ENDOGENOUS GROWTH AND STOCK RETURNS VOLATILITY IN THE LONG RUN

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ABSTRACT

We develop an endogenous growth model that incorporates random technological shocks to the economy. These random technological shocks affect both production and the depreciation of capital. We show the existence of a long-run steady-state growth path, and characterize it. An optimal growth rate for the economy and the long-run expected stock return are both derived. We then turn to study the volatility of expected stock returns around this steady state. Once tested, the model shows that deviations of de-trended capital stock, deviations of shocks from expected values and deviations of labor force growth from steady state together explain about 20% of the deviations of stock returns from long-term expected values. Our estimates also implies that investors have levels of risk aversion consistent with the literature, and that labor growth fluctuations are not significant, due to crowding out effects.

1. INTRODUCTION

The efficient markets hypothesis implies that stock market prices should follow a random walk and thus, stock returns should be unpredictable. However, many recent studies such as Fama and French (1988a, b), Keim and Stambaugh

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(1986), French, Schwert and Stambaugh (1987), Campbell and Shiller (1988), Chen, Roll and Ross (1986), Lo and MacKinlay (1988) and Fama (1990) document returns predictability. These studies have shown that state variables such as aggregate production growth and yield spreads are empirically useful in predicting stock and bond returns.

In an attempt to explain the time-varying behavior of stock returns, two broad categories of asset pricing models emerged. Consumption-based asset pricing models such as in Merton (1973), Lucas (1978), Breeden (1979) and Ceccetti, Lam and Mark (1990) relate the returns on financial assets to the intertemporal marginal rate of substitution of consumers using a consumption growth function. Production-based models on the other hand, relate the marginal rate of transformation to asset returns using production functions as in Cochrane (1991), Balvers, Cosimano and McDonald (1990) and Restoy and Rockinger (1994).

Cochrane (1991) finds that historical stock returns are related to economic variables such as growth rates of GNP and the investment to capital ratios. Unfortunately however, he does not provide a formal structure for explaining the real source and nature of economic fluctuations that might impact expected asset returns. Shawky and Peng (1995) use a real business cycle model with exogenous technical progress and show that technological shocks are critical factors in explaining asset returns.

We develop an endogenous growth model with technological shocks that affect both the final output and the depreciation rate of capital. We characterize the properties of a particular steady state growth path, where expected growth is constant in the long run. We then examine the volatility of stock returns around the steady state as a function of deviations of the de-trended capital stock, deviations from expected values of labor growth, and deviations of shocks from their long-term mean. This leads to an empirically testable hypothesis.

Our empirical results indicate that deviations of capital stock and technological shocks from their long run mean are significant variables, but that the deviations in labor growth are not, due perhaps to crowding out effects. The theoretical model predicts both the optimal growth rate of the economy as well as the long-term average stock market return. These are empirically testable implications. In fact, our results imply that investors exhibit a fair degree of consumption smoothing behavior. We also find that to be in accordance with the sample's expected stock return, our technology must exhibit some degree of increasing returns to capital.

This paper is organized in five sections. In Section 2 we set up the model and derive optimality conditions that are consistent with endogenous economic growth. We develop the concept of steady-growth in Section 3 and proceed to derive its equilibrium conditions. The time-series data, the methodology and the empirical results are presented and analyzed in Section 4. The final section provides a summary and some concluding remarks.

2. A MODEL OF ENDOGENOUS GROWTH

Consider a stochastic growth model similar to that in Brock-Mirman (1972), where technology is affected by a random shock every period. We also assume that growth is self-sustaining in a sense defined later on. Our goal is to investigate how stock returns are affected by long run technological trends.

Formally our goal is to search for the optimal policy that solves

$$J(Y_0) = \max E_0\left[\sum_{t=0}^{\infty} \rho^t U(C_t)\right]$$
$$C_t = (1 - i_t) \times Y_t \tag{1}$$
$$Y_t = \theta_t f(v_t, K_t)$$

Where C_t, K_t and Y_t are *per-unit-of-labor* consumption, capital stock and output, and v_t is the rate of capacity utilization. Labor L_t is assumed to evolve exogenously over time. The control variable is the investment rate i_t. The variable θ_t is a multiplicative random shock that is i.i.d. and is defined over a compact range $[\underline{\theta}, \overline{\theta}]$. The value of θ_t is realized at the beginning of period t. We assume that consumers' preferences are represented by $U(C_t) = C_t^{1-\gamma}/(1-\gamma)$, with $\gamma > 0$ representing the coefficient of relative risk aversion (CRRA).

We assume that the production function has constant returns to scale in capital and labor. In particular we use the following *per-capita* formulation:

$$f(v_t, K_t) = (A(v_t K_t)^{\alpha} + B)^{1/\alpha} \tag{2}$$

with $0 < \alpha < 1$ for now.[1] A crucial advantage of this formulation is that it allows the economy to grow at an endogenously sustained rate. In the traditional economic growth literature, an economy can only sustain growth by resorting to exogenous technical progress. In a sense, the fundamental source of economic growth is determined outside the model. The endogenous growth literature however, has sought to incorporate the sources of economic growth by featuring externalities (public spending, learning by doing) or certain factors of production that can be accumulated forever (human capital).

A critical feature for achieving endogenous growth is that these externalities counteract the natural tendency for decreasing returns to capital. In our model, given a stream of technological shocks, the economy will generate endogenous growth due to sufficiently high marginal returns to capital in the long run.[2] Capital utilization rates are included in the model because it is a way to measure the actual flow of services provided by the capital stock in place. These rates are exogenously determined and incorporating them in the model leads to a better estimate of the production function.[3]

We assume that capital depreciates at a stochastic rate δ_t, *and evolves according to*:[4]

$$K_{t+1} = [i_t Y_t + (1 - \delta_t) K_t] \frac{L_t}{L_{t+1}} \tag{3}$$

It is further assumed that the depreciation rate δ_t, is perfectly negatively correlated with the shocks θ_t, and hence we write $\delta_t = 1 - \mu\theta_t$, so that the above relationship becomes:

$$K_{t+1} = [i_t Y_t + \mu\theta_t K_t] \frac{L_t}{L_{t+1}} \tag{4}$$

The parameter μ must be such that $\mu\theta_t < 1$. The intuition for having a stochastic depreciation rate is that the outstanding stock of capital is generally subjected to the same type of transitory technological shocks as output.[5] For example, the productivity of labor measured in output/hour might be temporarily raised as a result of corporate downsizing. The productivity of capital might also be temporarily raised as a result of a credit crunch. A rise in productivity might induce some firms to slow down the rate of depreciation of certain capital goods.[6]

Next, we are solving for the social planner's optimum, as a way to characterize the optimal paths of consumption and investment in this economy.

A. Optimality Conditions

The standard first order condition is:[7]

$$\rho E_t \left\{ \frac{U'(C_{t+1})}{U'(C_t)} \theta_{t+1} f_2(v_{t+1}, K_{t+1}) \right\} \frac{L_t}{L_{t+1}} = 1 \tag{5}$$

Letting $\theta_{t+1} f_2(v_{t+1}, K_{t+1}) = (1 + R_{t+1})$ measure one plus the stock market return, we get:

$$\rho E_t \left\{ \frac{U'(C_{t+1})}{U'(C_t)} (1 + R_{t+1}) \right\} \frac{L_t}{L_{t+1}} = 1 \tag{6}$$

If we define $X_{t+1} = \dfrac{C_{t+1}}{C_t}$ and let $Z_{t+1} = X_{t+1}^{-\gamma}(1 + R_{t+1})$, then the first order condition becomes:

$$\rho E_t\{Z_{t+1}\}\,\frac{L_t}{L_{t+1}} = 1 \tag{7}$$

We will assume as in Hansen-Singleton (1983) that Z_{t+1} is log normally distributed $\ln(Z_{t+1}) \sim N(\mu_t, \sigma^2)$ conditional on the information available at t. Following their approach we can deduce a new first order condition as:

$$E_t\{R_{t+1}\} = \gamma E_t\left\{\ln\left(\frac{C_{t+1}}{C_t}\right)\right\} + \ln\left(\frac{L_{t+1}}{L_t}\right) - \ln(\rho) - \sigma^2/2 \tag{8}$$

Therefore:

$$R_{t+1} = \gamma E_t\left\{\ln\left(\frac{C_{t+1}}{C_t}\right)\right\} + \ln\left(\frac{L_{t+1}}{L_t}\right) - \ln(\rho) - \sigma^2/2 + \varepsilon_{t+1} \tag{9}$$

Where $\varepsilon_{t+1} = R_{t+1} - E\{R_{t+1}\}$. Equation (9) is identical to Hansen-Singleton (1983), except for the term involving labor growth. Hansen and Singleton state that it was not their goal to solve for an explicit representation of equilibrium prices in terms of the underlying shocks to technology. We take their model a step further by looking at the determinants of consumption growth in terms of technological progress and shocks.

3. STEADY STATE GROWTH

A characteristic of most industrialized economies is that per-capita real variables exhibit sustained growth over long periods of time. We will use the concept of steady state growth to describe a situation in which all state variables grow at the same constant *expected* rate. This is a novel approach in a growth model with stochastic shocks. Traditionally, the long-term stability of the economy refers to the convergence of cumulative distributions of shocks to a stationary distribution, as in Brock and Mirman (1972).

In order to characterize steady state growth, we need to transform the economy by detrending real variables.[8] Let g denote a particular growth rate and define new normalized variables as:

$$y_t = Y_t/(1+g)^t \quad c_t = C_t/(1+g)^t \quad k_t = K_t/(1+g)^t$$

We define a *Fulfilled Expectations Steady state* (FESS) as a vector $(\theta, g, n, \bar{i}, \bar{k}, \bar{y})$, where θ is the expected value of the random shock, g is the long-run expected growth rate of consumption, n is the long run growth rate of the labor force, and the vector $(\bar{i}, \bar{k}, \bar{y})$ is defined as follows:

$$\lim_{t\to\infty} \ln(i_t) = \ln(\bar{i}); \lim_{t\to\infty} \ln(k_t) = \ln(\bar{k})$$

$$\lim_{t\to\infty} \ln(y_t) = \lim_{t\to\infty} E_t\{\ln(y_{t+1})\} = \ln(\bar{y})$$

$$\text{with } \lim_{t\to\infty} E_t\{\ln(C_{t+1}/C_t)\} = g \text{ and } \lim_{t\to\infty} E_t\{\ln(L_{t+1}/L_t)\} = n$$

$$\text{and } \lim_{t\to\infty} \ln(\theta_t) = E_t\{\ln(\theta_{t+1})\} = \ln(\theta)$$

A Fulfilled Expectations Steady state is an equilibrium where the sequence of shock realizations converge to the expected value of the shock, and the long run expected growth rate is actually realized.[9] Our next proposition proves the existence of such a steady state.

Proposition 1: Assume there is a sequence of ex-post shocks which converges to θ, such that capacity utilization rates converge to a constant and the stock of capital grows at a constant rate in the long run, then a FESS $(\theta, g, n, \bar{i}, \bar{k}, \bar{y})$ exists and:

$$g = \lim_{t\to\infty} E_t\{\ln(C_{t+1}/C_t)\} = (1/\gamma)\left[\ln\left(\frac{\rho A^{1/\alpha}\bar{v}\theta}{1+n}\right) + \sigma^2/2\right]$$

$$\bar{R} = \lim_{t\to\infty} E_t\{R_{t+1}\} = \ln(A^{1/\alpha}\bar{v}\theta) = \gamma \ln(1+g) + \ln(1+n) - \ln(\rho) - \sigma^2/2$$

Proof: Assume that there exists a growth rate $\kappa > 0$ such that:

$$\lim_{t\to\infty} \ln(\theta_t) = \ln(\theta) \text{ and } \lim_{t\to\infty} \ln(\tilde{k}_t) = \ln(\tilde{k})$$

With $\tilde{k}_t = K_t/(1+\kappa)^t$. Therefore actual sequences of capital stocks grow to infinity. As $\lim_{t\to\infty} \ln(v_t) = \ln(\bar{v})$ this implies that:

$$\lim_{t\to\infty} E_t\{R_{t+1}\} = \lim_{t\to\infty} E_t\{\ln(\theta_{t+1} f_2(v_{t+1}, K_{t+1}))\} = \ln(A^{1/\alpha}\bar{v}\theta) \qquad (10)$$

Where $\ln(\theta) = E_t\{\ln(\theta_{t+1})\}$. We conclude from the Euler equation that:

$$\lim_{t\to\infty} E_t\{\ln(C_{t+1}/C_t)\} = (1/\gamma)\left[\ln\left(\frac{\rho A^{1/\alpha}\bar{v}\theta}{1+n}\right) + \sigma^2/2\right] = g \qquad (11)$$

So that consumption grows at a constant expected growth rate.[10] From Eqs (10) and (11) we can easily derive the second equality expressing the long-term expected rate $\bar{R}$ as a function of the growth rate and other parameters. From the capital accumulation equation we know that:

$$\lim_{t\to\infty} \ln(\tilde{k}_{t+1}) = \lim_{t\to\infty} \ln(i_t(Av_t^{\alpha} + BK_t^{-\alpha})^{1/\alpha} + \mu) + \lim_{t\to\infty} \ln(\tilde{k}_t) + \lim_{t\to\infty} \ln(\theta_t) - \kappa - \lim_{t\to\infty} \ln(L_{t+1}/L_t) \quad (12)$$

As $\lim_{t\to\infty} \ln(\theta_t) = \ln(\theta)$ this implies

$$\lim_{t\to\infty} \ln(A^{1/\alpha} i_t v_t + \mu) = n + \kappa - \ln(\theta) \quad (13)$$

so that $\lim_{t\to\infty} \ln(i_t v_t) = \text{constant}$. Since $\ln(v_t)$ converges to a constant $\ln(\bar{v})$ thus the log of the rate of investment $\ln(i_t)$ converges to a constant $\ln(\bar{i})$. If output is to grow at the same rate as consumption we have to set $\kappa = g$. In order to fulfill the last condition of existence of an FESS:

$$\lim_{t\to\infty} \ln(y_t) = \lim_{t\to\infty} E_t\{\ln(y_{t+1})\} = \lim_{t\to\infty} \ln(\bar{y}) \quad (14)$$

We need the following to hold true:

$$\lim_{t\to\infty} \ln(\theta_t) + \alpha^{-1} \lim_{t\to\infty} \ln[A + B(v_t K_t)^{-\alpha}] + \lim_{t\to\infty} \ln(v_t) + \lim_{t\to\infty} \ln(k_t) = \lim_{t\to\infty} E_t \ln(\theta_{t+1}) + \alpha^{-1} \lim_{t\to\infty} \ln[A + B(v_{t+1} K_{t+1})^{-\alpha}] + \lim_{t\to\infty} \ln(v_{t+1}) + \lim_{t\to\infty} \ln(k_{t+1}) = \ln(\bar{y}) \quad (15)$$

For some $\bar{y}$. Again, this is true when $\lim_{t\to\infty} \ln(\theta_t) = \ln(\theta)$. *Q.E.D.*

For the FESS to exist we impose a transversality condition that $\lim_{t\to\infty} (\rho(1+g)^{\gamma})^t = 0$. In other words, we need $\rho(1+g)^{\gamma} < 1$.[11] Proposition 1 gives exact closed form solutions for the optimal expected growth rate and the long run expected stock return. The long run expected stock return equals the long run expected productivity of capital. From a comparative statics perspective, we see that the long run rate of growth would rise with larger expected productivity, discount factor, and variance σ^2. The expected growth rate would drop with faster population growth, and a larger degree of risk aversion.[12]

A. Deviations from the Steady State

We follow the Real Business Cycle literature (Kydland-Prescott, 1982), and linearize the economy around the steady state (FESS). Even though an economy subjected to arbitrary shocks does not necessarily converge to the FESS, this steady state offers an interesting benchmark to look at macroeconomic fluctuations. It reproduces the stylized facts of actual economies, while still accounting for the random nature of shocks. One additional advantage is that by linearizing, we can construct a simple testable hypothesis about these fluctuations, without having to know the actual shape of optimal solutions.

The first step is to rewrite the first order conditions using normalized variables. Let us recall that $c_t = C_t/(1+g)^t$, and then we have:

$$R_{t+1} = \gamma E_t\left\{\ln\left(\frac{c_{t+1}}{c_t}\right)\right\} + \ln\left(\frac{L_{t+1}}{L_t}\right) - \ln\left(\frac{(1+g)^\gamma}{\rho}\right) - \sigma^2/2 + \varepsilon_{t+1} \qquad (16)$$

Let $e_t = C_t/Y_t$ be the consumption rate, then we have:

$$R_{t+1} = \gamma E_t\left\{\ln\left(\frac{e_{t+1}}{e_t}\right)\right\} + \gamma E_t\left\{\ln\left(\frac{y_{t+1}}{y_t}\right)\right\} + \ln\left(\frac{L_{t+1}}{L_t}\right)$$
$$- \ln\left(\frac{(1+g)^\gamma}{\rho}\right) - \sigma^2/2 + \varepsilon_{t+1} \qquad (17)$$

We denote with a 'hat' variables that represent deviations from the FESS. Thus $\hat{R}_{t+1} = (R_{t+1} - \bar{R})$ is the deviation of the stock market return, from its long run trend $\bar{R} = \ln(A^{1/\alpha}\bar{v}\theta)$. The variable $\hat{l}_{t+1} = (\ln(L_{t+1}/L_t) - n)$ is the deviation of labor force growth rate from its long-term value. We also define $\hat{y}_t = \ln(y_t/\bar{y})$, $\hat{k}_t = \ln(k_t/\bar{k})$, $\hat{v}_t = \ln(v_t/\bar{v})$, and $\hat{\theta}_t = \ln(\theta_t/\bar{\theta})$.

Because the representative agent's problem can be solved *after* we normalize the variables, analogous first order conditions imply that the optimal consumption rate decision can be rewritten as $e_t = 1 - i_t(y_t) = e(y_t)$.[13] If we define a monotonic transformation $\ln(e_t) = \mathrm{Q}(\ln(y_t))$, then the function $\ln(e_t)$ can be linearized around the FESS so that in effect we have:

$$\hat{e}_t = \ln(e_t/\bar{e}) \approx a \times \ln(y_t/\bar{y}) = a \times \hat{y}_t \text{ and } E_t \ln(e_{t+1}/\bar{e}) \approx a \times E_t \ln(y_{t+1}/\bar{y}) \qquad (18)$$

Where $a = \mathrm{Q}'(\ln(\bar{y}))$. The variable a represents the elasticity of the *rate* of consumption with respect to income, along the FESS.[14] In the long run, we obtain the following expression for the return on the market:

$$\hat{R}_{t+1} = \gamma(1+a)E_t\left\{\ln\left(\frac{y_{t+1}}{y_t}\right)\right\} + \hat{l}_{t+1} + \varepsilon_{t+1} \qquad (19)$$

This equation is similar to the Balvers et al. (1990), which suggests that the rate of return is mainly conditioned by output growth. The main difference is that we have labor growth as another explanatory variable, and we explicitly model the technological sector. Next, we expand the first term on the right hand side of (19) substituting in the specific production function:

$$E_t\left\{\ln\left(\frac{y_{t+1}}{y_t}\right)\right\} = E_t\{\ln(\theta_{t+1})\} + \alpha^{-1}\ln(A + B(v_{t+1}K_{t+1})^{-\alpha})$$

$$+ \ln(v_{t+1}) + \ln\left(\frac{k_{t+1}}{y_t}\right) \tag{20}$$

The last term in (20) becomes:

$$\ln\left(\frac{k_{t+1}}{y_t}\right) = \ln(i_t + (\mu\theta_t k_t/y_t)) - \ln\left((1+g)\,\frac{L_{t+1}}{L_t}\right) \tag{21}$$

Thus, as the capital stock K_t grows without bounds, the previous expression (21) becomes:

$$E_t\left\{\ln\left(\frac{y_{t+1}}{y_t}\right)\right\} = \ln(A^{1/\alpha}\theta) + \ln(i_t v_t + \mu/A^{1/\alpha})$$

$$+ \hat{v}_{t+1} - \hat{v}_t - \hat{l}_{t+1} - \ln((1+g)(1+n)) \tag{22}$$

Similar to the argument made previously about the consumption function, we can deduce that the optimal investment rate policy $i_t = \mathrm{I}(y_t)$ is a function of the normalized variable y_t, and $\mathrm{I}(y_t)$ is continuous. We also know that $y_t = \theta_t(A(v_t k_t)^{\alpha} + B(1+g)^{-\alpha t})^{1/\alpha}$. We can linearize this last function around the FESS, and express the second term on the RHS of (22) as:

$$\ln(i_t v_t + \mu/A^{1/\alpha}) = \ln(\overline{iv} + \mu/A^{1/\alpha}) + b_1\hat{\theta}_t + b_2\hat{k}_t + b_3\hat{v}_t \tag{23}$$

Where the coefficients b_is represent the elasticities of the *effective rate* of investment *iv*, with respect to θ, *k* and *v*, along the FESS. Finally, inserting this back into the linearized Euler condition (19) leads to:

$$\hat{R}_{t+1} = \gamma(1+a)(b_1\hat{\theta}_t + b_2\hat{k}_t + \hat{v}_{t+1} + (b_3 - 1)\hat{v}_t) + (1 - \gamma(1+a))\hat{l}_{t+1}$$

$$+ \gamma(1+a)\ln\left(\frac{\theta(A^{1/\alpha}\overline{iv} + \mu)}{(1+g)(1+n)}\right) + \varepsilon_{t+1} \tag{24}$$

Once we substitute the value for the growth rate g into this equation we obtain:

$$\hat{R}_{t+1} = \gamma(1+a)(b_1\hat{\theta}_t + b_2\hat{k}_t + \hat{v}_{t+1} + (b_3 - 1)\hat{v}_t) + (1 - \gamma(1+a))\hat{l}_{t+1} + (1+a)$$
$$\times \left[\gamma \ln(\overline{iv} + \mu/A^{1/\alpha}) - \gamma \ln(\bar{v}) - \ln(\rho) - (1-\gamma) \ln\left(\frac{\theta\bar{v}A^{1/\alpha}}{1+n}\right) - \sigma^2/2 \right]$$
$$+ \varepsilon_{t+1} \tag{25}$$

We will present some evidence in the next section that the rate of capacity utilization is related to labor growth, and that labor growth deviations from steady state are autocorrelated. In fact, we are making the following assumptions:

$$\hat{l}_{t+1} = \delta_1\hat{l}_t \text{ and } \hat{v}_{t+1} = \delta_2\hat{l}_{t+1} \tag{26}$$

Thus Eq. (25) becomes:

$$\hat{R}_{t+1} = \gamma(1+a)(b_1\hat{\theta}_t + b_2\hat{k}_t) + [1 - \gamma(1+a)[1 - \delta_2(1 + (b_3 - 1)/\delta_1)]] \cdot \hat{l}_{t+1}$$
$$+ (1+a)\left[\gamma \ln(\overline{iv} + \mu/A^{1/\alpha}) - \gamma \ln(\bar{v}) - \ln(\rho) - (1-\gamma) \ln\left(\frac{\theta\bar{v}A^{1/\alpha}}{1+n}\right) - \sigma^2/2 \right]$$
$$+ \varepsilon_{t+1} \tag{27}$$

Equation (27) is the fundamental result of the paper. It shows how stock return deviations from their long run mean are predicted by deviations of detrended capital stock, labor growth and technological shocks away from their long-term means. The rationale behind this equation is that stock return fluctuations around their mean are conditioned by fluctuations in real activity or the business cycle, around a growing trend. Our model essentially takes the stance that the sources of linearity in the relationship (19) come from a near steady state analysis, as well as the linearity of the technology in the long run. This equation is in a reduced form that lends itself easily to empirical testing.

4. EMPIRICAL RESULTS

A. Time Series Variables

All economic data series are obtained from Datastream International, spanning the period 1959–1998. All variables are quarterly, beginning first quarter 1959 to third quarter of 1998. The GDP (USGDP. . D) and consumption expenditures (USCONEXPD) are deseasonalized real variables. Real investment is measured by real private non-residential investment (USNRINVD). Real wage is calculated by dividing nominal wage (USWAGSALB) by the CPI (USCP. . . . F) normalized to be 100 in 1992. Labor is measured as total civilian population employed (USEMPTOTE). Capacity utilization in all industries (USOPERATE), is constructed for the missing years 1959–1967, by regressing capacity utilization over the period 1967–1998 onto the rate of employment and projecting that relationship backward over the missing years. Total stock returns were obtained using Datastream and Tradetools, for the S&P 500 and the dividend yield. Total annual real returns are calculated using the CPI index as deflator.

B. Construction of Capital Stocks and Technological Shocks

The capital stock is constructed using the permanent inventory approach. It is determined using quarterly data from 1959 to 1998. A feature of the model is that technological shocks influence depreciation. Thus there is a nested determination of shocks and capital stocks next period. The initial index of total factor productivity is derived using Baumol et al. (1986).[15] The subsequent indexes of technical shocks are obtained by using Thornqvist's formula given in Barro et al. (1994), which calculates discrete increments in the Solow residuals.[16] We select the depreciation parameter μ through a numerical procedure to get an average annual depreciation rate equal to 9.6%, over the sample period.[17]

C. The Production Function

Our regression is for the period 1966–1998. The production function is estimated with OLS. Here we follow a separate approach for constructing the capital stock. The capital stock is constructed based on the assumption that the depreciation rate is non-stochastic and *constant* at 9.6% per year.[18] In each period; the capital stock is weighted by the corresponding rate of capacity utilization. Our initial capital stock is arbitrarily chosen, thus the estimated stocks of capital will be unreliable for the first few quarters starting in 1959. However, as the stock is further accumulated and depreciated, future estimates become progressively more accurate. It was necessary to re-construct the sequence of random shocks, to avoid the problem of deriving shocks from time series of capital stocks, leading to serial correlation. We accomplish that by using a dual approach as in Barro (1998).[19] The series is then regressed on a linear time trend and detrended.

We use an iterative procedure where we estimate the parameters *A* and *B*, based on a chosen value for α (Table 1, Panel A). The second regression (Panel B) insures that the generated shock sequence indeed corresponds to the estimated residuals over the sample. This will be true only if, after substituting in the values for *A* and *B*, the coefficient on the exogenous variable is close to 1, and the constant term is close to zero. The parameter α is iteratively modified to achieve that outcome. The list of parameters of the production function is given in Table 4. From Table 1 we see that the adjusted $R^2 = 0.82$. The value for α is equal to 3.16, which implies increasing marginal returns to capital.[20] Even though the production function we chose conforms to the neoclassical theory of factor income distribution, the share of income going to capital rises, as the economy grows to the steady state, and then levels off. This is consistent with

Table 1. Production Function.

This regression is based on 128 quarterly observations for the period 1966–1998. The initial capital stock is chosen to be $1 trillion (1992 dollars). The shocks θ_t are derived from a dual approach (Barro (1998)). The exponent α equals 3.16, and the depreciation rate equals a constant 2.4% per quarter.

Panel A:

$$\left[\frac{Y_t}{\theta_t}\right]^{\alpha} = A(v_t K_t)^{\varepsilon} + B$$

	Constant	$(v_t K_t)^{\alpha}$
Coefficient	4.71×10^{12}	0.03
T-values	(28.79)	(24.62)
Adjusted R^2	0.82	

Panel B:

$$\ln(Y_t^{\alpha}) = \rho_1 \ln[A(v_t K_t)^{\alpha} + B] + \rho_2 + u_t$$

where u_t is the residual. It should be true that $\exp(u_t) \approx \theta_t$ when the coefficient α is chosen appropriately so that $\rho_1 = 1$ and $\rho_2 = 0$. Here $\alpha = 3.16$.

	Constant	$\ln[A(v_t K_t)^{\alpha} + B]$
Coefficient	–0.39	1.01
T-values	(–1.1)	(84.01)
Adjusted R^2	0.98	

the labor productivity slowdown observed from the 1970s to the late 1980s (Baumol et al. (1991)).

D. Detrending the Variables

Recall that all our variables are detrended log deviations from a steady state path. In our case we detrend the variables using the average per-capita consumption growth rate for the period 1966–1997. We estimate it to be 1.23% annually. Deviations are defined with respect to the sample means.

E. Discussion of Results

Table 2 presents the results for our stock return regression corresponding to Eq. (27). In panel A, we are using 128 quarterly observations, from 1966 to 1997. The adjusted R^2 is 8%. In panels B, we use only second quarter observations at yearly intervals. The adjusted R^2 is 20%.[21] These results are consistent with the findings of Balvers, Cosimano and McDonald (1990), related to alternative

Table 2. Stock Returns, Productivity Shocks and other Economic Variables.

The rate of growth of the economy is 0.307% per quarter. The depreciation rate *averages* 2.4% per quarter.

$$\hat{R}_{t+1} = A_0 + A_1\hat{\theta}_t + A_2\hat{k}_t + A_3\hat{l}_{t+1} + \varepsilon_{t+1}$$

Panel A: This regression is based on 128 quarterly observations from the first quarter of 1966 to the first quarter of 1998.

	Constant	$\hat{\theta}_t$	$\hat{k}_t$	$\hat{l}_{t+1}$
Coefficient	0.57	197.60	24.26	–124.64
T-Values	(0.59)	(3.17)	(3.54)	(–0.83)
Adjusted R^2	0.08			

Panel B: This regression is based on 32 observations for the period 1966–1997. Each observation uses second quarter data at yearly intervals.

	Constant	$\hat{\theta}_t$	$\hat{k}_t$	$\hat{l}_{t+1}$
Coefficient	1.41×10^{-13}	575.84	122.06	–170.31
T-Values	(-5.5×10^{-14})	(2.25)	(3.22)	(–0.84)
Adjusted R^2	0.20			

return horizons. In the case of yearly intervals, we find that 20% of the volatility of stock returns around a steady state can be explained by macroeconomic variables such as volatility in capital stocks, technical shocks and labor growth fluctuations.

Looking at *t*-statistics, the only variables that are statistically significant are the detrended deviations of the capital stock and the Solow residual shocks. Labor growth deviations are not significant at the 95% confidence level. A possible interpretation for this result, is that the direct effects of employment growth are crowded out by the adjustments made to capacity utilization.[22] In fact, we find that rates of capacity utilization are correlated on a year to year basis with current labor growth, with an adjusted R^2 of 40% (see Table 3 panel B). As for labor growth, we find that it is autocorrelated on a quarterly basis with an adjusted R^2 of 27% (Table 3 Panel C). Initially, positive fluctuations in labor growth have a negative impact on stock returns, but because the rate of capacity utilization rises with the growth in labor, this tends to crowd out the first effect.[23]

We can also test our model using the predicted optimal growth rate for the economy given in Eq. (11). We derive the CRRA preference parameter that is consistent with having the theoretical optimal growth rate equal the sample average per capita consumption growth rate of 1.23% over the period 1966–1997. We find that the parameter value equals 2.84, which implies a fair degree of consumption smoothing behavior. The value of 0.965 for the discount rate is found by imposing the transversality condition, given the growth rate of the economy and the CRRA parameter. This is low compared to the range of estimates (0.988, 0.993) used in the standard RBC literature (King-Plosser & Rebelo, 1988; Ambler & Paquet, 1994).

Another interesting result is that the variance of the joint distribution of returns and marginal rates of substitution in consumption contributes for about 32% of the value of the expected growth rate of the economy.

The other important value derived from this exercise is the long-term interest rate found here to be equal to 7.75%, which is close to the annual mean return of 7.97%, over the sample period.[24] These results offer corroboration for our analysis.

5. CONCLUSION

Using an endogenous growth model we derived a theoretical relationship between the stock market returns deviations from long run expected value, expressed as a function of the deviations of aggregate macroeconomic variables

Table 3. Consumption Rate, Labor Growth and Capacity Utilization.

The rate of growth of the economy is 0.307% per quarter. The depreciation rate *averages* 2.4% per quarter.

Panel A: This regression is based on 32 observations at yearly intervals, from the second quarter of 1966 to the second quarter of 1997.

$$\hat{e}_t = a\hat{y}_t + b$$

	Constant	$\hat{y}_t$
Coefficient	7.19×10^{-3}	-10^{-5}
T-values	(3.87)	(–8.97)
Adjusted R^2	0.72	

Panel B: This regression is based on 32 observations for the period 1966–1997. Each observation is using second quarter data.

$$\hat{v}_{t+1} = \delta_2 \hat{l}_{t+1} + b$$

	Constant	$\hat{l}_{t+1}$
Coefficient	3×10^{-4}	1.95
T-values	(0.05)	(4.63)
Adjusted R^2	0.40	

Panel C: This regression is based on 128 quarterly observations for the period 1966–1997.

$$\hat{l}_{t+1} = \delta_1 \hat{l}_t + b$$

	Constant	$\hat{l}_t$
Coefficient	-1.56×10^{-6}	0.53
T-values	(-4×10^{-3})	(7.04)
Adjusted R^2	0.27	

Panel D: This regression is based on 32 observations for the period 1966–1997.

$$\ln(i_t v_t + \mu/A^{1/\alpha}) = \ln(\overline{iv} + \mu/A^{1/\alpha}) + b_1\hat{\theta}_t + b_2\hat{k}_t + b_3\hat{v}_t$$

	Constant	$\hat{\theta}_t$	$\hat{k}_t$	$\hat{v}_t$
Coefficient	–0.61	0.02	0.20	0.10
T-Values	(–331.62)	(0.13)	(5.71)	(1.81)
Adjusted R^2	0.66			

Table 4. Summary of Estimated and other Derived Parameters.

Avge yrly θ	A	B	Capacity Ut.	Quart. Dep.
4	0.03	4.71×10^{12}	82.24%	2.4%
α	ρ	n	σ^2	g
3.16	0.965	1.82%	2.22%	1.23%
δ_1	δ_2	μ	γ	Sample $\bar{R}$
0.079	1.95	0.154	2.84	7.97%
Derived = $\bar{R} = \ln(A^{1/\alpha}\hat{\nu}\theta) = 7.75\%$				

and technological shocks from their steady state values. The novelty of this model is that it shows the predictability of returns in the case when economic growth is endogenous, without appealing to outside exogenous progress. We also derived a closed form solution for the expected long run growth rate of the economy and the long-term expected stock market return, as functions of the underlying parameters of the economy.

The model implies that deviations of capital stocks and technological shocks and labor growth rate from their long run means, account for about 20% of the predictability of deviations of stock returns from long run trend. Labor growth does not play a significant role, because of a crowding out effect with capacity utilization.

The estimated parameters are consistent with the literature. We derive a value for the CRRA parameter well within the range of estimates in the literature. That value implies a fair degree of consumption smoothing behavior. In order to obtain a long run expected stock return close to the sample mean over the period 1966–1998, we are led to adopt a technology that has increasing marginal returns to capital. This assumption has some limitations, as it may lead to some indeterminacy of equilibria (Benhabib Farmer, 1994). Explicitly modeling externalities, might resolve that issue.

Thus, further research could encompass a look at alternate specifications for the way growth is embodied into the model. For instance, one possible way is to incorporate public goods or human capital, in the production function. Another interesting extension for this research is to use this model to examine the presence of consumption smoothing behavior along a sustained growth path in other economies.

NOTES

1. Note that labor is implicitly part of formulation (2) as K_t represents the capital/labor ratio. When the coefficient is greater than *1*, the production function has increasing marginal returns to capital. In our case though, the marginal productivity of capital is bounded above.

2. See Barro and Sala-i-Martin (1994).

3. See Solow (1957) and Paquet and Robidoux (1997).

4. This capital accumulation equation involves the term $\frac{L_t}{L_{t+1}}$ as all variables are normalized in terms of labor units in two subsequent periods.

5. Ambler and Paquet (1994) use stochastic depreciation in the context of a real business cycle model.

6. This approach is consistent with the use-factor method of depreciation used in accounting. Another example is when oil or mining corporations revise their estimate of the amount of recoverable units, as a result of further discoveries. This makes the depreciation rate a function of the rate of extraction. Note that there are instances in which technological progress calls for the use of a new generation of capital goods and the scrapping of the old equipment (for example the switch from analog to digital networks). This is referred to in the literature as the process of creative destruction. Our model does not contradict that process, as technological progress is embodied in the inputs here, and our technological shocks reflect short-term adjustments of productivity outside the scope of long-term productivity growth.

7. See for example Hansen-Singleton (1983).

8. We use a simple deterministic trend. Controversies abound on the complexity of the relationship between trend and cycle components. See Canova (1998). Our approach is similar to King, Plosser and Rebelo's (1988). In their real business cycle model, they define a stationary equilibrium for the detrended economy. They work from a certainty equivalence perspective. They posit a particular stochastic process for the random shocks and replace the sequence of random shocks by their conditional expectations.

9. Even though the probability of obtaining such a sequence is extremely small, it is still a useful concept as it implies the known stylized facts about actual economic time series. Nelson and Plosser (1982) have given evidence that macro time series have important stochastic trends. Our notion of steady state does not contradict these findings as it looks at expected trends.

10. In the case where $\alpha > 1$, we know that the technology has increasing marginal returns to capital. Thus the first order conditions might describe a minimum rather than a maximum. But in fact, a simple argument shows that this cannot be the case. The reason is that in our steady state, the marginal productivity of capital has reached its peak, and from the consumer's standpoint this maximizes the growth of consumption over time.

11. This condition is analogous to King-Plosser and Rebelo (1988).

12. Note that the variance σ^2 and the CRRA parameter are inversely related.

13. A method used by King-Plosser and Rebelo (1988). As the sequence of realizations of consumption and output levels are bounded in the appropriate sup-norm topology, paralleling an argument from Danthine-Donaldson (1981), we deduce that the

optimal policy $e(y_t)$ is continuous. We will assume that this optimal policy is actually differentiable.

14. The value $(1+\alpha)$ is the elasticity of consumption with respect to income.

15. The formula is $TFP = Y/[s_L L + (1 - s_L)K]$, where K represents the capital stock (in their notation), and s_L is the income share of labor.

16. The formula is

$$\ln\left(\frac{\theta_{t+1}}{\theta_t}\right) = \ln\left(\frac{Y_{t+1}}{Y_t}\right) - \left[\bar{s}_{Lt}\ln\left(\frac{L_{t+1}}{L_t}\right) + (1 - \bar{s}_{Lt})\ln\left(\frac{K_{t+1}}{K_t}\right)\right]$$

where $\bar{s}_{Lt}$ is the average share of labor between period t and $t+1$.

17. This is within the range of usual values from 8.4% to 10% (see Ambler and Paquet 1994).

18. If we were to use a stochastic depreciation rate, as we did for the stock returns regression, the adjusted $R^2 = 0.61$ for the first regression (Panel A), and $R^2 = 0.77$ for the second regression (Panel B). The parameter is equal to 4.05. The implied long run expected stock return is 8%, and the CRRA parameter is 3.08.

19. The formula is

$$\ln\left(\frac{\theta_{t+1}}{\theta_t}\right) = \ln\left(\frac{Y_{t+1}}{Y_t}\right) - \left[\bar{s}_{Lt}\ln\left(\frac{W_{t+1}}{W_t}\right) + (1 - \bar{s}_{Lt})\ln\left(\frac{R_{t+1}}{R_t}\right)\right]$$

where $\bar{s}_{Lt}$ is the average share of labor between period t and $t+1$ and W_t is the real wage.

20. Our model does not generate a large enough expected return for smaller αs. On the other hand, with the values shown for α and the average depreciation rate, we are able to generate a long-term expected return of 7.75%, which is close to the sample mean of 7.97%. Having increasing marginal returns to capital might lead to the indeterminacy of equilibria as explored in Benhabib and Farmer (1994). This means that our steady state might not be globally stable, when the economy is shocked away from it.

21. OLS Regressions for other quarters are slightly weaker, and still confirm the same significance levels for our variables.

22. Shawky and Peng (1995) incorporate technological shocks in a standard exogenous growth model. They express the expected return on capital assets as a function of relative growth in capital stocks, labor and Solow residuals. They find that labor growth is highly significant. Our analysis differs by focusing near the steady state. We find that labor growth does not play a significant role, due to the correlation between labor growth and capacity utilization.

23. Table 3 shows regressions for the processes hypothesized for the dynamics of the consumption rate, of labor growth and capacity utilization. We find an adjusted R^2 of 0.72 for the relationship (18) between the log of the rate of consumption and the log of the detrended real GDP (see Table 3, panel A). The relationship between effective capacity utilization and other variables expressed in (23) gives an adjusted R^2 of 0.66 (Panel D).

24. This in part is due to our assumptions made on the elasticity α of the production function. It is important to point out that all regression results are *insensitive* to the choice of the initial value for the capital stock in 1959.

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