, a, y, \tilde{y}, a_p, \tilde{y}_p)$ denote the value function of a person whose parent is dead and alive, respectively.⁸ In the former case, a_p and \tilde{y}_p are not in the state space any more.

$$(8) \quad V^I(t, a, y, \tilde{y}) = \max_{c, a'} U(c) + \beta E(V^I(t+1, a', y', \tilde{y}'))$$

subject to (4), (5), and (6).

(9)

$$V(t, a, y, \tilde{y}, a_p, \tilde{y}_p) = \max_{c, a^+} U(c) + \beta E(p_{t+7} V(t+1, a^+, y', \tilde{y}', a'_p, \tilde{y}'_p) + (1-p_{t+7}) V^I(t+1, a', y', \tilde{y}'))$$

subject to (5), (6), and

$$(10) \quad c + a^+ = (1 - \tau_l) w \epsilon_t y - \tau_{ss} \min(w \epsilon_t y, e_{max}) + (1 + r(1 - \tau_a)) a$$

$$(11) \quad a' = a^+ + b,$$

where a^+ denotes the financial assets at the end of the period before receiving bequests.

(iii) In the sub periods $t = 8$ to $t = 9$ (from 55 to 60 years of age), no more inheritances are expected because the agent's parent is already dead. The agent does not face any survival uncertainty.

$$(12) \quad V(t, a, y, \tilde{y}) = \max_{c, a'} U(c) + \beta E(V(t+1, a', y', \tilde{y}'))$$

⁸Since parents are retired, parent's productivity y_p is not a state variable any more, and Social Security contributions for parents, \tilde{y}_p , do not change over time.

subject to (4), (5), and (6).

(iv) From $t = 10$ to $t = 14$ (from 65 to 85 years of age), the agent does not work and does not inherit any more, but faces a positive probability of dying. In case of death, the agent derives utility from bequeathing his/her assets. Households receive Social Security benefits $P(\tilde{y})$.

$$(13) \quad V(t, a, \tilde{y}) = \max_{c, a'} U(c) + \beta p_t (V(t+1, a', \tilde{y})) + (1 - p_t) \phi(b)$$

subject to (5) and

$$\begin{aligned} c + a' &= (1 + r(1 - \tau_a))a + P(\tilde{y}) \\ b &= a' - \tau_b * \max(a' - ex_b, 0). \end{aligned}$$

2.6 Definition of the stationary equilibrium

I focus on an equilibrium concept where factor prices and age-wealth distribution are constant over time. Each agent's state is denoted by x . An equilibrium is described as follows.

Definition 1 *A stationary equilibrium is given by government tax rates, transfers, and spending $(\tau_{ss}, \tau_l, \tau_a, \tau_b, ex_b, P(\tilde{y}), G)$; an interest rate r and a wage rate w ; value functions $V(x)$, allocations $c(x)$, $a'(x)$; and a constant distribution of people $m^*(x)$, such that the following conditions hold:*

(i) *Given government tax rates and transfers, the interest rate, and the wage rate, the functions $V(x)$, $c(x)$ and $a'(x)$ solve the above described maximization problem for a household in a state x .*

(ii) *m^* is the invariant distribution of households over the state variables for this economy.⁹*

⁹I normalize m^* so that $m^*(X) = 1$, which implies that $m^*(\chi)$ is the fraction of people alive that are in a state χ .

(iii) All markets clear.

$$C = \int cm^*(dx), \quad K = A = \int am^*(dx), \quad L = \int \epsilon ym^*(dx),$$

$$C + (1+n)K - (1-\delta)K + G = Y = F(K; L)$$

(iv) The price of each factor is equal to its marginal product.

$$r = F_1(K, L) - \delta, \quad w = F_2(K, L).$$

(vi) Government budget constraint is balanced at each period.

$$G = \tau_a r A + \tau_l w L + \int \tau_b (1 - p_t) I_{t>9} \max(a' - ex_b, 0) m^*(dx).$$

(vii) Social Security budget is balanced at each period.

$$\int I_{t>9} P(\tilde{y}) m^*(dx) = \tau_{ss} \int I_{t\leq 9} \min(w\epsilon_t y, e_{max}) m^*(dx)$$

3 Calibration

The model has twenty parameters. I pick sixteen of them from other empirical studies and choose the remaining four parameters so that the model matches the bequest-output ratio, the capital-output ratio, the average bequest left by people in the lowest 80th bequest percentile, and government spending.¹⁰

I set the rate of population growth, n , to the average value of population growth from 1950 to 1997 from the Economic Report of the President (1998). The p_t 's are the vectors of conditional survival probabilities for people older than 65 and is set to the mortality probabilities for people born in 1965 (Bell, Wade and Goss (1992)).

I take α , the share of income that goes to capital, to be 0.36 (Prescott (1986), and Cooley and Prescott (1995)). I take depreciation to be 6% (Stokey and Rebelo

¹⁰Since one period in this model corresponds to 5 years in real life, I adjust parameters in the model accordingly. I report parameters at annual frequency, unless stated otherwise.

Parameters	Calibrations
Demographics	
n annual population growth	1.2%
p_t survival probability	see text
Technology	
α capital share in National Income	0.36
δ annual depreciation rate of capital	0.06
Endowment	
ϵ_t age-efficiency profile	see text
ρ_y AR(1) coefficient of 5-year productivity process	0.85
σ_y^2 innovation of 5-year productivity process	0.30
ρ_{yh} AR(1) coefficient of productivity inheritance process	0.67
σ_{yh}^2 innovation of productivity inheritance process	0.37
Government policy	
τ_a tax on capital income	20%
τ_b tax on bequest	10%
ex_b exemption level on bequest tax	40
G government spending	0.18
$P(\tilde{y})$ Social Security earnings	see text
τ_{ss} Social Security tax	11.4%
τ_l tax on labor income	21%
Preference	
η risk aversion coefficient	1.5
β discount factor	0.96
ϕ_1 weight of bequest in utility function	-20
ϕ_2 shifter of bequest in utility function	15

Table 1: Parameters used in the benchmark model

(1995)). Given the calibration for the production function, the before-tax interest rate on capital net of depreciation r , is 6%.

The deterministic age-profile of labor productivity ϵ_t is taken from Hansen (1993).¹¹ The persistence ρ_y and variance σ_y^2 of the stochastic productivity process are estimated from PSID data (Altonji and Villanueva (2002)).¹² The persistence is low and variance is high because this refers to income in a 5-year period. I take persistence ρ_{yh} of the productivity inheritance process from Zimmerman (1992), and variance σ_{yh}^2 from De Nardi (2004).

¹¹Since I impose mandatory retirement at the age of 65, I set $\epsilon_t = 0$ for $t > 9$.

¹²De Nardi (2004) provides a detailed discussion of the estimation process.

The capital income tax τ_a is set at 20% (Kotlikoff, Smetters and Walliser (1999)). The rate τ_b is the tax rate on estates that exceed the exemption level ex_b . I choose these two parameters from De Nardi (2004) who matches the observed ratio of estate tax revenues to GDP, and the proportion of estates that pay estate taxes. G is total government expenditure and gross investment excluding transfers, and is chosen to be 18% of GDP (Council of Economic Advisors (1998) for 1996).

The retirement benefit is calculated to mimic the Old Age and Survivor Insurance component of Social Security system. The Social Security benefit is equal to

$$\left\{ \begin{array}{ll} 0.9\tilde{y} & \tilde{y} \leq 0.2 \\ 0.18 + 0.32(\tilde{y} - 0.2) & 0.2 \leq \tilde{y} < 1.24 \\ 0.5128 + 0.15(\tilde{y} - 1.24) & 1.24 \leq \tilde{y} \leq e_{max} \end{array} \right\}.$$

The Social Security earnings cap is $e_{max} = 2.47$. The bend points and Social Security income cap, expressed as average earnings, and marginal rates are from Huggett and Ventura (1999). The corresponding payroll tax rate that balances the Social Security funds budget is 11.4%. The labor income tax, excluding Social Security tax, that balances the government budget is 21%.

I take the risk aversion coefficient, η , to be 1.5, from Attanasio, Banks, Meghir and Weber (1999), and Gourinchas and Parker (2002), who estimate it from consumption data. This value is in the commonly used range (1-5) in the literature.

I choose β, ϕ_1, ϕ_2 , and τ_l to match the capital-output ratio of 3, bequest-output ratio of 2.64% (Gale and Scholz (1994)), the average bequest left at the lowest 80th percentile of 2.78 (Hurd and Smith (2001)), and to balance government budget.¹³

¹³I use distribution for single decedents instead of the one for all decedents. Typically a surviving spouse inherits a large share of the estate, consumes part of it, and only leaves the remaining to the couple's children.

4 Numerical Results

This section examines the long-run effect of eliminating pay-as-you-go (PAYG) Social Security. To answer this question, I compare the benchmark model with one without PAYG Social Security. The tax rates on capital income, labor income, and bequest are held constant across steady states. Government purchases might change to balance government budget. Details about the computation of the equilibrium are in the Appendix 6.1.

In the benchmark model, Social Security provides the following benefits. Social Security partially substitutes for the missing private annuity market. In addition, since retirement benefits are partially linked to lifetime contributions, the system insures against labor-income risk. However, Social Security imposes the following costs. Payroll tax distorts consumption and saving behavior, especially when borrowing constraints are binding. In addition, Social Security provides a positive return in an economy with population growth, but under current parameters, the return is lower than the return to physical capital. Furthermore, Social Security lowers capital stock, which decreases aggregate steady-state consumption at an economy with capital stock below its golden rule level. Finally, Social Security affects the market interest rate and causes households to change consumption over the life cycle, which may either decrease or increase welfare.

I will show the aggregate statistics and the distributional and welfare effects. The welfare effects of eliminating PAYG Social Security system can be measured by the compensating variations, the fraction of consumption that should be given to a household in the steady state in all future periods and all contingencies to make the household as well off as in the steady state of the benchmark economy without Social Security. The welfare gain for an unborn agent before the realization of all

contingencies, denoted as w , is defined as,

$$(14) \quad w = \left(\frac{\int I_{t=1} V_n(x) dx}{\int I_{t=1} V_b(x) dx} \right)^{\frac{1}{1-\eta}} - 1,$$

where V_b and V_n refer to the value in the model with and without Social Security, respectively.¹⁴ Note that a negative number indicates that the agent experiences a welfare loss after eliminating Social Security.

To better understand the aggregate welfare effects, I further classify new-born agents by the types of shocks to labor productivity they receive at the beginning of the first age. Agents who receive initial productivity shock y_i , are referred to as type- i agents. For a newborn type- i agent, the welfare gain of a new policy reform, denoted as w_i , is

$$(15) \quad w_i = \left(\frac{\int I_{t=1, y=y_i} V_n(x) dx}{\int I_{t=1, y=y_i} V_b(x) dx} \right)^{\frac{1}{1-\eta}} - 1.$$

4.1 Impure Altruism

In the model with one-sided altruism, parents and their children are linked by bequests, both voluntary and accidental. When I change Social Security system, the interest rate and the wage rate will adjust to clear the labor and capital markets. To disentangle the general equilibrium effect, I report two different types of experiments. In the first experiments I keep prices fixed, thus the U.S. is treated as a small open economy. In the second experiment, I adjust the interest rate and the wage rate to clear the markets.

Aggregate statistics in the alternative Social Security systems are summarized in Table 2. We see that eliminating Social Security leads to several changes. Eliminat-

¹⁴The expected life-time utility may include utility from leaving bequests. Therefore, strictly speaking, w defined in this way is not equal to the consumption variation. This is of minor importance since for most households the discounted utility from leaving bequests is small compared with the discounted utility from consumption.

ing Social Security boosts household savings. Under PAYG Social Security system, the nonlinear relation between Social Security contribution and Social Security benefits provides within-cohort income risking. Furthermore, the Social Security annuity partially insures against mortality risk. Thus eliminating Social Security, by eliminating the above two effects of Social Security, leads to a dramatic increase of saving. If prices are fixed, eliminating Social Security increases aggregate wealth by 53%.¹⁵ Consumption increases by 10.6%. Households hold on average more assets at retirement. Since the bequest is modeled as a luxury good, those households who receive larger inheritances will choose a higher saving rate, and pass more bequests to the next generation. Therefore the aggregate bequest increases dramatically.

	r	w	Y	A	C	B
Initial	6.0%	1.00	1.00	3.00	0.62	0.0264
Final						
Fix prices	6.0%	1.00	1.00	4.60	0.69	0.0414
Change prices	4.2%	1.11	1.11	4.06	0.66	0.0292

Table 2: Aggregate statistics (with bequest motive)

If the interest rate and the wage rate are allowed to adjust to clear the markets, then the interest rate decreases by 1.8 percentage points and wage rate increases by 11%. The decline of interest rate discourages saving. Thus aggregate wealth increases by 35%, smaller than when prices are fixed. Although young households consume more, elderly households consume less, therefore aggregate consumption increases by 5.3%.

Table 3 summarizes the wealth distribution in the alternative Social Security systems. I present shares for the quintiles, the 80-95%, the 95-99%, the top 1%, and

¹⁵Output is defined as GDP. Since labor supply is inelastic, output does not change.

the Gini coefficient for wealth. The model with intergenerational links of bequest and human capital can generate a skewed wealth distribution.¹⁶ Social Security is redistributive since benefits are regressive with respect to contribution and payroll taxes are proportional for earnings below Social Security earnings cap. However, such a redistributive Social Security system can lead to higher wealth dispersion. Eliminating Social Security decreases overall wealth inequality. The Gini coefficient of wealth decreases from 0.76 in the benchmark economy to 0.69 if the price is fixed and 0.70 if the price adjusts. If prices are allowed to adjust, the lower interest rate increases borrowing. Thus households at the bottom 40% save less compared with the case of fixed prices. Households at the top 5% mainly save for bequest and are less affected by the interest rate, thus the share of wealth held by them barely changes by the decrease of interest rate.

Wealth among retirees is less unevenly distributed than wealth for the whole economy both with and without Social Security. This shows that a large amount of wealth dispersion in the economy is due to differences in age. Eliminating Social Security decreases wealth inequality for retirees more dramatically than for the whole economy. Households in the bottom 4 quintiles hold more assets without Social Security. This is because, Social Security has an intragenerational redistribution effect by giving households with lower social security earnings a higher replacement rate, which reduces wealth holding at the bottom of the distribution among retirees relatively more.

Table 4 reports the overall welfare effects of privatizing Social Security for the economy as a whole, as well as for each type of agent. Living in an economy without a Social Security system, an unborn agent experiences a welfare gain. The reason is that, the benefit from Social Security as insurance against income shocks and as an

¹⁶De Nardi (2004) provides more detailed explanation on how the existence of intergenerational links of bequest and human capital increases wealth concentration.

	Gini	1st	2nd	3rd	4th	5th	80-95	95-99	99-100
All									
With Social Security	0.76	0.0	0.54	4.2	15	80	19	27	13.0
Without Social Security									
Fix prices	0.69	0.0	1.4	7.4	20	71	17	23	9.7
Change prices	0.70	0.0	1.2	7.1	21	71	18	23	9.3
Age 65 and above									
With Social Security	0.63	0.5	3.7	9.2	21	65	17	20	7.5
Without Social Security									
Fix prices	0.47	3.6	8.1	14	23	52	13	14	5.2
Change prices	0.47	3.6	8.1	14	23	52	13	14	5.1

Table 3: Wealth distribution (with bequest motive)

annuity against mortality risk is outweighed by the loss from low capital accumulation and tighter borrowing constraints.

The welfare measure for a new-born agent masks differences at the individual level. Households with high initial productivity are more in favor of eliminating a progressive Social Security system. In this model, two forces work in the opposite directions. Social Security has a redistribution effect since the Social Security benefit is regressive with respect to past contributions. When income shocks are quite persistent, households with high initial productivity are more likely to have high lifetime income and pay more Social Security tax than households with low initial productivity. Thus households with low productivity lose from progressive pension income after retirement. However, they benefit from the elimination of Social Security tax which limits households' ability to smooth consumption. Low ability households are more likely to be borrowing constrained. Proportional Social Security

tax on income decreases consumption dollar for dollar when liquidity constraints are binding.¹⁷ Households with high initial productivity are less likely to be borrowing constrained.

If prices are allowed to adjust, households with low initial productivity seem to be much more in favor of eliminating Social Security. This is caused by the decrease in interest rate and increase in wage rate. The decrease in interest rate has a positive income effect for net borrowers and a negative income effect for net lenders. Households with low initial productivity, as net borrowers especially when young, benefit from both the lower interest rate and the higher wage rate. Households with high initial productivity receive less benefit from eliminating Social Security, since the positive effect from increasing wage rate is more than offset by the effect of the interest rate decrease on net savers.

	w ₁	w ₂	w ₃	w ₄	w ₅	w ₆	w
Fix price	14.0	13.6	13.1	12.6	12.0	8.8	13.0
Change price	22.6	21.3	19.7	17.6	14.5	8.2	18.9
Parent's	22.3	20.6	18.9	17.1	14.8	9.69	
product-	23.0	21.3	19.6	17.6	15.1	9.76	
ivity	23.3	21.6	19.9	18	15.4	9.92	
	23.1	21.4	19.8	18	15.4	9.96	
	22.2	20.5	18.9	17.1	14.8	9.56	
	19.8	17.9	16	14.1	11.8	6.95	

Table 4: Average welfare gain (with bequest motive)

I further classify all new-born agents by the types of shocks to labor productivity they receive at age 20 and their parents receive at age 55. Households whose parents

¹⁷The effect of borrowing constraint on the welfare and aggregate effect of Social Security is illustrated by Hubbard and Judd (1987).

have medium productivity are more in favor of eliminating progressive Social Security system, regardless of the children’s initial productivity. This is because, after eliminating Social Security, parents having medium productivity increase asset holding relatively more and their children may receive more bequests relative to lifetime income than those whose parents are of other productivity types.

	Children’s productivity					
Parent’s	2.31	1.45	0.93	0.59		
product-	4.36	2.69	1.66	1.07	0.62	
ivity	6.72	4.29	2.66	1.70	0.95	0.69
	4.63	5.01	3.53	2.16	1.34	0.77
		4.36	2.58	1.54	0.915	0.45
			-11.0	-7.76	-4.65	-2.65

Table 5: increase of mean lifetime inheritance relative to lifetime earnings in percentage (with bequest motive)

To elaborate the different changes of inheritance by parents’ productivity, I simulate 1,500,000 households in the benchmark economy with and without Social Security. Lifetime earnings is defined to be the total earnings from age 20 to 60, discounted to age 65 using the after-tax interest rate. Lifetime inheritance is defined analogously. I then calculate the increase in the expected lifetime inheritance relative to lifetime earnings for all new-born agents by the types of shocks to labor productivity they receive at the beginning of the first age and their parents receive at age 55. The results are shown in Table 5.¹⁸ We observe that the increase of inheritance relative to life income is monotonically decreasing with children’s productivity level.

¹⁸Due to the intergenerational transmission of productivity, the measures of new-born agents who receive high productivity shocks and whose parents receive low productivity shocks, and of those who receive low productivity shocks and whose parents receive high productivity shocks, are very small. The results from simulation for those types of households are subject to severe simulation error and thus are omitted.

This is because, children’s lifetime income is monotonically increasing with children’s productivity, and expected inheritance, after controlling for parents’ productivity, is independent of children’s productivity. However, the increase of inheritance relative to life income is not monotonic with parent’s productivity level. After elimination of Social Security, those parents who receive the medium productivity levels at age 55 will increase wealth holding after retirement; therefore their children receive on average more inheritance relative to lifetime earnings. Those parents who receive the highest productivity shocks decrease wealth holding after retirement; therefore their children receive less inheritance.

4.2 No bequest motive

To better understand the interaction between bequest motive and Social Security system, I compare the benchmark model with bequest motives and intergenerational transfers with a standard life-cycle model without altruism. I turn off all intergenerational links and assume that accidental bequests are equally redistributed among 45-year-old people.¹⁹ Then I look at a model where parents do not care about bequests but there is intergenerational transfer of accidental bequest and human capital. In each experiment, I recalibrate β and τ_l accordingly to match the capital-output ratio of 3 and to balance government budget in the steady state with Social Security.

Table 6, row two shows the aggregate statistics in an environment without intergenerational links after elimination of Social Security system when prices have adjusted. The interest rate decreases by 2.06 percentage points and the wage rate increases by 13%. Aggregate saving rises by 42% and aggregate consumption increases by 5.9%. Without Social Security to provide imperfect annuity to partially

¹⁹According to the demographics in the model, the expected life expectancy is 84 and the expected age of receiving inheritance is 49. Distributing bequests to 50-year-old people gives quantitatively similar results.

	r	w	Y	A	C	B
No links						
With Social Security	6.0%	1.00	1.00	3.00	0.62	0.0183
No Social Security	3.94%	1.13	1.13	4.26	0.66	0.0281
With links						
With Social Security	6.0%	1.00	1.00	3.00	0.62	0.0176
No Social Security	3.97%	1.13	1.13	4.24	0.66	0.0267

Table 6: Aggregate statistics (no bequest motive)

insure against mortality risk, most elderly households hold positive assets. Thus total accidental bequest increases by 54%.

Table 6, row three shows the aggregate statistics after elimination of Social Security system in an environment with intergenerational links but without bequest motives. The results are very similar to the case without intergenerational links. Aggregate saving rises by 41%. Aggregate consumption increases by 5.9%. Total accidental bequests increase by 52%.²⁰ This comparison shows that the unequal distribution of involuntary bequests and intergenerational transfer of earnings ability are not important for understanding the aggregate effect of Social Security.

In the benchmark economy, an unfunded Social Security system crowds out only 35% of the capital stock, which is smaller than the results obtained in pure life-cycle models. This difference is partially due to the existence of a bequest motive in the benchmark model. In pure life-cycle models, Social Security redistributes income away from middle-aged agents with higher marginal propensities to save to old agents with lower marginal propensities to save, therefore reduces the saving rate.

²⁰Total accidental bequests in the model with links are smaller than in the model without links. This is because the calibrated β is slightly lower in the model with links and thus retirees hold less wealth on average.

In a model with bequest motive, old agents might save for bequest motives, which increases marginal propensities to save. As a result, Social Security in my model has a smaller effect on the aggregate saving rate and on capital accumulation than in a pure life-cycle model.

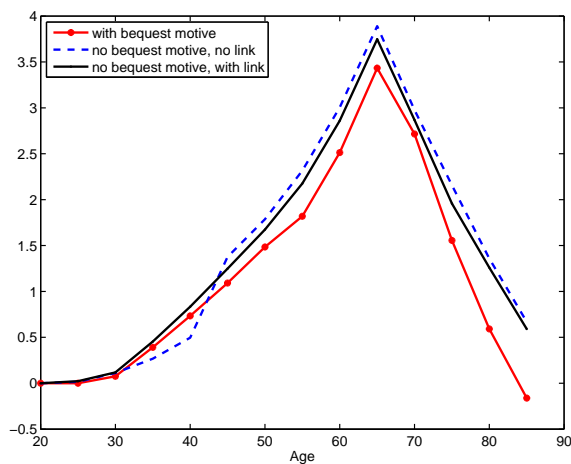


Figure 2: Assets increased after privatization

To elaborate the differences in responses in saving by age groups to changes in Social Security, I show, in Figure 2, the amount of asset increase at each age group after eliminating Social Security, in the benchmark model with impure bequest motive and life-cycle models without bequest motive. Eliminating Social Security increases asset holding for households 45 years old and older more in pure life-cycle models than in a model with impure bequest motive. The responses in saving by age groups in the model without intergenerational links is very similar to those in the model with intergenerational links, except for agents aged 30-40. This is due to the uncertainty of inheritance in the latter model. In an economy without links, agents aged 30-40, in anticipating receiving bequest at age 45 with certainty, reduce saving accordingly.

Table 7 summarizes the wealth distribution in the alternative Social Security systems. Life-cycle models without bequest motive, regardless of the existence of in-

tergenerational transfer of bequest, do not generate a skewed distribution of wealth as in the benchmark model. As in the benchmark model, eliminating Social Security boosts aggregate wealth holding but decreases overall wealth inequality. This comparison shows that the unequal distribution of involuntary bequests and inter-generational transfer of earnings ability are not important for understanding the distributional effect of Social Security.

	Gini	1st	2nd	3rd	4th	5th	80-95	95-99	99-100
All									
No links									
Initial	0.73	0.0	1.0	6	17	76	19	26	10.9
Final	0.69	0.0	1.1	8	21	70	17	22	8.9
With links									
Initial	0.74	0.0	0.8	5	17	77	19	26	11.0
Final	0.68	0	1.4	8	21	69	17	22	8.6
Age 65 and above									
No links									
Initial	0.58	1.7	5.3	11	21	61	16	18	6.2
Final	0.44	4.7	8.7	14	23	50	13	14	4.5
With links									
Initial	0.60	0.78	4.5	10	22	63	16	19	6.4
Final	0.46	3.8	8.5	14	23	50	13	14	4.6

Table 7: Wealth distribution (no bequest motive)

Table 8 reports the overall welfare effects of eliminating Social Security for the economy as a whole, as well as for each type of agent. An unborn agent has higher expected welfare in an economy without Social Security system. The welfare difference

in the benchmark model with impure altruism is smaller than in life-cycle models. In a model with intended altruism, middle-aged households receive larger bequests than in a pure life-cycle model. This relaxes borrowing constraints. Thus with altruism the benefit of removing Social Security from relaxing borrowing-constraints is smaller. In addition, aggregate consumption increases less than in pure life-cycle models.

	w ₁	w ₂	w ₃	w ₄	w ₅	w ₆	w
No links	25.2	23.4	21.5	19.3	16.4	10.2	20.8
With links	24.2	22.8	21.2	19.1	16.3	10.2	20.5

Table 8: Average welfare gain (no altruism)

In the model without links, households with low initial productivity seem to be more in favor of eliminating Social Security than in the benchmark economy. Since accidental bequests are evenly distributed, they benefit from the increase in accidental bequest more than in the benchmark economy. This even distribution of bequest also explains why the welfare gain by low initial productivity type in the model without intergenerational links is higher than in the model with intergenerational links, despite the similar aggregate effects in both models.

4.3 Two-sided Altruism

Now I show the effect of two-sided altruism in analyzing the effect of Social Security. I look at an economy that is populated by overlapping generations of individuals that are linked through altruistic transfers. Individuals derive utility from their own lifetime consumption, and from the well-being of their predecessors and descendants. As in Laitner (1992), the parent and the children constitute a single decision unit, called a household, by pooling their resources and jointly solving a maximization

problem. I assume that households face borrowing constraints and cannot hold negative assets at any age. There is no estate taxation since assets are jointly held by parent and children. In all other regards, the model is identical to the one presented in Section 2. More details of the model are provided in Appendix 6.2.²¹ I recalibrate β and τ_l accordingly.

Aggregate statistics after eliminating Social Security system are summarized in Table 9. Eliminating Social Security increases aggregate capital by 10.5%. Consumption increases by 1.8%. Eliminating Social Security increases the capital stock less than in life-cycle models with and without bequest motives.

	r	w	Y	A	C
Initial	6.0%	1.00	1.00	3.00	0.62
Final	5.3%	1.04	1.04	3.31	0.63

Table 9: Aggregate statistics (two-sided altruism)

The result that Social Security has a small crowding-out effect under two-sided altruism is consistent with Fuster, Imrohoroglu and Imrohoroglu (2003) who show that in a model with two-sided altruism, the transfer induced by Social Security is partially undone by altruistic transfers from parent to children and thus Social Security has a small impact on saving. In the benchmark model, households only have one-sided impure altruism which provides less insurance among generations. The elimination of Social Security does not impact decisions on the timing, direction, and the size of bequests. Therefore Social Security has a bigger crowding-out effect on capital stock.

Table 10 summarizes the wealth distribution in the alternative Social Security

²¹The main difference between this two-sided altruism model and Fuster, Imrohoroglu and Imrohoroglu (2003) is that this model incorporates idiosyncratic income shocks. The main difference between this model and Fuster, Imrohoroglu and Imrohoroglu (2007) is that this model abstracts from labor supply choice and differences in income and mortality by education.

systems. Compared with life-cycle models with and without bequest motive, the model with two-sided altruism generates less wealth concentration. Parent and children are pooling income together, which reduces the income risk that households face and thus reduces wealth dispersion. As in life-cycle models, eliminating Social Security decreases overall wealth inequality.²²

	Gini	1st	2nd	3rd	4th	5th	80-95	95-99	99-100
With Social Security	0.67	0.11	2.04	7.82	21.6	22.8	17.6	20.3	7.79
No Social Security	0.63	0.37	3.18	9.47	22.6	21.9	16.4	19.0	7.08

Table 10: Wealth distribution (two-sided altruism)

Table 11 reports the overall welfare effects of eliminating Social Security. An unborn agent has lower expected welfare in an economy without Social Security system. The reason is that, in contrast to a life-cycle model, the loss caused by a Social Security system from low capital accumulation and tighter borrowing constraints is outweighed by the benefit from Social Security as insurance against income shocks and as annuity against mortality risk. The loss from low capital accumulation caused by Social Security is much less than in a life-cycle model. Insurance among family members mitigates borrowing constraints, thus the loss caused by a Social Security system from tighter borrowing constraints is small. Since individuals also value the well-being of their predecessors and descendants, Social Security as insurance against income shocks and as annuity against mortality risk is valued more in the model with two-sided altruism.

I classify all agents by the types of shocks to labor productivity they receive at age 20 and their parents receive at age 55. Households in which children have higher initial productivity and parents have lower productivity are more in favor of

²²Fuster (1999) finds similar results.

	Children's productivity						w
Parent's	-12.2	-6.6	-1.2	3.3	6.8	1.3	
product-	-11.5	-6.8	-2.0	2.3	5.8	6.0	
ivity	-9.4	-5.9	-2.0	1.7	5.0	5.5	
	-5.9	-3.7	-1.1	1.7	4.4	4.9	
	-3.1	-1.9	-0.3	1.6	3.6	4.1	
	-6.5	-2.6	-1.5	-0.2	1.3	2.1	
	-11.7	-6.3	-1.7	1.7	3.7	3.2	-2.0

Table 11: Average welfare gain (two-sided altruism)

eliminating a progressive Social Security system.²³ Those households are expecting to pay higher taxes because the children are of higher ability, and to get lower pension because the parents are of lower ability. Therefore, the return from Social Security is lower for them than for other types of households.

5 Conclusion

This paper studies how the long-run effect of eliminating Social Security depends on the existence of impure altruism in a life-cycle model. The model that I am using mainly incorporates two basic forces, bequests and human capital transmission, into an otherwise standard life-cycle model in which households face uninsurable labor income risk, uncertain lifetimes and a borrowing constraint. In this life-cycle model with impure altruism, Social Security affects saving for retirement but not saving for bequests. An unfunded Social Security system crowds out less capital stock than in pure life-cycle models, but more than in a model with two-sided altruism. The

²³Fuster, Imrohoroglu and Imrohoroglu (2003) find similar results in an environment without idiosyncratic shocks.

welfare gain of eliminating Social Security under impure altruism is smaller than in a pure life-cycle model, and bigger than in a model with two-sided altruism.

In this paper, I have abstracted from some important issues in order to make the model manageable. Now I discuss these simplifications and their likely implications. I calibrate my model to U.S. data and examine the stationary equilibrium. It is interesting to look at the transition path of the policy changes. However, due to the computational complexity, I leave this interesting issue for future research.

This paper also abstracts from housing. Housing is the single largest investment made by consumers over their lifetime. The median household owns a house valued about twice its annual earnings. As it is shown in Yang (forthcoming), abstracting from housing might bias the study of life-cycle consumption and asset accumulation. It will be interesting to extend this model to look at the effect of eliminating Social Security in an environment with housing.

6 Appendix

6.1 Computation of the Benchmark Model

I discretize both the productivity and the productivity inheritance processes to six-state Markov chains according to Tauchen and Hussey (1991). Since I want the possible realizations for the initial inherited productivity level to be the same as the possible realizations for productivity during the lifetime, I choose the quadrature points jointly for the two processes. The resulting grid points for the productivity process y are [0.1464, 0.3356, 0.7002, 1.4283, 2.9801, 6.8306]. The transition matrix Q_y is

$$\begin{bmatrix} 0.6099 & 0.3537 & 0.0357 & 0.0007 & 0.0000 & 0.0000 \\ 0.1307 & 0.5327 & 0.3026 & 0.0333 & 0.0006 & 0.0000 \\ 0.0086 & 0.1974 & 0.5193 & 0.2528 & 0.02175 & 0.0001 \\ 0.0002 & 0.0218 & 0.2528 & 0.5193 & 0.1974 & 0.0086 \\ 0.0000 & 0.0006 & 0.0333 & 0.3026 & 0.5327 & 0.1307 \\ 0.0000 & 0.0000 & 0.0007 & 0.0357 & 0.3537 & 0.6099 \end{bmatrix}.$$

The transition matrix Q_{yh} is

$$\begin{bmatrix} 0.3668 & 0.4788 & 0.1426 & 0.0116 & 0.0002 & 0.0000 \\ 0.0922 & 0.4240 & 0.3855 & 0.0928 & 0.0054 & 0.0000 \\ 0.0134 & 0.1887 & 0.4615 & 0.2899 & 0.0454 & 0.0011 \\ 0.0011 & 0.0454 & 0.2899 & 0.4615 & 0.1887 & 0.0134 \\ 0.0000 & 0.0054 & 0.0928 & 0.3855 & 0.4240 & 0.0922 \\ 0.0000 & 0.0002 & 0.0116 & 0.1426 & 0.4788 & 0.3668 \end{bmatrix}.$$

The transition matrices Q_y and Q_{yh} also induce an initial distribution of earnings.

The distribution of accumulated productivity at each age is approximated on a grid of 36. The state space for asset holdings is discretized. Using this grid, I can store the value functions and the distribution of households as finite-dimensional arrays.

I solve the approximated optimal consumption and saving plans recursively. Households surviving to the last period T have an easy problem to solve. Based on the period T policy functions, I solve the consumption and saving decisions that maximize the period $T - 1$ value function. The same procedure is carried back until decision rules in the first period are computed for a large number of states.

For a given set of parameters, I solve for the steady state equilibrium as follows:

1. Make an initial guess of τ_l .
2. Given an initial guess of interest rate r , use the equilibrium conditions in the factor markets to obtain the wage rate w .
3. Solve the value function for the last period of life for each of the points of the grid.
4. By backward induction, repeat step 3 until the first period in life.
5. Guess an initial joint distribution of parents and children at the beginning of the life cycle.
6. Given the initial distribution and policy functions, compute the associated stationary distribution of households.
7. Compute the implied joint distribution of parents and children at the beginning of the life cycle. If the distributions converge, go to step 8; otherwise go to step 5.
8. Given the stationary distribution and prices, compute factor input demands and supplies and check whether market clearing conditions hold. If all markets clear, go to step 9. If not, go to step 2 and update interest rate r .
9. If the government budget is balanced, an equilibrium is found. If not, go to step 1 and update τ_l .

6.2 Model with two-sided Altruism

The demographics in the model are the same as in the benchmark model. Each individual faces positive death shock after age 65. A household lasts 35 periods and is constituted by the “parent”, and his $m = (1 + n)^{35}$ adult children. In a given household, all children are identical regarding their labor abilities and age. The composition of a household changes when the parent dies. There are two types of households: those where the parent has died, and those where the parent is still

alive. Let $h \in \{0, 1\}$ indicate household composition that takes the value unity if the parent is alive and 0 otherwise. A dynasty is a sequence of households that belong to the same family line. Once the children reach age 55, each of them becomes a parent in the next-generation household of the dynasty.

The state of a household is given by the age of the youngest adult member j ; the demographic type h ; the asset holding a ; labor productivity of children and parent y_s, y_f , and the average Social Security earnings of children and parent \tilde{y}_s, \tilde{y}_f .

From $t = 1$ to $t = 6$ (from 20 to 50 years of age), the household solves the following problems,

$$(16) \quad V(t, h, a, y_s, y_f, \tilde{y}_s, \tilde{y}_f) = \max_{c_s, c_f, a'} mU(c_s) + hU(c_f) + \beta E(V(t+1, h', a', y'_s, y'_f, \tilde{y}'_s, \tilde{y}'_f))$$

subject to

$$(17) \quad mc_s + hc_s + a' = (1 + r(1 - \tau_k))a + e(t, h, y_s, y_f, \tilde{y}_s, \tilde{y}_f)$$

$$(18) \quad a' \geq 0, \quad c \geq 0,$$

$$(19) \quad \tilde{y}'_s = ((t-1)\tilde{y}_s + \min(w\epsilon_t y_s, e_{max}))/t,$$

$$(20) \quad \tilde{y}'_f = ((t+6)\tilde{y}_f + \min(w\epsilon_{t+7} y_f, e_{max}))/t,$$

where c_s and c_f are the consumption of the child and the parent, and a' denotes the asset holdings to be carried over to age $t+1$. The term $e(t, h, y_s, y_f, \tilde{y}_s, \tilde{y}_f)$ denotes the after-tax household earnings,

$$(21) \quad e(t, h, y_s, y_f, \tilde{y}_s, \tilde{y}_f) = \left. \begin{array}{ll} m((1 - \tau_l)w\epsilon_t y_s - \min(w\epsilon_t y_s, e_{max})) + hP(\tilde{y}_f) & \text{if } t \geq 3 \\ m((1 - \tau_l)w\epsilon_t y_s - \min(w\epsilon_t y_s, e_{max})) + & \text{otherwise} \\ h((1 - \tau_l)w\epsilon_{t+7} y_f - \min(w\epsilon_{t+7} y_f, e_{max})) & \end{array} \right\}.$$

From $t = 3$ to $t = 7$, the parent is between 65 to 85 years of age and does not work. $P(\tilde{y}_f)$ denotes the Social Security benefit, which is a function of the parent's average lifetime earnings \tilde{y}_f .

At $t = 7$, the household solves the following problems,

$$(22) \quad V(t, h, a, y_s, y_f, \tilde{y}_s, \tilde{y}_f) = \max_{c_s, c_f, a'} mU(c_s) + hU(c_f) + m\beta E(V(1, 1, a', y'_{ss}, y'_s, 0, \tilde{y}'_s))$$

subject to (18), (19), (21), and

$$mc_s + hc_s + ma' = (1 + r(1 - \tau_k))a + e(t, h, y_s, y_f, \tilde{y}_s, \tilde{y}_f).$$

In the next period, m new households are constituted in the dynasty and by construction the parent is alive. The ability of the new generation of the dynasty, y'_{ss} , is correlated with the ability of the parent, y'_s .

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