Social Security Reform with Impure Intergenerational Altruism*

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Abstract

This paper studies the long-run aggregate and welfare effects of eliminating Social Security in a quantitative dynamic general equilibrium life-cycle model where parents and their children are linked by voluntary and accidental bequests. Social Security in this model with impure altruism has a smaller effect on capital accumulation than in a pure life-cycle model, a bigger effect than in a model with two-sided altruism. The welfare gain of eliminating Social Security system under impure altruism is smaller than that in a pure life-cycle model, and bigger than that in a model with two-sided altruism.

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1 Introduction

There is considerable evidence supporting the hypothesis that bequest motives are important both for aggregate capital accumulation and to better match the individuals savings profiles over the life cycle (see, among others, Kotlikoff and Summers (1981), Gale and Scholz (1994), and De Nardi (2004)). Using a structural model with a survey instrument, Ameriks, Caplin, Laufer, and Van Nieuwerburgh (forthcoming) find strong support for warm-glow bequest motives. Lockwood (2010) also finds strong support for bequest motives from the patterns of wealth and long-term care insurance holdings. Lockwood (forthcoming) shows that bequest motives play a central role in explaining the lack of demand for annuities. Despite this mounting amount of empirical evidence, not only is there a lot of uncertainty on how to best model bequest motives, but much of the literature on Social Security reform abstracts from voluntary bequest motives altogether.[1] This abstraction has the potential of leading to a biased conclusion regarding, for example, the effects of reforming or eliminating Social Security. On the other extreme, analysis of Social Security that allows for altruism uses two-sided pure altruism models which, as shown in this paper, fail to generate skewed wealth inequality.[2] This paper adds to the literature by quantitatively assessing whether impure warm-glow bequest motives and intergenerational links in a life-cycle model are important for understanding the long-run effects of Social Security on aggregate allocations and on the distribution of welfare across households.

This paper uses a life-cycle model with two additional basic forces, bequests and human capital transmission, which can generate a highly concentrated wealth distribution and a realistic relationship between lifetime earnings and wealth at retirement (De Nardi (2004), and Yang (2008)). In this model, agents care about the total bequests left to their chil-

[1] Contributors to this literature include, among others, Feldstein (1985), Auerbach and Kotlikoff (1987), Hubbard and Judd (1987), Imrohoroglu, Imrohoroglu and Joines (1995, 1999), Huggett and Ventura (1999), Storesletten, Telmer and Yaron (1999), Conesa and Krueger (1999), Krueger and Kubler (2006), Nishiyama and Smetters (2007), Pries (2007), Rojas and Urrutia (2008), Andolfatto and Gervais (2008), and Chen (2010).

[2] Papers that examine the effect of Social Security under two-sided altruism include, among others, Laitner (1988), Altig and Davis (1993), Fuster (1999), and Fuster, Imrohoroglu, and Imrohoroglu (2003, 2007).

dren, but not about the consumption of their children. Facing uninsurable labor income risk, uncertain lifetimes and a borrowing constraint, households save to self-insure against labor earning shocks and life-span risk, for retirement, and possibly to leave bequests to their children. Pay-as-you-go Social Security provides insurance against income risk and lifespan risk. The benchmark model with impure bequest motives differs from pure life-cycle models and two-sided altruism models in several dimensions. In a pure life-cycle model without bequest motives, individuals save during middle age when they receive high income and dissave after retirement in order to maintain smoothed consumption. In the life-cycle model with impure bequest motives, in addition to life-cycle saving and precautionary saving motives, individuals save to leave bequests. In a two-sided altruism model, households care about their predecessors, descendants as well as themselves, and thus use bequests and intervivos transfers to smooth consumption within the dynasty. The need to saving for retirement is substantially reduced since retirees' children provide financial support to them. In addition, through bequests and intervivos transfers, households are better able to insure against income and mortality risks.

I compare the steady state with Social Security to one in which the replacement rate is zero. The results indicate that there are significant differences in how Social Security reform affects individuals under different assumptions of bequest motives. The main findings can be summarized as follows. In the benchmark model, the steady-state values of aggregate consumption, output, and capital are higher after the elimination of Social Security system. An unfunded Social Security system crowds out only 37 percent of the capital stock, less than the amount of 44 percent in pure life-cycle models, regardless of whether accidental bequests are inherited by the descendent of the deceased, or are evenly distributed among agents of the same age group. This is because, in the benchmark model, old individuals might save for bequest motives and respond less to the elimination of Social Security. However, the effect of Social Security on aggregate saving is much bigger than that in a model with two-sided altruism. In a model with two-sided altruism, the transfer induced by Social Security is partially undone by altruistic transfers from parents to children, thus Social Security has a

small impact on saving, and the aggregate wealth increases only by 10.7 percent. In the benchmark model with one-sided impure altruism, many households do not receive transfers and there is less insurance across generations. Therefore Social Security has a bigger crowding-out effect on the capital stock.

The model with intergenerational links of bequests and human capital can generate more wealth inequality than the models without a bequest motive and the model with two-sided altruism. A redistributive Social Security system always leads to higher wealth dispersion in all models: Households with lower Social Security earnings has a higher replacement rate, which reduces wealth holding at the bottom of the distribution relatively more. If Social Security wealth is added, wealth inequality is higher after the reform, indicating that poor households receive more Social Security benefits than what they would save on their own after the elimination. Abolishing Social Security decreases consumption inequality in the life-cycle models. This can be explained by the fact that, the reduction of taxes and the increase of wage rate enables borrowing-constrained poor households to increase consumption while does not affect rich households who save for retirement. However, eliminating Social Security increases consumption inequality in the two-sided altruism model. This is because, Social Security benefit is fixed income and provides an insurance against labor income risk for children who live with their retired parents.

I then look at the overall long-run welfare effect of abolishing Social Security for an unborn agent in the economy. The welfare gain of eliminating Social Security system under impure altruism (22 percent in equivalent consumption) is smaller than that in pure life-cycle models (24 percent) mainly due to a smaller increase in aggregate consumption after the elimination. The welfare gain in the benchmark model is much bigger than that in a model with two-sided altruism (-1.3 percent). In the benchmark model, households have one-sided impure altruism which does not provide much insurance across generations. Besides, the crowding-out effect of Social Security in the benchmark model is bigger. Therefore, as in the vast majority of the life-cycle models, the large negative effect of Social Security on capital accumulation and on consumption smoothing leads to an important reduction in the steady-state welfare.

This paper contributes to the literature that studies the effect of Social Security in a life-cycle framework with intergenerational transfers of bequests or bequest motives. Abel (1985, 1886) focuses on a fully funded Social Security system and abstracts from idiosyncratic income shocks. De Nardi, Imrohoroglu and Sargent (1999) study the effect of demographics on Social Security in an environment with voluntary bequest motives, abstracting from intergenerational transfers of bequests. Michel and Pestieau (1998), and Caballe and Fuster (2000) focus on the effects of Social Security system on the distribution of altruistic transfers.

The paper is organized as follows. In Sections 2 and 3, I present the benchmark model and the model with two-sided altruism, respectively. The calibration of the model is presented in Section 4. In Section 5, I show the quantitative implications of eliminating Social Security in the benchmark model, in pure life-cycle models without altruism, and in a two-sided altruism model. Brief concluding remarks are provided in Section 6.

2 The Benchmark Model

The economy is a discrete-time overlapping generations world with an infinitely-lived government. There are idiosyncratic earnings shocks which are uninsurable: The only financial instrument is a one-period bond. Households cannot engage in unsecured borrowing; net assets must be non-negative.[3] There is mortality risk but private annuity markets do not exist.[4] Members of successive generations are linked by the bequests and children's inheritance of part of their parent's productivity. At age 20, each agent enters the model and starts consuming, working, and paying labor and capital income taxes.[5] At age 35, the agent procreates. After retirement, the agent no longer works but receives interest from accumulated assets and benefits from Social Security. The government taxes labor earnings, capital income and estates, pays Social Security benefits to retirees, and provides government consumption.

[3]Rojas and Urrutia (2008) show that endogenizing borrowing constraints is not quantitatively important for understanding the effect of Social Security.

[4]Although private annuity market does exist in the U.S., due to reasons such as the lack of actuarially fair prices, and the existence of bequest motives, the demand for annuities is quite low.

[5]In order to focus on the effect of Social Security on capital accumulation, I assume inelastic labor supply, omitting the distortion caused by Social Security payroll tax on labor supply.

2.1 Demographics

During each model period, which is 5 years long, a continuum of people is born. Since there are no inter-vivos transfers, all agents start their working life with no assets.[6] I denote age $t = 1$ as 20 years old, age $t = 2$ as 25 years old, and so on. At the beginning of period 4, the agent's children are born, and four periods later (when the agent is 55 years old) the children are 20 years old and start working. The agents retire at $t = 10$ (when they are 65 years old) and die by the end of age $T = 14$ (before turning 90 years old). From $t = 10$ (when they are 65 years old), each person faces a positive probability of dying, given by $(1 - p_t)$. The probability of dying is exogenous and independent of other household characteristics. The population grows at rate n.

2.2 Government

The government taxes labor earnings, capital income and estates to finance the exogenous public expenditure, G. Income from labor is taxed at a flat rate τ_l.[7] Income from capital is taxed at a flat rate τ_a. Government taxes bequests at the rate τ_b for the proportion above the exemption level ex_b.

The structure of the Social Security system is the following: Government taxes labor earnings below e_{max}, at a flat rate of τ_{ss}. Retired households receive Social Security benefits from the government each period until they die. The Social Security benefits that individuals receive are linked to their average lifetime earnings according to a piecewise linear function, as in the US Social Security program.

2.3 Technology

There is one type of good produced according to the aggregate production function $F(K; L) = K^{\alpha} L^{1-\alpha}$, where K is the aggregate capital stock and L is the aggregate labor input. The final

[6] Data from the Health and Retirement Study suggests that observed inter-vivos monetary transfers are fairly small (Cardia and Ng (2000)). Given the small size of observed inter-vivos monetary transfers, I doubt that this simplification would affect the quantitative predictions of my model much.

[7] In the model, labor earnings are estimated using after-tax earnings. Thus all the progressive features of the tax system are already reflected in the calibrated after-tax earnings distribution. I introduce a constant tax rate τ_l to balance the government budget.

goods can be either consumed or invested into physical capital. Physical capital depreciates at rate δ. Households rent capital and efficient labor units to the representative firm each period.

2.4 Labor Productivity

In this economy, all agents of the same birth cohort face the same exogenous age-efficiency profile, ϵ_t. Each worker i also faces stochastic productivity shocks y_t^i, which follows a Markov process Q_y: $\ln y_t^i = \rho_y \ln y_{t-1}^i + \mu_t^i,\ \mu_t^i \sim N(0, \sigma_y^2)$. This Markov process is the same for all households, so that there is no uncertainty over the aggregate labor endowment. The total productivity of a worker at age t is given by the product of the worker's age-t productivity shock and age-t deterministic efficiency index: $y_t^i \epsilon_t$.

To capture the intergenerational correlation of earnings, I assume the parent's productivity shock at age 55 is transmitted to children at age 20 according to the following transition function Q_{yh}: $\ln y_1^i = \rho_{yh} \ln y_{h,8}^i + \nu_1^i,\ \nu_1^i \sim N(0, \sigma_{yh}^2)$. What the children inherit is only their first draw; from age 20 on, their productivity y_t evolves stochastically according to Q_y.

2.5 Consumer's Maximization Problem

2.5.1 Preferences

Individuals derive utility from consumption and from bequests transferred to their children upon death. Preferences are assumed to be time separable, with a constant discount factor β. The momentary utility function from consumption is of the constant relative-risk aversion class given by $U(c) = c^{1-\eta}/(1-\eta)$.

Following De Nardi (2004), the utility from leaving a bequest b is denoted by $\phi(b) = \phi_1(1 + b/\phi_2)^{1-\eta}$. The term ϕ_1 reflects the parent's concern about leaving bequests to his/her children, while ϕ_2 measures the extent to which bequests are luxury goods.[8]

[8]With the specifications, bequests are luxury goods. The scalar 1 inside the parameters ensures that the marginal utility of small bequests is bounded, while the marginal utility of large bequests declines more slowly than the marginal utility of consumption.

2.5.2 The Household's Recursive Problem

In a stationary equilibrium, the interest rate is constant at r and the wage rate is at w.[9] I assume that children have full information about their parents' state and children solve their decision problems after observing their parents' decisions. Children infer the size of the bequests they are likely to receive based on this information. The household's state variables are given by $(t, a, y, \widetilde{y}, S_p)$. The first four variables denote the agent's age, financial assets carried from the previous period, the agent's productivity, and average Social Security earnings, respectively. The last term S_p denotes the agent's parent's state variables and differs in each of the following four cases.

(i) From $t = 1$ to $t = 2$ (from 20 to 25 years of age), the agent survives for sure until next period and does not expect to receive a bequest because his/her parent is younger than 65.

$$V(t, a, y, \widetilde{y}, a_p, y_p, \widetilde{y_p}) = \max_{c,a'} U(c) + \beta E\big[V(t+1, a', y', \widetilde{y'}, a'_p, y'_p, \widetilde{y'_p})\big] \tag{1}$$

subject to

$$c + a' = (1-\tau_l) w \epsilon_t y - \tau_{ss} \min\left(w\epsilon_t y, e_{max}\right) + \big[1 + r(1-\tau_a)\big] a \tag{2}$$

$$a' \geq 0, \; c \geq 0, \tag{3}$$

$$\widetilde{y'} = \big[(t-1)\widetilde{y} + \min\left(w\epsilon_t y, e_{max}\right)/5\big]/t, \tag{4}$$

$$\widetilde{y'_p} = \big[(t+6)\widetilde{y_p} + \min\left(w\epsilon_{t+7} y_p, e_{max}\right)/5\big]/(t+7), \tag{5}$$

$$a'_p = a'_p(a_p, y_p, \widetilde{y_p}). \tag{6}$$

The expected value of the value function is taken with respect to (y', y'_p), conditional on (y, y_p). At any subperiod, the agent's resources depend on asset holdings, a, and labor endowment, $\epsilon_t y$. Asset holdings pay a risk-free rate r and labor receives a real wage w. Average Social Security earnings for children and parents accumulate according to equations (4) and (5), respectively. The law of motion of assets for the parents, which is their decision rule, is given in equation (6).

(ii) From $t = 3$ to $t = 7$ (from 30 to 50 years of age), the worker survives for sure until the next period. However, the agent's parent is at least 65 years old and faces a positive

[9] A formal definition of the stationary equilibrium is provided in Appendix 7.1.

probability of dying at any period; hence, a bequest might be received at the beginning of the next period. Let $V^I(t,a,y,\widetilde{y})$ and $V(t,a,y,\widetilde{y},a_p,\widetilde{y_p})$ denote the value function of a person whose parent is dead and alive, respectively.[10]

$$V^I(t,a,y,\widetilde{y}) = \max_{c,a'} U(c) + \beta E\big[V^I(t+1,a',y',\widetilde{y'})\big] \tag{7}$$

subject to (2), (3), and (4). In the latter case,

(8)

$$V(t,a,y,\widetilde{y},a_p,\widetilde{y_p}) = \max_{c,a^+} U(c)+\beta E\big[p_{t+7}V(t+1,a^+,y',\widetilde{y'},a_p',\widetilde{y_p})+(1-p_{t+7})V^I(t+1,a',y',\widetilde{y'})\big]$$

subject to (3), (4), (6), and

$$c + a^+ = (1-\tau_l)w\epsilon_t y - \tau_{ss}\min\left(w\epsilon_t y, e_{max}\right) + \big[1+r(1-\tau_a)\big]a, \tag{9}$$

$$a' = a^+ + b(a_p,\widetilde{y_p})/N, \tag{10}$$

where a^+ denotes the financial assets at the end of the period before receiving bequests, b is a function of the parent's state variables, and N is the average number of children.

(iii) The sub periods $t = 8$ to $t = 9$ (from 55 to 60 years of age) are the periods before retirement, during which no more inheritances are expected because the agent's parent is already dead by that time. The agent does not face any survival uncertainty.

$$V(t,a,y,\widetilde{y}) = \max_{c,a'} U(c) + \beta E\big[V(t+1,a',y',\widetilde{y'})\big] \tag{11}$$

subject to (2), (3), and (4).

(iv) From $t = 10$ to $t = 14$ (from 65 to 85 years of age), the agent does not work and does not inherit any more, but faces a positive probability of dying. In case of death, the agent derives utility from bequeathing his/her assets. Households receive Social Security benefits $P(\widetilde{y})$.

$$V(t,a,\widetilde{y}) = \max_{c,a'} U(c) + \beta p_t V(t+1,a',\widetilde{y}) + (1-p_t)\phi(b) \tag{12}$$

subject to (3) and

$$\begin{aligned} c + a' &= \big[1+r(1-\tau_a)\big]a + P(\widetilde{y}) \\ b &= a' - \tau_b * \max(a' - ex_b, 0). \end{aligned}$$

[10] In both cases, since parents are retired, productivity for parents, y_p, is not in the state space and average Social Security earnings for parents, $\widetilde{y_p}$, does not change over time. In the former case, a_p and $\widetilde{y_p}$ are not in the state space any more.

3 Model with Two-sided Altruism

The economy is populated by overlapping generations of individuals that are linked through altruistic transfers. Individuals derive utility from their own lifetime consumption, and from the well-being of their predecessors and descendants. As in Laitner (1992), the parent and the children constitute a single decision unit, called a household, by pooling their resources and jointly solving a maximization problem. I assume that households face borrowing constraints and cannot hold negative assets at any age. There is no estate taxation since assets are jointly held by the parent and their children. In all other regards, this model is identical to the benchmark model presented in Section 2.[11]

The demographics in the model are the same as in the benchmark model. Each individual faces positive death shocks after age 65. A household lasts 35 years and is constituted by the "parent", and its $m = (1+n)^{35}$ adult children. In a given household, all children are identical regarding their labor abilities and ages. The composition of a household changes when the parent dies. There are two types of households: those where the parent has died, and those where the parent is still alive. Let $h \in \{0, 1\}$ indicate household composition that takes the value unity if the parent is alive and 0 otherwise. A dynasty is a sequence of households that belong to the same family line. Once the children reach age 55, each of them becomes a parent in the next-generation household of the dynasty.

The state of a household is given by the age of the youngest adult member j; the demographic type h; the asset holding a; labor productivity of the children and the parent y_s, y_f, and the average Social Security earnings of the children and the parent $\widetilde{y}_s, \widetilde{y}_f$.

From $t = 1$ to $t = 6$ (from 20 to 50 years of age), the household solves the following problems,

$$V(t, h, a, y_s, y_f, \widetilde{y}_s, \widetilde{y}_f) = \max_{c_s, c_f, a'} mU(c_s) + hU(c_f) + \beta E\big[V(t+1, h', a', y_s', y_f', \widetilde{y}_s', \widetilde{y}_f')\big] \tag{13}$$

[11] The main difference between this two-sided altruism model and Fuster (1999) and Fuster, Imrohoroglu and Imrohoroglu (2003) is that this model incorporates idiosyncratic income shocks. The main difference between this model and Fuster, Imrohoroglu and Imrohoroglu (2007) is that this model abstracts from labor supply choice and differences in income and mortality by education.

subject to

$$mc_s + hc_s + a' = \big[1 + r(1-\tau_a)\big]a + e(t, h, y_s, y_f, \widetilde{y_s}, \widetilde{y_f}) \tag{14}$$

$$a' \geq 0,\ c \geq 0, \tag{15}$$

$$\widetilde{y_s'} = \big[(t-1)\widetilde{y_s} + \min\left(w\epsilon_t y_s, e_{max}\right)/5\big]/t, \tag{16}$$

$$\widetilde{y_f'} = \big[(t+6)\widetilde{y_f} + \min\left(w\epsilon_{t+7} y_f, e_{max}\right)/5\big]/(t+7), \tag{17}$$

where c_s and c_f are the consumption of the child and the parent, respectively, and a' denotes the asset holdings to be carried over to age $t+1$. The term $e(t, h, y_s, y_f, \widetilde{y_s}, \widetilde{y_f})$ denotes the after-tax household earnings,

(18)

$$e(t, h, y_s, y_f, \widetilde{y_s}, \widetilde{y_f}) = \left\{ \begin{array}{ll} m\big[(1-\tau_l)w\epsilon_t y_s - \tau_{ss}\min\left(w\epsilon_t y_s, e_{max}\right)\big] + hP(\widetilde{y_f}) & \text{if } t \geq 3 \\ m\big[(1-\tau_l)w\epsilon_t y_s - \tau_{ss}\min\left(w\epsilon_t y_s, e_{max}\right)\big] + & \\ h\big[(1-\tau_l)w\epsilon_{t+7} y_f - \tau_{ss}\min\left(w\epsilon_{t+7} y_f, e_{max}\right)\big] & \text{otherwise} \end{array} \right\}.$$

From $t = 3$ to $t = 7$, the parent is between 65 to 85 years of age and does not work. $P(\widetilde{y_f})$ denotes the Social Security benefit, which is a function of the parent's average lifetime earnings $\widetilde{y_f}$.

At $t = 7$, the household solves the following problems,

$$V(t, h, a, y_s, y_f, \widetilde{y_s}, \widetilde{y_f}) = \max_{c_s, c_f, a'} mU(c_s) + hU(c_f) + m\beta E\big[V(1, 1, a', y_{ss}', y_s', 0, \widetilde{y_s}')\big] \tag{19}$$

subject to (15), (16), (18), and

$$mc_s + hc_s + ma' = \big[1 + r(1-\tau_a)\big]a + e(t, h, y_s, y_f, \widetilde{y_s}, \widetilde{y_f}).$$

In the next period, m new households are constituted in the dynasty and by construction the parent is alive. The ability of the new generation of the dynasty, y_{ss}', is correlated with the ability of the parent, y_s' according to Q_{yh}.

4 Calibration

I set the rate of population growth, n, to the average value of population growth from 1950 to 1997 from the Economic Report of the President (1998).[12] Vectors of conditional survival probabilities for people older than 65, p_t's, are set to the survival probabilities for people

[12]Since one period in this model corresponds to 5 years in real life, I adjust parameters in the model accordingly. I report parameters at annual frequency, unless stated otherwise.

born in 1965 (Bell, Wade and Goss (1992)).

Parameters		Calibrations
Demographics		
n	annual population growth	1.2%
p_t	survival probability	see text
Technology		
α	capital share in National Income	0.36
δ	annual depreciation rate of capital	6%
Endowment		
ϵ_t	age-efficiency profile	see text
ρ_y	AR(1) coefficient of 5-year productivity process	0.85
σ_y^2	innovation of 5-year productivity process	0.30
ρ_{yh}	AR(1) coefficient of productivity inheritance process	0.67
σ_{yh}^2	innovation of productivity inheritance process	0.37
Government policy		
τ_a	tax on capital income	20%
τ_b	tax on bequests	10%
ex_b	exemption level on bequest tax	40
e_{max}	Social Security earnings cap	2.47
$P(\widetilde{y})$	Social Security benefit	see text
τ_{ss}	Social Security tax	11.4%
τ_l	tax on labor income	21.81%
Preference		
η	risk aversion coefficient	1.5
β	discount factor	0.954
ϕ_1	weight of bequest in utility function	-20
ϕ_2	shifter of bequest in utility function	15

Table 1: Parameters used in the benchmark model

I take α, the share of income that goes to capital, to be 0.36 (Cooley and Prescott (1995)). I take depreciation to be 6% (Stokey and Rebelo (1995)). Given the calibration for the production function, the before-tax interest rate on capital net of depreciation r is 6%.

The deterministic age-profile of labor productivity ϵ_t is taken from Hansen (1993).[13] The persistence ρ_y and variance σ_y^2 of the stochastic productivity process are estimated from PSID data (Altonji and Villanueva (2002)).[14] The persistence is low and variance is high because this refers to income in a 5-year period. I take persistence ρ_{yh} of the productivity inheritance process from Zimmerman (1992), and variance σ_{yh}^2 from De Nardi (2004).

[13]Since I impose mandatory retirement at the age of 65, I set $\epsilon_t = 0$ for $t > 9$.

[14]De Nardi (2004) provides a detailed discussion of the estimation process.

The capital income tax τ_a is set at 20% (Kotlikoff, Smetters and Walliser (1999)). The rate τ_b is the tax rate on estates that exceed the exemption level ex_b. These two parameters are chosen from De Nardi (2004) who matches the observed ratio of estate tax revenues to GDP, and the proportion of estates that pay estate taxes. The Social Security earnings cap e_{max} is 2.47. The retirement benefit is calculated to mimic the Old Age and Survivor Insurance component of Social Security system:

$$P(\widetilde{y}) = 0.9min(\widetilde{y}, 0.2) + 0.32max(0, min(\widetilde{y}, 1.24) - 0.2) + 0.15max(0, min(\widetilde{y}, e_{max}) - 1.24).$$

The bend points and Social Security earnings cap, expressed as average earnings, and marginal rates are from Huggett and Ventura (1999). The corresponding payroll tax rate that balances the Social Security funds budget is 11.4%.

I take the risk aversion coefficient, η, to be 1.5, from Attanasio, Banks, Meghir and Weber (1999), and Gourinchas and Parker (2002), who estimate it from consumption data. This value is in the commonly used range (1-5) in the literature.

I choose β, ϕ_1, ϕ_2, and τ_l to match the capital-output ratio of 3, bequest-output ratio of 2.64% (Gale and Scholz (1994)), the amount of bequest left at the lowest 80th percentile of 3.02 (Hurd and Smith (2001)), and a ratio of government spending to GDP of 18% (Council of Economic Advisors (1998)).[15]

5 Numerical Results

This section quantitatively assesses the long-run effects of eliminating pay-as-you-go (PAYG) Social Security. In the model with impure altruism, parents and their children are linked by bequests, both voluntary and accidental. To better understand the interaction between intergenerational altruism and Social Security system, I run several experiments. First, I study the benchmark model with bequest motives and intergenerational transfers of bequests and human capital. Then, I look at a standard life-cycle model without altruism by turning off all intergenerational links and assuming that accidental bequests are equally redistributed

[15]I use distribution for single decedents instead of the one for all decedents. Typically a surviving spouse inherits a large share of the estate, consumes part of it, and only leaves the remaining to the couple's children.

among 45-year-old people.[16] Next I examine a model where parents do not care about bequests but there are intergenerational transfers of accidental bequests and human capital. Finally I show the model with two-sided altruism. In each experiment, I recalibrate β and τ_l accordingly to generate a capital-output ratio of 3 and a ratio of government spending to GDP of 0.18 in the steady state with Social Security.[17] Details about the computation of the benchmark equilibrium are provided in the Appendix 7.2.

For each model mentioned above, I compare the model with one without PAYG Social Security in which Social Security tax τ_{ss} and benefits are set to zero simultaneously. The tax rates on capital income and bequests and the level of government purchases G are kept constant. The tax on labor income is adjusted to balance the government budget. I will compare the aggregate statistics, wealth and consumption inequality, and welfare.

5.1 Aggregate Statistics

Benchmark model When I change the Social Security system, the interest rate and the wage rate will adjust to clear the capital and labor markets. To disentangle the general equilibrium effect, I report two different types of experiments. In the first experiment, I keep prices fixed and thus the U.S. is treated as a small open economy. In the second experiment, I adjust the interest rate and the wage rate to clear the markets.

Aggregate consequences of eliminating Social Security are summarized in Table 2, Line 1. Eliminating Social Security boosts household savings and leads to an increase in the capital stock. If prices are fixed, aggregate wealth increases by 64 percent.[18] Consumption increases by 17 percent. Households hold on average more assets at retirement. Therefore, the aggregate bequest increases dramatically. Government tax revenue from capital income increases. Hence to maintain a balanced budget, tax rate on labor income is reduced by 4 percentage points.

[16] According to the demographics in the model, the life expectancy is 84 and the expected age of receiving inheritance is 49. Distributing bequests to 50-year-old people gives quantitatively similar results.

[17] The resulting β is 0.961,0.959, 0.934 in the model without intergenerational links, the one without bequest motives, and the one with two-sided altruism, respectively. τ_l is 0.2187 in all three models.

[18] Output is defined as GDP. Since labor supply is inelastic, output does not change.

If the interest rate and the wage rate are allowed to adjust to clear the capital and labor markets, the interest rate decreases by 1.87 percentage points and wage rate increases by 12.0 percent. The decline of interest rate discourages saving. Thus aggregate wealth increases by 37.2 percent, smaller than that when prices are fixed. A higher stock of capital raises the aggregate output by 12.0 percent. Aggregate consumption increases by 7.63 percent.

	τ_l[a]	r	w	Y	A	C	B
1, Impure altruism							
With Social Security	21.8	6.00	1.000	1.000	3.000	0.623	0.0264
No Social Security							
Fix prices	-4.0	0	0	0	+64.0	+17.0	+76.9
Change prices	-1.9	-1.87	+12.0	+12.0	+37.2	+7.63	+8.72
2, No links of accidental bequests and human capital							
With Social Security	21.8	6.00	1.000	1.000	3.000	0.623	0.0183
No Social Security	-2.1	-2.11	+13.9	+13.9	+43.5	+8.57	+51
3, With links of accidental bequests and human capital							
With Social Security	21.8	6.00	1.000	1.000	3.000	0.623	0.0176
No Social Security	-2.2	-2.14	+14.2	+14.2	+44.8	+8.76	+53
4, 2-sided altruism							
With Social Security	21.8	6.00	1.000	1.000	3.000	0.623	-
No Social Security	-0.6	-0.67	+3.7	+3.7	+10.7	+2.61	-

[a]Tax rate and interest rate are already expressed in percent, changes after the reform are in levels.

Table 2: Aggregate statistics (% change from the corresponding model with Social Security)

No bequest motives Line 2 in Table 2 shows the aggregate statistics in an environment without intergenerational links. Aggregate accidental bequest generated in this economy without altruism is much smaller than that the benchmark model with impure altruism. This shows that a voluntary bequest motive is the key to generate a large amount of aggregate bequest as is observed in the data.[19] After the elimination of Social Security system, when prices have adjusted, the interest rate decreases by 2.11 percentage points and the wage rate increases by 13.9 percent. Aggregate saving rises by 43.5 percent and aggregate consumption increases by 8.57 percent. The total accidental bequest increases by 51 percent. The reduction in interest rate and payroll tax rate is larger compared with the benchmark economy after the elimination of Social Security.

Table 2, Line 3 shows the aggregate statistics in an environment with intergenerational

[19]More comparison of the model with and without bequest motives are provided in Yang (2008).

links but without bequest motives. The results are very similar to the case without intergenerational links.[20] Aggregate saving rises by 44.8 percent and aggregate consumption increases by 8.76 percent. Total accidental bequests increases by 53 percent. The comparison between experiment 2 and experiment 3 shows that the unequal distribution of involuntary bequests and intergenerational transfer of earnings ability are not important in understanding the aggregate effect of Social Security reform.

In the benchmark economy, an unfunded Social Security system crowds out only 37 percent of the capital stock, which is smaller than the results obtained in pure life-cycle models. This difference is partially due to the existence of a bequest motive in the benchmark model. In pure life-cycle models, Social Security redistributes income away from middle-aged agents with higher marginal propensities to save to old agents with lower marginal propensities to save, therefore reduces the saving rate. In a model with bequest motives, old agents might save for leaving bequests, which increases marginal propensities to save. As a result, Social Security in my model has a smaller effect on the aggregate saving rate and on capital accumulation than in a pure life-cycle model.

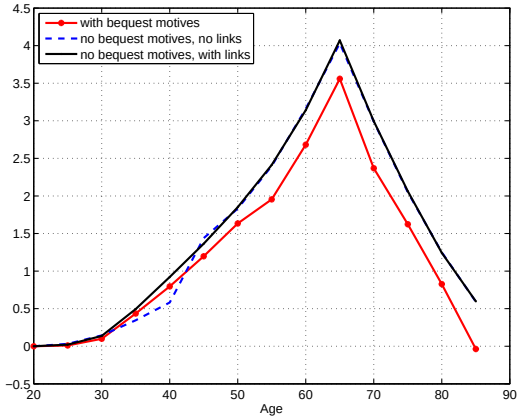

Figure 1: Increase of average assets holdings after privatization

I now elaborate this point by showing the responses in savings by age group to the change in Social Security. Figure 1 shows the increase of assets after eliminating Social Security at each age group. Eliminating Social Security increases assets holdings from age 30, with a

[20]Total accidental bequests in the model with links are a bit smaller than in the model without links. This is because the calibrated β is slightly lower in the model with links and thus retirees hold less wealth on average.

magnitude significantly larger around the retirement age. This is because, young households save mainly for precautionary reasons, and as households age, retirement saving becomes more important. We notice that, after the elimination of Social Security, asset holdings for households at ages 45 and older increase more in pure life-cycle models than in a model with impure bequest motives. This is because, old agents might save for bequest motives and respond less to a change in Social Security. The responses in those two models without bequest motives are very similar.[21]

Two-sided altruism Now I show the effect of two-sided altruism in analyzing the effect of Social Security in Line 4 in Table 2. We observed that the aggregate implications are much smaller compared with those in the benchmark model. Aggregate capital increases by 10.7 percent and consumption increases by 2.61 percent. Eliminating Social Security increases the capital stock much less than that in life-cycle models.

The result that Social Security has a very small crowding-out effect under two-sided altruism is consistent with Fuster, Imrohoroglu and Imrohoroglu (2003) who show that in a model with two-sided altruism, the transfer induced by Social Security is partially undone by altruistic transfers from parent to children and thus Social Security has a small impact on saving. In the benchmark model, households only have one-sided impure altruism which provides less insurance across generations. The elimination of Social Security does not impact decisions on the timing and direction of bequests. Therefore Social Security has a bigger crowding-out effect on capital stock.

5.2 Wealth and Consumption Inequality

Next I discuss the effect of Social Security on wealth and consumption inequality. Those results are important to understand the welfare effect of Social Security reform.

Benchmark model Line 1 in Table 3 reports the Gini coefficient of wealth in the alternative Social Security systems. The model with intergenerational links of voluntary

[21]The responses in the model without intergenerational links are smaller at ages 30 to 40, and bigger at ages 45 and after, compared with those in the model with intergenerational links. This is due to the difference in redistributing inheritances. In the economy without links, the amount of inheritances at age 45 increases after the reform, while in the economy with links, inheritances increase from ages 30 to 55.

bequests and human capital can generate a skewed wealth distribution with a Gini coefficient of 0.765.[22] Wealth inequality among retirees is smaller than that among workers. This shows that a large amount of wealth dispersion in the economy is due to differences in age. In an economy with Social Security, retirees implicitly have Social Security wealth in addition to net worth a. Thus, for retirees, I add the present value of accrued future Social Security benefits. This reduces Gini among retirees substantially, indicating that poor retirees have more Social Security wealth relative to their net worth than rich retirees do.

Social Security is redistributive since benefits are regressive with respect to contribution. By discouraging wealth accumulation of the poor relative to lifetime earnings more than that of the rich, such a redistributive Social Security system leads to higher wealth dispersion.[23] Accordingly, abolishing Social Security decreases wealth inequality, especially for retirees. However, the Gini coefficient of 0.523 among retirees is actually bigger than that after adding Social Security wealth (0.514). This is because, poor households receive more Social Security benefits than what they would save on their own after the elimination.

	wealth			+SS wealth
	all	workers	retirees	retirees
1, Impure altruism				
With Social Security	0.765	0.782	0.69	0.514
No Social Security	0.694	0.741	0.523	
2, No links				
With Social Security	0.732	0.760	0.625	0.472
No Social Security	0.688	0.739	0.502	
3, With links				
With Social Security	0.738	0.763	0.647	0.481
No Social Security	0.684	0.731	0.516	
4, 2-sided altruism				
With Social Security	0.669	-		
No Social Security	0.631	-		

Table 3: Gini coefficient of wealth

Table 4 shows the Gini coefficients of consumption at each age group. Four age groups are chosen according to cases in the benchmark model. The benchmark model generates an increase of consumption inequality by age. After the reform, the distribution of consumption

[22]De Nardi (2004) provides more detailed explanation on how the existence of intergenerational links of bequests and human capital increases wealth concentration.

[23]The results when fixing prices are very similar and are available upon request.

becomes less skewed with a Gini coefficient reduced from 0.477 to 0.459. This indicates that households are able to smooth consumptions better. Gini coefficients at ages 20 to 30 and at ages 30 to 55 are reduced noticeably. This is because, the reduction of taxes and the increase of wage rate enables borrowing-constrained poor households to increase consumption while does not affect rich households who save for retirement. Gini coefficients among retirees are increased significantly. This is due to the fact that Social Security is redistributive. Poor households receive more Social Security benefits than what they would save for themselves without Social Security. After the Social Security reform, poor retirees reduce consumption relatively more than the rich does, generating a bigger Gini coefficient.

No bequest motives Lines 2 and 3 in Table 3 show the wealth inequality in the two versions of life-cycle models without bequest motives. Life-cycle models without bequest motives, regardless of the existence of intergenerational transfer of bequests, do not generate a skewed distribution of wealth as in the benchmark model. As in the benchmark model, abolishing Social Security decreases wealth inequality. Consumption inequality in the model with links is similar to that in the benchmark model, indicating that bequest motive along does not change the distribution of consumption very much. Consumption inequality is lower in the model without links, especially among those after age 30, because bequest is evenly distributed. As in the benchmark model, abolishing Social Security decreases consumption inequality among young workers and increases that among retirees.

	all	[20, 30)	[30, 55)	[55, 65)	[65, 90)
1, Impure altruism					
With Social Security	0.477	0.424	0.463	0.459	0.460
No Social Security	0.459	0.417	0.445	0.461	0.475
2, No links					
With Social Security	0.462	0.416	0.440	0.441	0.446
No Social Security	0.445	0.408	0.427	0.446	0.461
3, With links					
With Social Security	0.476	0.418	0.460	0.459	0.458
No Social Security	0.458	0.411	0.443	0.461	0.475
4, 2-sided altruism					
With Social Security	0.407	0.400	0.397	0.400	0.391
No Social Security	0.420	0.405	0.412	0.405	0.414

Table 4: Gini coefficient of consumption by age

Two-sided altruism In this model, wealth is jointly held by parent and children in the same household thus I only report Gini of wealth for all. Compared with the life-cycle models, the model with two-sided altruism generates less wealth concentration with a Gini coefficient of 0.669. Parent and children are pooling income together, which reduces the income risk that households face and thus reduces wealth dispersion. In addition, there is less variation of wealth by age. As in life-cycle models, abolishing Social Security decreases wealth inequality.[24]

The consumption inequality under two-sided altruism is very different from that in a life-cycle model. First, since parent and children are pooling income together, consumption inequality barely varies along the life cycle.[25] Secondly, after the reform, the distribution of consumption becomes more skewed at every age group, especially among retirees and among those aged 30 to 50. This is because, Social Security benefit is fixed income and provides an insurance against labor income risk for children who live with their retired parents. For example, with Social Security, the consumption Gini among those aged 30 to 55 with parents (0.391) is lower than that among those without parents (0.414). Without Social Security, the pattern is reversed with corresponding numbers of 0.412 and 0.398, respectively. Those who live with their retired parents need to provide consumption for their parents who no longer receive Social Security benefits, resulting in more volatile consumption.

5.3 Welfare

The long-run welfare effects of abolishing PAYG Social Security system can be measured by the compensating variation, the fraction of consumption that should be given to a household in the steady state in all future periods and all contingencies to make the household as well off as in the steady state of the economy without Social Security. A caveat to this steady-state welfare measure is that it abstracts from costs of transition associated with such a policy

[24]In a model with 2-sided altruism and without idiosyncratic shocks, Fuster (1999) finds that eliminating Social Security decreases overall wealth inequality.

[25]The Gini coefficient at age 20 (25) equals that at age 55 (60) because the parent and their children in the same household have the same amounts of consumption.

reform.[26] The welfare gain for an unborn agent before the realization of all contingencies, denoted as w, is defined as,

$$\mathrm{w} = \left(\frac{\int_{t=1} V_n(x) m_n(dx)}{\int_{t=1} V_p(x) m_p(dx)} \right)^{\frac{1}{1-\eta}} - 1, \tag{20}$$

where V_p and m_p (V_n and m_n) refer to the value and the measure in the model with (without) Social Security, respectively.[27] Note that a negative number indicates that the agent experiences a welfare loss after eliminating Social Security.

To better understand the aggregate welfare effects, I further classify new-born agents by the types of shocks to labor productivity they receive at the beginning of the first age. Agents who receive initial productivity shock y_i, are referred to as type-i agents. For a newborn type-i agent, the welfare gain of a policy reform, denoted as w_i, is

$$\mathrm{w}_i = \left(\frac{\int_{t=1, y=y_i} V_n(x) m_n(dx)}{\int_{t=1, y=y_i} V_p(x) m_p(dx)} \right)^{\frac{1}{1-\eta}} - 1. \tag{21}$$

Social Security is beneficial as an annuity against mortality risk and insurance against labor-income shocks. However, Social Security imposes the following costs. Payroll tax distorts consumption and saving behavior when borrowing constraints are binding. In addition, Social Security lowers the capital stock and decreases aggregate steady-state consumption when capital stock is below its golden rule level. Those benefits and costs depend crucially on how generations are altruistically linked.

Benchmark model Table 5 reports the overall welfare impacts of privatizing Social Security for the economy as a whole, as well as for each type of agents. Living in an economy without a Social Security system, an unborn agent experiences a welfare gain. This indicates that, the benefit from Social Security as insurance against income shocks and as an annuity against mortality risk is outweighed by the loss from low consumption and tighter borrowing constraints. If prices are allowed to adjust, the average welfare gain increases further. In

[26] To eliminate unfunded liability, the government has to either cut benefits or raise taxes and make households in the transition period worse off. It is interesting to look at the transition path of the policy changes. However, due to the computational complexity, I leave this interesting issue for future research.

[27] The expected life-time utility may include utility from leaving bequests. Therefore, strictly speaking, w defined in this way is not equal to the consumption variation. This is of minor importance since for most households the discounted utility from leaving bequests is small compared with the discounted utility from consumption.

fact, the total welfare gain of 22.2 percent is much bigger than the percentage increase of aggregate consumption of 7.63 percent. This is because, as in shown in Table 4, consumption becomes less volatile after the reform. In addition, the large increase of wage rate enables better consumption smoothing across time. As is shown in Figure 2, the life-cycle profile of consumption is sharply different in the partial equilibrium and in the general equilibrium.

	w	w_1	w_2	w_3	w_4	w_5	w_6
1, Impure altruism							
Fix prices	18.5	19.7	19.2	18.7	18.1	17.5	14.5
Change prices	22.2	26.2	24.8	23.1	20.7	17.3	10.8
2, No links	23.8	28.3	26.4	24.5	22.2	19.2	12.9
3, With links	24.3	28.4	26.9	25.1	22.8	19.7	13.4
4, 2-sided altruism	-1.3	-11.1	-5.7	-1.0	2.5	4.5	4.0

Table 5: Average welfare gain (in percent)

In this model, consumption tracks income in the earlier life because of the existence of borrowing constraints and income shocks. Holding prices fixed, consumption increases parallelly after the reform and the peaks of consumption is similar to the counterpart in the steady state with Social Security. In the general equilibrium, consumption profile becomes more smooth. The large increase of wage rate relaxes borrowing constraints and leads to a higher consumption when young, which generates a welfare gain.

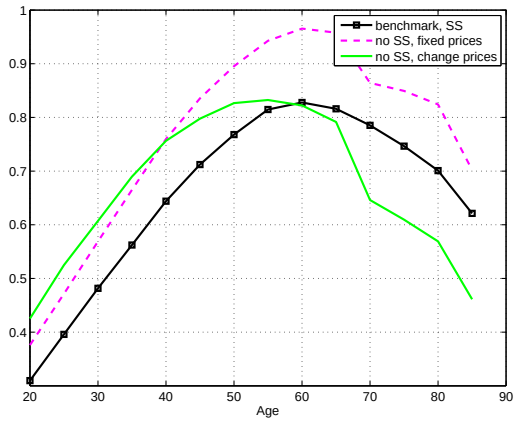

Figure 2: Average consumption over the life cycle (benchmark model)

The welfare measure for a new-born agent masks differences at the individual level. Households with low initial productivity are more in favor of eliminating a progressive Social Security system. To understand this observation, note that in this model two forces work in the opposite directions. Social Security has a redistribution effect since the Social Security benefit

is regressive with respective to past contributions. When income shocks are quite persistent, households with high initial productivity have lower replacement rates than households with low initial productivity. Thus households with low productivity suffer from the loss of Social Security benefits more. However, they benefit from the elimination of Social Security tax and a reduction of labor income tax. Social Security tax decreases consumption dollar for dollar when liquidity constraints are binding.[28] Low ability households has a steeper life-cycle profile of earnings thus are more likely to be borrowing constrained.

When prices adjust, the welfare gain for households with low initial productivity increases further while that for those with high initial productivity decreases. This is again because the increase in wage rate relaxes borrowing constraints. I simulate 1,000,000 households in the partial equilibrium and in the general equilibrium. I then calculate the average consumption at each age by initial productivity. Table 6 shows that consumption increases more for households with lower initial productivity after the price adjustment.

	Age													
type	20	25	30	35	40	45	50	55	60	65	70	75	80	85
1	1.12	1.13	1.14	1.12	1.08	1.04	1.00	0.99	0.95	0.92	0.71	0.81	0.79	0.73
2	1.12	1.15	1.15	1.11	1.07	1.03	0.99	0.97	0.93	0.90	0.73	0.78	0.76	0.71
3	1.12	1.16	1.14	1.10	1.05	1.01	0.97	0.94	0.91	0.87	0.75	0.75	0.73	0.68
4	1.18	1.16	1.11	1.08	1.03	0.99	0.95	0.91	0.88	0.83	0.76	0.72	0.69	0.66
5	1.18	1.16	1.07	1.06	1.00	0.95	0.92	0.86	0.83	0.78	0.74	0.68	0.65	0.63
6	1.18	1.09	1.06	1.00	0.98	0.92	0.89	0.83	0.80	0.75	0.74	0.68	0.64	0.63

Table 6: Ratio of mean consumption in the general equilibrium to that in the partial equilibrium

I further classify all new-born agents by the types of shocks to labor productivity they and their parents receive when they are at age 20 and their parents are at age 55. Table 7 reports the welfare effects of privatizing Social Security for each type of agent. Households whose parents have medium productivity are more in favor of eliminating progressive Social Security system, regardless of the children's initial productivity.

One measure that can be used to understand the welfare results is the changes of inheritances by parents' productivity, I simulate 1,000,000 households in the benchmark economy

[28]The effect of borrowing constraint on the welfare and aggregate effect of Social Security is illustrated by Hubbard and Judd (1987).

	Children's productivity (from low to high)					
Parent's	25.8 (1.64)	24.1 (1.27)	22.3 (0.70)	20.3 (0.26)	17.7 (—)	12.4 (—)
product-	26.7 (2.81)	24.8 (1.87)	23.0 (1.18)	20.9 (0.83)	18.1 (0.54)	12.5 (—)
ivity	27.0 (3.77)	25.2 (2.70)	23.4 (1.81)	21.3 (1.22)	18.4 (0.72)	12.7 (-0.09)
(from low	26.4 (7.77)	24.7 (1.62)	23.0 (2.00)	21.0 (1.37)	18.2 (0.86)	12.6 (0.64)
to high)	25.5 (—)	23.7 (1.21)	22.0 (0.79)	20.1 (0.66)	17.5 (0.56)	12.1 (0.39)
	23.1 (—)	21.0 (—)	19.1 (-11.3)	17 (-3.49)	14.5 (-2.99)	9.46 (-1.61)

Table 7: Welfare gain and increase of mean lifetime inheritance relative to lifetime earnings in percentage in parenthesis (with bequest motives)

with and without Social Security. Lifetime earnings is defined to be the total earnings from age 20 to 60, discounted to age 65 using the after-tax interest rate. Lifetime inheritance is defined analogously. I then calculate the increase in the expected lifetime inheritance relative to lifetime earnings for all new-born agents by labor productivity shocks they receive at the beginning of the first age and their parents receive at age 55. The results are shown in Table 7 in parenthesis.[29]

We observe that the increase of inheritance relative to life income is monotonically decreasing with children's productivity level. This is because, children's lifetime income is monotonically increasing with children's productivity, and expected inheritance conditional on parents' productivity is independent of children's productivity. However, the increase of inheritance relative to lifetime income is not monotonic with respect to parent's productivity level. After elimination of Social Security, those parents who receive the medium productivity levels at age 55 will increase wealth holding after retirement; therefore their children on average receive more inheritances relative to lifetime earnings. In contrast, those parents who receive the highest productivity shocks decrease wealth holding after retirement due to a reduction in the interest rate; therefore their children receive less inheritances.[30]

No bequest motives Table 5 reports the overall welfare effects of abolishing Social Security for the economy as a whole, as well as for each type of agent. An unborn agent has

[29]Due to the intergenerational transmission of productivity, the measures of new-born agents who receive high productivity shocks and whose parents receive low productivity shocks, and of those who receive low productivity shocks and whose parents receive high productivity shocks, are very small. The results from simulation for those types of households are subject to severe simulation error and thus are omitted.

[30]In the partial equilibrium where interest rate and wage rate are fixed, parents' wealth increases monotonically by productivity shocks at age 55.

higher expected welfare in an economy without Social Security system. The welfare gain is bigger than that in the benchmark model with impure altruism because aggregate consumption increases more. As in the benchmark model, households with low initial productivity are more in favor of eliminating a Social Security system.

Two-sided altruism Line 4 in Table 5 reports the overall welfare effects of eliminating Social Security in the two-sided altruism. An unborn agent experiences a welfare loss of about 1.3 percent of consumption at each state if he/she were born in an economy without Social Security system.[31] One reason is that, the loss from low capital accumulation caused by Social Security is much less than that in a life-cycle model. In addition, as is shown in Table 4, consumption becomes more volatile after the reform. Since individuals also value the well-being of their predecessors and descendants, Social Security as insurance against income shocks and as annuity against mortality risk is valued more in the model with two-sided altruism.

Moreover, now households in which children have higher initial productivity are more in favor of abolishing Social Security system than those with lower productivity children. This is the opposite of life-cycle model. This comparison highlights the effect of intergenerational risk-sharing in two-sided altruism model which enhances consumption smoothing. In the two-sided altruism model, the adverse effect of Social Security tax from tighter borrowing constraints is offset by intergenerational transfers. Thus the difference in welfare gain from the reform by initial productivity is mainly driven by the regressivity of Social Security benefit, which the lower productivity children value more. In life-cycle models, intergenerational transfers is limited and the cost of Social Security system from tighter borrowing constraints is quite sever. Thus the difference in welfare gain from the reform by initial productivity is mainly driven by relaxing borrowing constraints, which is more welfare enhancing for the lower productivity children.

I further classify all agents by the types of shocks to labor productivity they and their

[31] Fuster, Imrohoroglu, and Imrohoroglu (2003) find similar results in an environment without idiosyncratic shocks. Fuster, Imrohoroglu, and Imrohoroglu (2006) find that, with elastic labor supply, eliminating Social Security generates a small welfare gain.

	Children's productivity (from low to high)					
Parent's	-11.6	-5.9	-0.5	4.1	7.7	2.3
product-	-10.9	-6.2	-1.3	3.1	6.7	6.9
ivity	-8.7	-5.2	-1.3	2.6	5.9	6.4
(from low	-5.2	-3.0	-0.4	2.5	5.2	5.7
to high)	-2.5	-1.2	0.4	2.4	4.4	4.9
	-6.1	-1.9	-0.8	0.5	2.1	2.9

Table 8: Welfare gain (two-sided altruism)

parents receive when they are at age 20 and their parents are at age 55. Table 8 reports the welfare effects of privatizing Social Security for each type of agent. Households in which children have higher initial productivity and parents have lower productivity are more in favor of eliminating a progressive Social Security system.[32] Those households are expecting to pay higher taxes because the children are of higher ability, and to get lower pension because the parents are of lower ability. Therefore, the return from Social Security is lower for them than for other types of households.

6 Conclusions

This paper studies how the long-run effect of eliminating Social Security depends on the existence of impure altruism in a life-cycle model. The model that I am using mainly incorporates two basic forces, bequests and human capital transmission, into an otherwise standard life-cycle model in which households face uninsurable labor income risk, uncertain lifetimes and a borrowing constraint. In this life-cycle model with impure altruism, Social Security affects saving for retirement but not saving for bequests. An unfunded Social Security system crowds out less capital stock than in pure life-cycle models, but more than in a model with two-sided altruism. The welfare gain of eliminating Social Security under impure altruism is smaller than that in a pure life-cycle model, and bigger than that in a model with two-sided altruism.

In this paper, I have abstracted from some important issues in order to make the model manageable. One assumption is there is no housing. Housing is the single largest investment made by consumers over their lifetime. As it is shown in Yang (2009), abstracting from

[32]Fuster, Imrohoroglu, and Imrohoroglu (2003) find similar results in an environment without idiosyncratic shocks.

housing might bias the study of life-cycle consumption and asset accumulation. It will be interesting to extend this model to look at the effect of eliminating Social Security in an environment with housing.

7 Appendix

7.1 Definition of the Stationary Equilibrium in the Benchmark Model

I focus on an equilibrium concept where factor prices and age-wealth distribution are constant over time. Each agent's state is denoted by x. An equilibrium is described as follows.

Definition 1 *A stationary equilibrium is given by government tax rates, transfers, and spending* $(\tau_{ss}, \tau_l, \tau_a, \tau_b, ex_b, P(\widetilde{y}), G)$*; an interest rate* r *and a wage rate* w*; value functions* $V(x)$*, allocations* $c(x)$*,* $a'(x)$*; and a constant distribution of people* $m^*(x)$*, such that the following conditions hold:*

(i) Given government tax rates and transfers, the interest rate, and the wage rate, the functions $V(x)$*,* $c(x)$ *and* $a'(x)$ *solve the above described maximization problem for a household in a state* x*.*

(ii) m^* *is the invariant distribution of households over the state variables for this economy.*[33]

(iii) All markets clear.

$$C = \int cm^*(dx),\ K = A = \int am^*(dx),\ L = \int \epsilon y m^*(dx),$$

$$C + (1+n)K - (1-\delta)\,K + G = Y = F(K;L)$$

(iv) The price of each factor is equal to its marginal product.

$$r = F_1(K,L) - \delta,\ w = F_2(K,L).$$

(vi) Government budget constraint is balanced at each period.

$$G = \tau_a r A + \tau_l w L + \int \tau_b (1-p_t) I_{t>9} \max(a' - ex_b, 0) m^*(dx).$$

(vii) Social Security budget is balanced at each period.

$$\int I_{t>9} P(\widetilde{y}) m^*(dx) = \tau_{ss} \int I_{t\leq 9} \min\left(w\epsilon_t y, e_{max}\right) m^*(dx)$$

7.2 Computation of the Benchmark Model

I discretize both the productivity and the productivity inheritance processes to six-state Markov chains according to Tauchen and Hussey (1991). Since I want the possible realizations

[33] I normalize m^* so that $m^*(X) = 1$, which implies that $m^*(\chi)$ is the fraction of people alive that are in a state χ.

for the initial inherited productivity level to be the same as the possible realizations for productivity during the lifetime, I choose the quadrature points jointly for the two processes. The resulting grid points for the productivity process y are [0.1464, 0.3356, 0.7002, 1.4283, 2.9801, 6.8306]. The transition matrix Q_y is

$$\begin{bmatrix} 0.6099 & 0.3537 & 0.0357 & 0.0007 & 0.0000 & 0.0000 \\ 0.1307 & 0.5327 & 0.3026 & 0.0333 & 0.0006 & 0.0000 \\ 0.0086 & 0.1974 & 0.5193 & 0.2528 & 0.02175 & 0.0001 \\ 0.0002 & 0.0218 & 0.2528 & 0.5193 & 0.1974 & 0.0086 \\ 0.0000 & 0.0006 & 0.0333 & 0.3026 & 0.5327 & 0.1307 \\ 0.0000 & 0.0000 & 0.0007 & 0.0357 & 0.3537 & 0.6099 \end{bmatrix}.$$

The transition matrix Q_{yh} is

$$\begin{bmatrix} 0.3668 & 0.4788 & 0.1426 & 0.0116 & 0.0002 & 0.0000 \\ 0.0922 & 0.4240 & 0.3855 & 0.0928 & 0.0054 & 0.0000 \\ 0.0134 & 0.1887 & 0.4615 & 0.2899 & 0.0454 & 0.0011 \\ 0.0011 & 0.0454 & 0.2899 & 0.4615 & 0.1887 & 0.0134 \\ 0.0000 & 0.0054 & 0.0928 & 0.3855 & 0.4240 & 0.0922 \\ 0.0000 & 0.0002 & 0.0116 & 0.1426 & 0.4788 & 0.3668 \end{bmatrix}.$$

The transition matrices Q_y and Q_{yh} also induce an initial distribution of earnings. The distribution of accumulated productivity at each age is approximated on a grid of 36. The state space for asset holdings is discretized. Using this grid, I can store the value functions and the distribution of households as finite-dimensional arrays.

For a given set of parameters, I solve for the steady state equilibrium as follows:

1. Guess an initial value of τ_l, and interest rate r, use the equilibrium conditions in the factor markets to obtain the wage rate w.

2. Solve the approximated optimal consumption and saving plans recursively.

3. Guess an initial joint distribution of parents and children at the beginning of the life cycle, compute the associated stationary distribution of households.

4. Compute the implied joint distribution of parents and children at the beginning of the life cycle. If the distributions converge, go to step 5; otherwise go to step 3.

5. Given the stationary distribution and prices, check whether all markets clear and whether the government budget is balanced. If so, an equilibrium is found. If not, go to step 1.

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