

# A VALUATION FORMULA FOR FIRMS IN THE EARLY STAGE OF THEIR LIFECYCLE

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## ABSTRACT

We develop a valuation formula for analyzing high growth firms using the stages of an industry lifecycle. Our model is best suited for start-up firms with low (or negative) earnings and low sales. Our formula uses start-up firm data and captures the firm's growth potential by incorporating data about two key stages along the lifecycle. One stage corresponds to the largest firm in the industry and the other to the firm situated at the inflection point of the S-shaped curve describing the lifecycle. We test the formula by examining the biotechnology industry in the late 1990s. An empirical analysis of the biotechnology industry reveals an important correlation between market values growth rates and assets growth rates, which is predicted by our formula. We find that, on average, our formula underestimates the actual market value of biotechnology start-up firms by about 15%.

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## I. INTRODUCTION

Valuation of common equity is perhaps one of the most researched topics in the finance literature. It has the unique quality of bringing together the art and the science of financial markets. Ever since the early works of Williams (1938), Durand (1957), Miller-Modigliani (1961), Gordon (1962), Solomon (1963), Malkiel (1963), and Mao (1966), equity valuation models have continued to advance and develop at a fairly rapid rate. One feature that has not changed much over time, however, is that virtually all of these models begin with a positive stream of dividends, earnings, or cash flows.

The recent emergence of several high technology industries characterized by a preponderance of firms with negative earnings, has presented a serious challenge to our traditional valuation methodology. Over the decade of the 1990s, many start-up technology firms were selling at spectacular market price multiples despite their continuing to accumulate large losses for several years and in many cases, since their inception. Although these multiples have since come down due to the year 2000 market correction, the state of our understanding of the valuation of these firms is still left wanting. Our primary objective in this paper is to develop a valuation model that is capable of valuing the equity of a company in its initial phases of development.<sup>1</sup>

Two recent articles have attempted to address the valuation problems of high growth firms. Schwartz and Moon (2000) apply real-option theory and capital budgeting techniques to valuing Internet stocks. They assume that the dynamics of revenues is such that the growth rate of an Internet company eventually reverts to the industry average. Using Monte Carlo simulations, they derive the value of an Internet company based on firm specific and industry specific parameters. Their model is essentially a stochastic continuous time version of the multistage valuation models used in the earlier literature.

Berk, Green, and Naik (1999) model a firm's cash flows based on a stochastic investment process. They are able to derive a firm's value as a collection of existing projects and growth options. One key aspect of their result is that they explicitly incorporate the changes in the firm's systematic risk over time. They use a simulation methodology and

calibrate their model using macroeconomic data. While their model appears to address several important market anomalies in the recent literature, it is not readily applicable to the valuation of *individual* firms.

Our approach is primarily based on the industry lifecycle hypothesis. We extend this hypothesis in a rather unconventional way to incorporate a new view of a firm's maturity in the valuation process. In the traditional industry lifecycle hypothesis, sales and earnings are typically the variables that mature over time. This hypothesis posits that an industry goes through a number of stages: Introduction, Growth, Maturity, and Decline phases as in Porter (1998). Inflection points in the rate of growth of sales usually define these stages. The industry lifecycle trajectory roughly follows an S-shaped curve. In the Introduction phase, start-up companies usually need external capital and are often characterized by negative earnings. In the Growth phase, firms are able to generate positive free cash flows. Finally, the Maturity and Decline phases occur as market growth is tapering off.

We use a novel approach in which the variable “sales” is used to characterize the maturity stage of a firm, and “assets” are the measurement variable that evolves with sales. Furthermore, we adopt a diffusion process commonly used in the biological science literature that provides a good representation for the S-shaped curve that describes the lifecycle hypothesis. Assuming that a typical start-up firm develops according to the S-shaped curve, we develop a closed form valuation formula that provides an estimate of its market value. Our model shows how firm-specific variables and other variables that capture key maturity stages of the industry lifecycle are relevant for the valuation of these firms. In particular, we show that two points on the lifecycle curve have particular relevance. One relevant point corresponds the “inflection” firm in the industry, and the other corresponds to the largest firm. The “inflection” firm is the firm that is located at the inflection point of the S-shaped curve.

The biotechnology industry is used to test our model. This choice is largely dictated by the availability of historical data necessary to implement the empirical tests and the applicability of our model to relatively new and emerging firms. The biotechnology industry has grown considerably since its early days in the 1970s. Total market capitalization of the industry in 1998 was \$97 billion. The

biotechnology industry in the United States remains fairly concentrated. At the end of 1998, the 20 leading biotech companies accounted for nearly two-thirds of the total market capitalization of the industry. While publicly held companies accounted for only 25% of the industry's total number, they represented 94% of its market capitalization.

Empirical evidence about the biotechnology industry points to a strong correlation between growth in market value and growth in assets for these firms especially during the early stages of their development.<sup>2</sup> Simulation results comparing our formula to actual market values in the biotechnology industry appear to be consistent with a general overpricing of these firms during the late nineties. We also show that our valuation model is fairly robust across a wide range of parameters.

This paper is organized into five sections. We introduce the theoretical model and describe the relevant variables in Section II. In Section III we present the data for firms in the biotechnology industry and provide a statistical analysis of the relevant variables. The empirical application and testing of the model is presented in Section IV. Concluding remarks are given in the final section.

## II. THE MODEL

Many existing valuation models have recognized that firms often go through three or more different growth stages. A good example is the three-stage valuation model developed by Mao (1966). The lifecycle model, characterized by an S-shaped curve tracing the behavior of sales over time, is particularly useful in describing this behavior. Unlike the traditional lifecycle model however, where sales is the variable that matures over time, we use a novel approach to characterize a firm's maturity. The variable "sales" is used to capture maturity while "assets" is the measurement variable that evolves parsimoniously with sales.

Using sales as the maturity variable is rather unconventional. We are accustomed to viewing a firm's maturity in terms of time. In practical terms however, one can easily find two firms reaching similar levels of maturity within vastly different time frames. Conversely, being in an industry for a given number of years does not insure a particular level of maturity. Hence, our notion of sales as a measure of

the condition of a company relative to its peers may be a reasonable proxy for maturity.<sup>3</sup> An apparent weakness of this approach might be that sales are fairly volatile in the short run. However, it is a firm's ability to *sustain* a given level of sales for a period of time, which we posit characterizes its maturity level.

It is important to recognize that this new approach used to characterize the lifecycle model is critical to the development of our valuation model and appears to be consistent with the data.<sup>4</sup> We posit that along the industry lifecycle, the sales level is related to the growth of a firm's assets in the following way,

$$g_j = \frac{1}{A_j} \frac{dA_j}{dS_j} = \beta \times \exp(-g \cdot S_j). \quad (2.1)$$

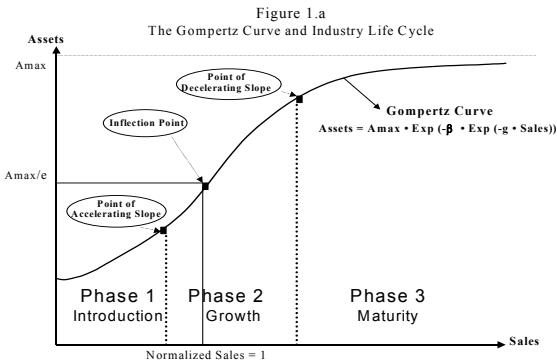
Equation (2.1) describes a diffusion process known as the Gompertz distribution, often used to model biological processes (Banks (1994)).<sup>5</sup> The variables  $S_j$  and  $A_j$ , respectively, are sales revenues and total assets of firm  $j$ . The level of sales  $S_j$  associated with the firm defines its maturity stage  $j$ . The variable  $S_j$  is deterministic with a range of values equal to  $[0, \infty]$ . The differential equation (2.1) states that along the industry lifecycle, the percentage growth rate of assets is positive and slows down exponentially as sales grow or as the firm becomes more mature. This differential equation can be solved to lead to the following logistic relationship between a firm's assets and sales:<sup>6</sup>

$$A_j = A_{\max}(J) \times \exp(-\beta \times \exp(-g \cdot S_j)), \quad (2.2)$$

where  $A_{\max}(J) = A(S_j)$  is the maximum assets employed by the largest firm in the industry having the maximum sales  $S_j$ . The parameter  $g$  is the percentage growth rate of assets of the *inflection* firm in the industry.<sup>7</sup> The coefficient  $\beta$  represents the difference in logs of the largest company to the smallest one, ranked by assets. It captures the average compounded growth rate of a small firm, over the entire lifecycle. Equation (2.1) can also be re-manipulated to show that asset growth (per dollar of sales) slows down, as a firm's assets size gets closer to the largest company's assets

$$g_j = g \times \ln \left( \frac{A_{\max}(J)}{A_j} \right). \tag{2.3}$$

Figure 1.a shows the Gompertz curve and the traditional three phases of industry lifecycle according to the classification offered by Porter (1998). The three phases are Introduction, Growth, and Maturity.<sup>8</sup>



The behavior implied by the Gompertz curve is such that in the Introduction phase, assets grow but not as fast as sales, thus the need for external capital. In other words, these firms greatly benefit from raising additional equity, as the impact on sales will be significant. For firms in the Growth phase, assets grow along with sales, as these firms are more able to transform earnings into assets. Finally, firms in the Maturity stage are unable to keep their asset base growing even though sales growth may be possible at least for a while.

Another important assumption we make is that a firm's return on assets  $R_j$  is also influenced by the stage of the lifecycle in which the firm is. We express the return on assets as

$$R_j = \alpha_2 + \alpha_1 \cdot \left( 1 - \frac{g_j}{g} \right) = \text{or, by using (2.3)} \quad R_j = \alpha_1 \cdot \ln \left( e \cdot \frac{A_j}{A_{\max}(J)} \right) + \alpha_2 \tag{2.4}$$

Equation (2.4) says that the return on assets is inversely related to how fast a company's assets grow along its lifecycle. The parameter  $\alpha_2$  is the return on assets for the inflection firm, and  $(\alpha_1 + \alpha_2)$  is the return on assets for the largest size firm. A convenient property of equation (2.4) is that it allows the return on assets to be negative for firms in the early stages of the industry lifecycle. Expected earnings are assumed to equal  $E_j = R_j \cdot A_j$ . In general, the market value of a company equals the present value of its future free cash flows to equity. Even though this may appear controversial, we purposely assume that the market value of a firm in the early stage of the lifecycle is equal to the present value of its future earnings:<sup>9</sup>

$$V_{0,J} = \int_0^{S_J} E_j \times e^{-k \cdot S_j} dS_j \quad (2.5)$$

Expression (2.5) is a distinguishing feature in the development of our model. As pointed out earlier, sales is used as a proxy for maturity. Analogous to computing the present value of earnings *over time*, our equation (2.5) is based on computing the present value of earnings using the path of sales along the industry lifecycle as the integrating variable. Equation (2.5) also involves applying a discount rate over the path of sales values. The parameter  $k$  is the discount rate, which is obtained by taking the periodic expected rate of return on equity for the start-up firm and *normalizing* it by the inflection firm's growth rate of sales. This procedure essentially transforms the usual time-based discount rate into a discount rate based on sales as the measurement unit.<sup>10</sup>

**Proposition 1.** Assume that the behavior of assets and the return on assets follow the processes expressed in equations (2.2) and (2.4). Also assume that the largest and most mature firm is such that  $S_j$  is infinite and  $A_{\max}(J)$  is finite, then the market valuation of a high growth firm in the early stage of its lifecycle is given by

$$V_0 = \frac{A_{\max} \cdot \left[ \alpha_1 e^{-\beta} + (\alpha_2 + \alpha_1 - \alpha_1 \cdot k / g) \beta^{-k/g} \cdot \gamma(k/g, \beta) \right]}{g},$$

where  $\gamma(\cdot; \cdot)$  is the incomplete gamma function. Moreover, when  $\beta$  is very large the above valuation formula becomes:

$$V_0 \approx \frac{A_{\max} \cdot [\alpha_1 e^{-\beta} + (\alpha_2 + \alpha_1 - \alpha_1 \cdot k / g) \beta^{-k/g} \cdot \Gamma(k/g)]}{g},$$

where  $\Gamma(\cdot)$  is the gamma function.

**Proof:**

From equations (2.2), (2.4) and (2.5), the market value of a start-up company is

$$V_{0,J} = \int_0^{S_j} E_j \cdot e^{-kS_j} dS_j = \int_0^{S_j} A_{\max}(J) \cdot \exp(-\beta e^{-gS_j} - kS_j) [\alpha_1 + \alpha_2 - \alpha_1 \beta e^{-gS_j}] dS_j. \quad (2.6)$$

Let  $X_j = \beta e^{-gS_j}$  then (2.6) can be expressed as

$$V_{0,J} = (\alpha_1 + \alpha_2) \int_0^{S_j} A_{\max}(J) \cdot e^{-X_j - kS_j} dS_j - \alpha_1 \beta \int_0^{S_j} A_{\max}(J) \cdot e^{-X_j - (k+g)S_j} dS_j. \quad (2.7)$$

Let the variable  $S_j = (-1/g) \cdot \ln(X_j / \beta)$  and its partial derivative be

$$dS_j = -dX_j / (gX_j). \text{ Let us define}$$

$$V_{0,J} = (\alpha_1 + \alpha_2) \cdot V_{0,J}^1 - \alpha_1 \beta \cdot V_{0,J}^2,$$

where:

$$V_{0,J}^1 = \int_0^{S_j} A_{\max}(J) \cdot e^{-X_j - kS_j} dS_j \quad (2.8)$$

and

$$V_{0,J}^2 = \int_0^{S_j} A_{\max}(J) \cdot e^{-X_j - (k+g)S_j} dS_j. \quad (2.9)$$

Changing variables and reversing the order of integration, we get

$$V_{0,J}^1 = \frac{\beta^{-k/g}}{g} \int_{X_j}^{\beta} A_{\max}(J) \cdot X_j^{(k/g)-1} e^{-X_j} dX_j \quad (2.10)$$

and

$$V_{0,J}^2 = \frac{\beta^{-(k/g)-1}}{g} \int_{X_J}^{\beta} A_{\max}(J) \cdot X_j^{(k/g)} e^{-X_j} dX_j \quad (2.11)$$

Let  $S_J \rightarrow \infty$  and assume that  $A_{\max}(J) \rightarrow A_{\max}$ , then (2.10) and (2.11) become

$$V_{0,J}^1 = \frac{\beta^{-k/g}}{g} A_{\max} \cdot \gamma(k/g, \beta) \text{ and } V_{0,J}^2 = \frac{\beta^{-(k/g)-1}}{g} A_{\max} \cdot \gamma((k/g) + 1, \beta), \quad (2.12)$$

where  $\gamma(\cdot, \cdot)$  is the incomplete gamma function. It is easy to show that:

$$\gamma(k/g + 1, \beta) = \frac{k}{g} \gamma(k/g, \beta) - \beta^{k/g} e^{-\beta}. \quad (2.13)$$

Thus substituting (2.13) into (2.12), we obtain

$$V_{0,J} = A_{\max} \left[ (\alpha_1 + \alpha_2) \frac{\beta^{-k/g}}{g} \gamma(k/g, \beta) - \alpha_1 \beta \left( \frac{\beta^{-k/g-1}}{g} (k/g) \cdot \gamma(k/g, \beta) - \frac{e^{-\beta}}{\beta g} \right) \right]. \quad (2.14)$$

After some simple manipulations, we obtain the first relationship in our proposition. Noticing that  $\gamma(k/g, \beta)$  converges to  $\Gamma(k/g)$  as  $\beta$  becomes large, gives us the second approximation. **Q.E.D.**

In order to make the second formula in Proposition 1 operational from an implementation standpoint, we take the further step of expressing it as a function of parameters that can be estimated *over time*. Let  $I$  be the expected return on equity for the start-up firm, and  $G_a$  and  $G_s$  respectively be the assets and sales growth rates for the inflection firm over time. Let us rename the return on assets of the top firm  $R_{\max} = (\alpha_1 + \alpha_2)$  and the return on assets of the inflection firm  $R = \alpha_2$ . Next we express our valuation formula as a function of these parameters.

**Corollary 1.** Under the assumptions of Proposition 1, the market value of a firm at the beginning of its lifecycle is

$$V_0 = \frac{G_s}{G_a} \left[ A_0 (R_{\max} - R) + A_{\max} \left( R_{\max} - (R_{\max} - R) \cdot \frac{I}{G_a} \right) \ln \left( \frac{A_{\max}}{A_0} \right)^{-1/G_a} \Gamma(I/G_a) \right].$$

**Proof:**

Using the second formula derived from Proposition 1, note that  $\beta = \ln \left( \frac{A_{\max}}{A_0} \right)$ . We also have normalized the sales of the inflection firm  $S_e$  so that  $S_e = 1$ . From the definition of  $g_j$  in (2.3), and by introducing time derivatives, we get that  $g = G_a / G_s$ . It then suffices to note that  $k/g = \frac{I/G_s}{G_a/G_s} = I/G_a$  by a simple change of variables. **Q.E.D.**

Corollary 1 gives the final version of the valuation formula for a start-up company based on the industry lifecycle hypothesis.<sup>11</sup> The formula states that the market value of the firm depends on three kinds of parameters. First, it depends on parameters related to the firm itself (return on equity  $I$  and initial levels of assets  $A_0$ ). Secondly, it depends on parameters related to the *inflection* firm in the industry (growth rate of sales  $G_s$ , growth rate of assets  $G_a$  and return on assets  $R$ ). Finally, it depends on parameters related to the *largest* firm in the industry (maximum level of assets  $A_{\max}$  and its return on assets  $R_{\max}$ ).<sup>12</sup>

The valuation formula is a function of two terms. The first term represents the sum of the present value of expected average profits that are based on the industry potential, holding the assets of the start-up firm constant. The second term, involving  $\ln(A_{\max}/A_0)$  and the gamma cumulative distribution, represents the present value of potential growth of assets and profits.<sup>13</sup> It is easy to check that the value of a start-up firm is decreasing as a function of the discount rate  $I$  and increasing in the growth of sales  $G_s$  and in the return on assets  $R_{\max}$ .<sup>14</sup> It is also straightforward to show from the first derivative that as long as the assets of the largest firm  $A_{\max}$  are much larger than the assets of the start-up firm  $A_0$ , then the value of the start-up firm will increase as  $A_{\max}$  increases.<sup>15</sup>

## I. DESCRIPTIVE STATISTICS

### A. Data and Sample Selection

We use the biotechnology industry to test the model. The sample includes all of the publicly traded firms in the biotechnology industry identified by SIC code 2836 along with 13 other pharmaceutical firms that were identified by Standard & Poor's Industry Survey as having significant presence in the biotechnology field. The total number of these firms that had sales in 1998 of over one million dollars was 98. This constitutes our sample.

The data for these firms is obtained from Research Insight (previously known as Compustat) and is comprised of two distinct sets. The first data set is the monthly stock market prices for all firms in the sample for the period July 1994 through July 1999. The second data set contains quarterly observations for sales, assets and market value of common stock for each of these biotechnology firms for the period Q1-1989 to Q2-1999.

### B. The Gompertz Curve and the Behavior of Returns on Assets

It is necessary to verify that the behavior implied by the Gompertz curve is consistent with the data for the biotechnology industry. In order to examine this point, we show in Figure 2 a scattered plot of annualized assets and sales for the sample of 98 biotechnology firms in the first quarter of 1999 versus the Gompertz relationship that we hypothesized between assets and sales. The Gompertz curve is obtained by estimating the following regression:<sup>16</sup>

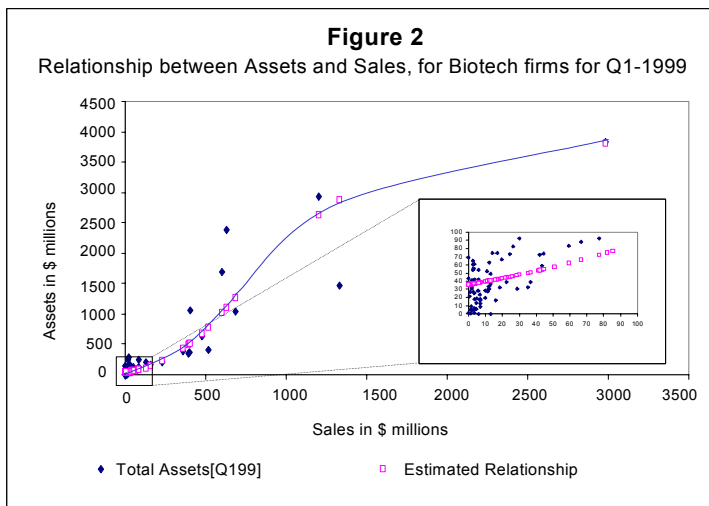
$$\ln(\ln(A_{\max}/A_j)) = \ln(\beta) - g \times S_j + \varepsilon$$

$$\ln(\ln(A_{\max}/A_j)) = 1.54 - 1.539 \times S_j$$

$$T\text{-stats} \quad (53.63) \quad (28.22)$$

$$\text{Number of obs. } 96 \quad \text{Adj. } R^2 = 89\%$$

The resulting estimates of the assets are then plotted against sales in Figure 2. The adjusted  $R^2$  is 89%, and both coefficients are highly significant. We also ran regressions for the same firms for Q1, Q2, Q3, and Q4 of 1998, and the adjusted R squares range from 88% to 91%, with the same levels of significance that are observed in the above regression.<sup>17</sup>



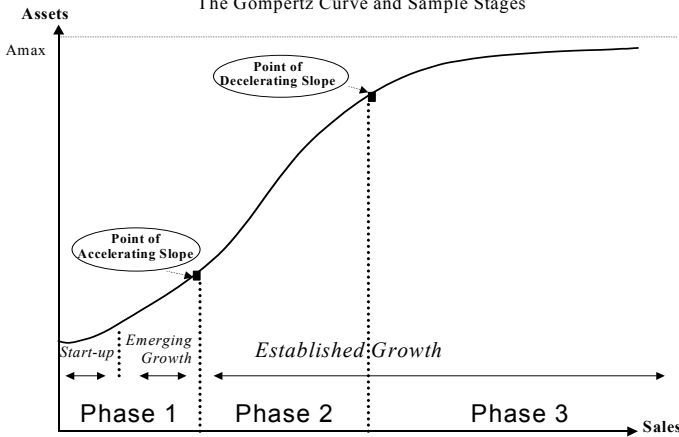
### C. Lifecycle Phases

In an effort to identify the various stages of an industry lifecycle, we divide the sample into three categories based on the companies' asset size.<sup>18</sup> The three categories are: Start-Up, Emerging Growth, and Established Growth.<sup>19</sup> To establish the ranking for these companies, we use data on assets for each of the following quarters, Q1-1998 to Q1-1999. Because there are significant movements over time in the rankings of small capitalization firms, we construct the Start-Up category by looking at firms that had less than \$50 million in assets as of Q1-1999. The Emerging Growth category applies to companies between \$50 million and \$240 million in Q1-1999.

The Established Growth phase is determined by taking companies that had over \$188 million in Q1-1998, \$200 million in Q2-1998, \$210 million in Q3-1998, \$233 million in Q4-1998, and \$240 million in Q1-1999. These cut-off points are obtained by finding the asset size consistent with an

acceleration of the slope of the Gompertz curve estimated for each of these quarters.<sup>20</sup> Figure 1.b shows how the three stages of the lifecycle are divided along the Gompertz curve. One important feature of this categorization scheme is that it overlaps with the “traditional” stages of the lifecycle. For example, our Start-Up and Emerging Growth stages are included in the Introduction phase. Finally, it should also be noted that since the biotechnology industry is still in its growth phase, it is difficult to identify many companies within that industry that can be classified as mature. For this reason, we grouped all of the largest companies into the Established Growth category for the purpose of our empirical analysis.

Figure 1.b  
The Gompertz Curve and Sample Stages



#### D. Descriptive Statistics

Table 1 presents monthly mean stock market returns, as well as their second and third moments. The sample statistics in the table provides results on the three phases of the industry, as well as for the overall sample. It should be noted that the sample size is reduced to a total of 88 firms since we only include firms with more than 30 monthly observations. As indicated in the table, the Start-Up sample exhibits extreme volatility relative to the other two groups as measured by the standard deviation, skewness and the range between the minimum and maximum values. Furthermore, the mean returns

for the Start-Up group is shown to be lower than the returns for the other two groups.

A visual presentation of the industry data seems to confirm our observations from Table 1.

Table 1. Annualized Monthly Mean Returns and Standard Deviations for Subsamples and Entire Biotechnology Industry for the period July 1994 through July 1999.

| Sampl              | Mean  | Std. Dev. | Skewnes | Min.   | Max. Value | No. of Obs. |
|--------------------|-------|-----------|---------|--------|------------|-------------|
| Start-up           | 28.6% | 17.6%     | 4.9     | -77.8% | 256.9%     | 43          |
| Emerging Growth    | 34.1% | 7.6%      | 1.2     | -13.5% | 99.2%      | 30          |
| Established Growth | 39.1% | 6.7%      | 2.5     | 7.7%   | 95.2%      | 15          |
| Entire Industry    | 32.3% | 13.3%     | 7.8     | -77.8% | 256.9%     | 88          |

Figure 3 depicts the behavior of the mean market returns for the sample firms scaled by the log of each firm's total assets, measured as of first quarter 1999. Assuming that asset size is a good proxy for a firm's maturity in its lifecycle, this graph shows the more homogenous values of the mean market returns as firms become more established.

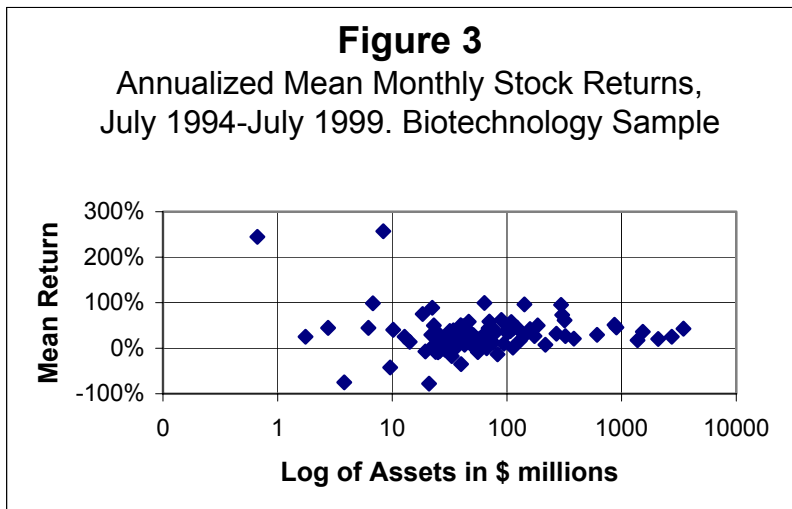
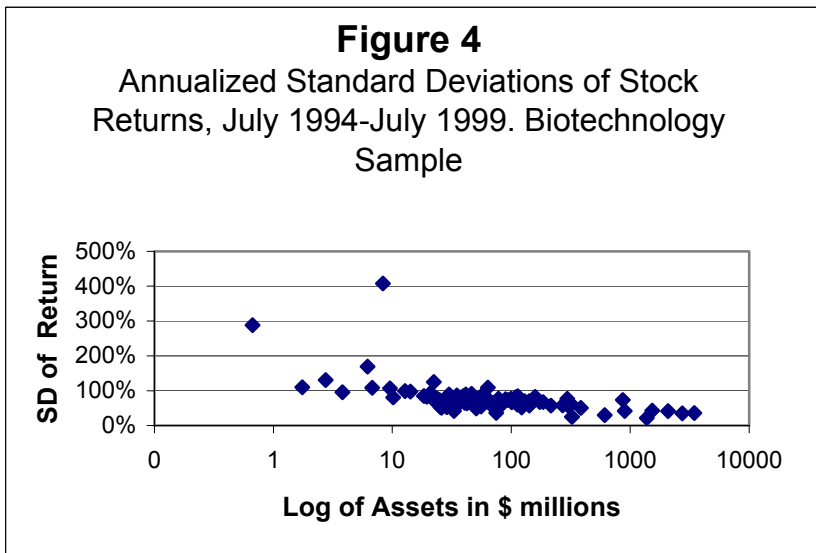


Figure 4 shows the degree of dispersion in the standard deviations of stock market returns of these firms, scaled by the log of their assets. Figure 4 again confirms the observations in Table 1 by showing a negative relationship between asset size and risk as measured by standard deviation of market returns.

As indicated earlier, the second data set used in this study is comprised of quarterly observations for sales, assets and market value of common stock for the period Q1-1989 to Q2-1999. Table 2 presents summary statistics for the estimated exponential growth rates in assets, sales, and market value for firms that had at least 30 available observations needed for the OLS regressions. The number of firms in that category is 49.



The exponential growth rates are obtained for each company by regressing the logs of each of these variables against time. The summary statistics presented in Table 2 includes only the statistically significant coefficients. The Established Growth sample exhibits growth rates of sales, assets and market value that have fairly similar magnitudes. In contrast, the Start-Up sample shows the least consistency among these three variables. More importantly, the standard deviations of these variables are shown to be orders of magnitudes higher for the Start-Up

sample than for the Established Growth sample. In both instances, the Emerging Growth sample results fall somewhere in between.

Another important observation from Table 2 concerns the behavior of the Emerging Growth sample. The average growth rates in this phase are shown to significantly accelerate relative to that observed in the Start-Up phase. The very large positive values for skewness for this phase are further evidence for the presence of hyper-growth that is generally consistent with the lifecycle hypothesis.

Table 3 presents a correlation matrix between these same variables. This table reveals important characteristics about the biotechnology industry. First, it is quite evident that growth in assets and growth in market value are very highly correlated. This is particularly significant when compared to the fairly weak association between the growth in sales and growth in market value. Second, it is even more important to note that the correlation between growth rates in assets and growth in market values is stronger for the Start-Up sample than for the more mature firms.

The strong correlation between growth rates in assets and market value appears to be a strong feature of the data. The traditional finance literature posits that as firms' revenues grows so will its earnings thus driving up its market price per share. This implies that there should be a strong correlation between the growth in sales and market values. However, it appears that for start-up firms, especially in high technology industries, growing their asset base through external financing is a function of market acceptance and hence is more likely to lead to appreciation in their market value. Interestingly, looking at the results for Established Growth firms, we see that their behavior is more consistent with the traditional view of valuation, as the correlation between market values and sales is essentially in the same range as that of market values and assets.

Table 2. Annualized Exponential Growth Rates in Assets, Sales, and Market Values for Subsamples and Entire Industry for the period Q1-89 through Q2-99.

| <b>Exponential Growth rates</b> |             |                  |             |            |            |                  |
|---------------------------------|-------------|------------------|-------------|------------|------------|------------------|
| <b>Sample/Phase*</b>            | <b>Mean</b> | <b>Std. Dev.</b> | <b>Skew</b> | <b>Min</b> | <b>Max</b> | <b># of Obs.</b> |
| <b>Start-up Phase</b>           |             |                  |             |            |            |                  |
| Sales                           | 25.86%      | 19.80%           | 104.90%     | -28.58%    | 119.80%    | 14               |
| Assets                          | 12.37%      | 15.17%           | 117.91%     | -41.61%    | 86.66%     | 14               |
| Market Value                    | 13.23%      | 30.32%           | 152.65%     | -100.99%   | 172.37%    | 14               |
| <b>Emerging</b>                 |             |                  |             |            |            |                  |
| Sales                           | 46.31%      | 21.53%           | 546.40%     | 9.74%      | 215.13%    | 20               |
| Assets                          | 30.94%      | 9.71%            | 194.78%     | 5.43%      | 79.40%     | 20               |
| Market                          | 29.40%      | 19.03%           | 365.31%     | -32.46%    | 165.44%    | 20               |
| <b>Established</b>              |             |                  |             |            |            |                  |
| Sales                           | 22.78%      | 5.03%            | 11.38%      | 8.95%      | 39.04%     | 15               |
| Assets                          | 21.14%      | 4.80%            | -13.69%     | 3.04%      | 36.60%     | 15               |
| Market                          | 25.30%      | 3.30%            | -238.63%    | 9.00%      | 31.65%     | 15               |
| <b>Entire</b>                   |             |                  |             |            |            |                  |
| Sales                           | 33.27%      | 18.09%           | 441.08%     | -28.58%    | 215.13%    | 49               |
| Assets                          | 22.63%      | 11.02%           | 59.53%      | -41.61%    | 86.66%     | 49               |
| Market                          | 23.53%      | 20.17%           | 186.53%     | -100.99%   | 172.37%    | 49               |

\*Note: All Firms in this sample had at least 30 observations, leading to a sample size of 49 firms.

**Table 3. Correlation Coefficients among Growth Rates in Sales, Assets, and Market Values; Q1-89 through Q2-99.**

| Industry Phase (#)             | Variables    | Sales | Assets |
|--------------------------------|--------------|-------|--------|
| <b>Start-up (14)</b>           | Assets       | 18.1% |        |
|                                | Market Value | 9.9%  | 80.5%  |
| <b>Emerging Growth (20)</b>    | Assets       | 23.1% |        |
|                                | Market Value | 18.9% | 69.2%  |
| <b>Established Growth (15)</b> | Assets       | 52.1% |        |
|                                | Market Value | 50.4% | 51.7%  |
| <b>Entire Industry (49)</b>    | Assets       | 27.9% |        |
|                                | Market Value | 16.8% | 74.6%  |

Note: 49 Biotechnology companies had growth rate estimates based on at least 30 observations.

## I. APPLICATION OF THE MODEL

### A-Valuation

The main theoretical model developed in the previous section under Corollary 1 presents a formula for the valuation of start-up firms. As shown in the formula, the value of a firm is given as a function of the relative growth of assets and sales for the individual firm, the average (inflection) firm and the largest firm. Furthermore, the formula also captures the present value of future earnings for the individual, the average and the largest firm in the industry. In essence, this formulation allows us to evaluate the potential earnings of emerging firms through the dynamic interaction between the growth rates in assets and sales that are likely to develop in the industry. More specifically, the formula that we shall empirically test is:

$$V_0 = \frac{G_s}{G_a} \left[ A_0 (R_{\max} - R) + A_{\max} \left( R_{\max} - (R_{\max} - R) \cdot \frac{I}{G_a} \right) \ln \left( \frac{A_{\max}}{A_0} \right)^{-1/G_a} \Gamma(I/G_a) \right] \quad (4.1)$$

where:

$V_0$ : the estimated market value of equity for the start-up firm.

$G_a$ : the rate of growth in assets for the inflection firm. This variable is estimated by averaging asset growth rates over the period Q4-96 through Q2-99 for five firms that fall closest to the inflection point. These firms are, Biogen, Centocor, Sigma-Aldrich, Genzyme General and Chiron Corporation.

$G_s$ : the rate of growth in sales for the inflection firm. This growth rate is derived from the growth rate in assets and the slope of the estimated Gompertz curve.<sup>21</sup>

$A_0$ : total assets of the individual firm. This variable is as of the latest available financial report, Q1-99.

$R_{max}$ : the return on assets for the largest firm in the industry. This variable is measured by the return on assets for Amgen, the largest firm in industry as of Q1-99.

$R$ : return on assets for the inflection firm. This is estimated by averaging the return on assets for the same five firms mentioned above for Q1-99.

$A_{max}$ : the total assets of the largest firm in the industry. This variable is represented by the total assets of Amgen, the largest firm in the industry as of Q1-99.

$I$ : the required return on capital for the individual firm. This variable is estimated for every firm in the sample using historical mean market returns over the period July 1994 through July 1999.

$\Gamma$ : is the Gamma distribution function with similar characteristics as the Chi-square distribution.

Having found reasonable estimates for these parameters, we are now in a position to examine the effectiveness of the model. Because of the several variables and parameters that are used as inputs, we use a spreadsheet simulation methodology to compare our estimated market values to actual market values.<sup>22</sup> In order to do so, we restrict the sample to the 82 companies that have asset figures and mean returns available for Q1-99. The sample is divided as follows: 37 Start-Up firms, 29 Emerging Growth firms, and 16 Established Growth firms.

Our approach in testing the model is to compute the percentage error for each company and average these errors for each of the phases of the lifecycle.<sup>23</sup> Even though our sample break-up that defines the Start-Up phase is somewhat arbitrary, it is important to emphasize that our valuation equation (4.1) is only appropriate for valuing firms with relatively small assets and sales. Hence, we focus our empirical analysis on the valuation of firms that are in this phase. As shown in Table 4, our formula on average underestimates the actual market value of Start-Up firms by only 15% for Q1 99.<sup>24</sup>

To further examine the sensitivity of the model to the choice of the period considered, we apply the model to four prior quarters. Table 4 displays a summary of the model's performance for Q1-1998 to Q1-1999. Based on the parameters obtained from the Gompertz curve regressions in each of these quarters and the estimates for the parameter inputs, we see that the average error for start-up companies varies between 15% and 60%. These results are generally consistent with the perceived overpricing of start-up companies in the biotechnology industry.

Table 4. Estimates of Market Values Compared with Actual Market Values, for Biotech Firms in Phase I (Start-up).

| Averages   | Assets       | Estim. MV    | Actual MV     | % Error     | Absolute Error |
|--|--------------|--------------|---------------|-------------|----------------|
| <b>Q1-99 (37 firms)</b>                              | <b>24.45</b> | <b>50.92</b> | <b>83.99</b>  | <b>15%</b>  | <b>88%</b>     |
| <i>Amax= 3850, ROA max = 24.18%, Ga/Gs = 154.02%</i> | <i>13.47</i> | <i>46.92</i> | <i>83.59</i>  | <i>102%</i> | <i>53%</i>     |
| <b>Q4-98 (40 firms)</b>                              | <b>25.14</b> | <b>43.67</b> | <b>82.89</b>  | <b>27%</b>  | <b>79%</b>     |
| <i>Amax= 3700, ROA max = 23.51%, Ga/Gs = 154.32%</i> | <i>13.19</i> | <i>32.57</i> | <i>75.98</i>  | <i>90%</i>  | <i>50%</i>     |
| <b>Q3-98 (40 firms)</b>                              | <b>24.96</b> | <b>47.67</b> | <b>79.71</b>  | <b>23%</b>  | <b>82%</b>     |
| <i>Amax= 3500, ROA max = 23.35%, Ga/Gs = 151.16%</i> | <i>13.13</i> | <i>44.07</i> | <i>74.36</i>  | <i>92%</i>  | <i>47%</i>     |
| <b>Q2-98 (39 firms)</b>                              | <b>26.35</b> | <b>47.98</b> | <b>106.18</b> | <b>45%</b>  | <b>81%</b>     |
| <i>Amax= 3300, ROA max = 20.30%, Ga/Gs = 151.70%</i> | <i>13.82</i> | <i>40.08</i> | <i>108.03</i> | <i>84%</i>  | <i>49%</i>     |
| <b>Q1-98 (38 firms)</b>                              | <b>26.85</b> | <b>46.24</b> | <b>118.91</b> | <b>60%</b>  | <b>87%</b>     |
| <i>Amax= 3100, ROA max = 21.14%, Ga/Gs = 148.54%</i> | <i>12.81</i> | <i>38.76</i> | <i>133.71</i> | <i>79%</i>  | <i>48%</i>     |

Note: Estimates are repeated for quarters Q1 98 to Q1 99, based on empirically estimated parameters. Growth in assets for the inflection firm is assumed equal to 23.75%, and ROA = 14.28% (Q1 99 parameters values). Assets, and Market Values are measured in \$ millions. Boldfaced values represent averages, italics represent standard deviations.

## B-Sensitivity Analysis of the Model

In this section, we examine how sensitive the valuation results are to changes in the model parameters. In the application of the model, there are

two sets of parameters that need to be estimated. The first set of parameters is company specific while the second set is more descriptive of the structure of the industry. The most critical company specific parameter is the required return on equity. Equally important are the industry specific parameters, such as return on assets and the growth rate of assets for the inflection firm, as well as the assets of the largest firm in the industry. All of these parameters are undoubtedly estimated with some degree of error.

Table 5 provides comparative statics results of the model for the Start-Up sample. The table shows how the mean errors and mean absolute errors are affected when changing the return on assets and the rate of growth in assets for the inflection firm. The columns represent range of values for the return on assets for the inflection firm. Our estimate for that variable is 14.28%. We examine an approximate range for that variable between 10% and 20% at 1.07% intervals. The rows represent range of values for the growth in assets for the inflection firm. The estimate for that variable is 23.75%, and we use 2% intervals over and below our estimate.

At extreme values of either parameter, we can immediately observe that our model either overestimates or underestimates the actual market values by a large error margin. More specifically if the inflection firm's rate of growth in assets is very high, the model overestimates market values (negative errors). In the case where this variable is very small, the model consistently underestimates market value.

As for the return on assets for the inflection firm, the diagonal elements in the table indicate that in order to maintain close estimates to the actual market values, there is a trade-off between return on assets and growth rates of assets. In other words, when the inflection firm's growth rate in assets is small, then the return on assets needs to be large, to keep our estimates in the same ballpark values. Where the model seems to be working best is around parameter values that are consistent with the position of the inflection firm in the industry. In other words, the growth rate of assets is large enough so that a large return on assets is guaranteed. This behavior is typical of a high technology industry.

Perhaps one of the more interesting insights from Table 5 is that we are able to empirically define the most reasonable ranges for both the return on assets and the growth in assets for the inflection firm in the biotechnology industry. A closer look at the figures in the table indicates that the reasonable

range of growth in assets for the inflection firm is between 19.75% and 27.75%. On the other hand, the range for the return on assets for the inflection firm appears to be anywhere above the 13% range. Furthermore, as observed along the diagonal, the model appears to be robust and produces small average errors for a reasonable range of parameter values.

Table 6 provides comparative statics results of the model when changing the value of the Start-Up phase mean return obtained from Table 1 and used in Table 4. In addition, the table contains a range of hypothetical values for the asset size of the largest firm in the industry. It is fairly obvious from the results in the table that the mean errors are not very sensitive to the size of the largest firm but far more sensitive to reasonable range of values for filtered sample mean returns. Given that we know that Amgen is the largest firm in the industry, with assets of approximately \$3.8 billion, the table shows that a reasonable range of estimates for the required return on equity for the biotech industry is between 20% and 33%.

**Table 5. Sensitivity Analysis of the Mean Errors and Mean Absolute Errors for Biotech Firms.  
Start-Up Sample Q1-1999. Parameters are ROA and Growth of Assets for Inflection Firm**

| Range of Growth<br>of Assets for<br>Inflection Firm | Range of Return on Assets for Inflection Firm |                            |                            |                            |                            |                            |                           |                           |                           |                           |
|---|---|----------------------------|----------------------------|----------------------------|----------------------------|----------------------------|---------------------------|---------------------------|---------------------------|---------------------------|
|   | 10%   | 11.07%                     | 12.14%                     | 13.21%                     | 14.28%                     | 15.35%                     | 16.42%                    | 17.49%                    | 18.56%                    | 19.63%                    |
| 31.75%  | <b>-26%</b><br><i>91%</i>                     | <b>-33%</b><br><i>92%</i>  | <b>-39%</b><br><i>92%</i>  | <b>-44%</b><br><i>92%</i>  | <b>-49%</b><br><i>93%</i>  | <b>-54%</b><br><i>94%</i>  | <b>-58%</b><br><i>95%</i> | <b>-62%</b><br><i>96%</i> | <b>-66%</b><br><i>96%</i> | <b>-69%</b><br><i>97%</i> |
| 29.75%  | <b>-9%</b><br><i>90%</i>                      | <b>-17%</b><br><i>90%</i>  | <b>-24%</b><br><i>90%</i>  | <b>-30%</b><br><i>90%</i>  | <b>-36%</b><br><i>91%</i>  | <b>-41%</b><br><i>91%</i>  | <b>-46%</b><br><i>92%</i> | <b>-50%</b><br><i>93%</i> | <b>-54%</b><br><i>94%</i> | <b>-58%</b><br><i>95%</i> |
| 27.75%  | <b>10%</b><br><i>92%</i>                      | <b>1%</b><br><i>89%</i>    | <b>-7%</b><br><i>88%</i>   | <b>-15%</b><br><i>88%</i>  | <b>-21%</b><br><i>89%</i>  | <b>-27%</b><br><i>89%</i>  | <b>-32%</b><br><i>89%</i> | <b>-37%</b><br><i>90%</i> | <b>-42%</b><br><i>91%</i> | <b>-46%</b><br><i>92%</i> |
| 25.75%  | <b>33%</b><br><i>98%</i>                      | <b>21%</b><br><i>93%</i>   | <b>12%</b><br><i>89%</i>   | <b>3%</b><br><i>86%</i>    | <b>-4%</b><br><i>86%</i>   | <b>-11%</b><br><i>87%</i>  | <b>-17%</b><br><i>87%</i> | <b>-22%</b><br><i>88%</i> | <b>-27%</b><br><i>88%</i> | <b>-32%</b><br><i>89%</i> |
| 23.75%  | <b>59%</b><br><i>107%</i>                     | <b>45%</b><br><i>100%</i>  | <b>34%</b><br><i>94%</i>   | <b>24%</b><br><i>91%</i>   | <b>15%</b><br><i>88%</i>   | <b>8%</b><br><i>85%</i>    | <b>1%</b><br><i>84%</i>   | <b>-5%</b><br><i>85%</i>  | <b>-11%</b><br><i>86%</i> | <b>-17%</b><br><i>86%</i> |
| 21.75%  | <b>88%</b><br><i>120%</i>                     | <b>71%</b><br><i>109%</i>  | <b>58%</b><br><i>102%</i>  | <b>47%</b><br><i>97%</i>   | <b>37%</b><br><i>93%</i>   | <b>28%</b><br><i>90%</i>   | <b>21%</b><br><i>87%</i>  | <b>13%</b><br><i>85%</i>  | <b>7%</b><br><i>84%</i>   | <b>1%</b><br><i>83%</i>   |
| 19.75%  | <b>120%</b><br><i>137%</i>                    | <b>100%</b><br><i>123%</i> | <b>85%</b><br><i>113%</i>  | <b>72%</b><br><i>106%</i>  | <b>61%</b><br><i>100%</i>  | <b>51%</b><br><i>96%</i>   | <b>42%</b><br><i>92%</i>  | <b>35%</b><br><i>90%</i>  | <b>27%</b><br><i>87%</i>  | <b>21%</b><br><i>85%</i>  |
| 17.75%  | <b>154%</b><br><i>160%</i>                    | <b>132%</b><br><i>142%</i> | <b>114%</b><br><i>129%</i> | <b>100%</b><br><i>119%</i> | <b>87%</b><br><i>111%</i>  | <b>76%</b><br><i>105%</i>  | <b>67%</b><br><i>100%</i> | <b>58%</b><br><i>96%</i>  | <b>50%</b><br><i>93%</i>  | <b>43%</b><br><i>90%</i>  |
| 15.75%  | <b>198%</b><br><i>202%</i>                    | <b>167%</b><br><i>171%</i> | <b>146%</b><br><i>151%</i> | <b>130%</b><br><i>137%</i> | <b>116%</b><br><i>127%</i> | <b>104%</b><br><i>119%</i> | <b>93%</b><br><i>112%</i> | <b>83%</b><br><i>106%</i> | <b>75%</b><br><i>102%</i> | <b>67%</b><br><i>97%</i>  |

Note: Mean errors are in bold and mean absolute errors in italics.

## I. CONCLUSION

In this paper, we use the concept of industry lifecycle to develop a valuation model that is suitable for a start-up firm. Assuming that a typical firm in a growth industry develops according to the S-shaped curve, we develop a closed form valuation formula that captures the value of a start-up firm. While in the traditional industry lifecycle model “sales” is the variable that matures over time, we use a novel approach to characterize firms' maturity. The variable “sales” is used to capture maturity while “assets” is the measurement variable that evolves parsimoniously with sales.

We choose the biotechnology industry to test our model. Data for all firms in the industry are obtained from Research Insight (formerly known as Compustat). The sample includes all of the publicly traded firms that had over \$1 million in sales in 1998. The total number of firms used in the study is 98. One of the advantages of using the biotechnology industry instead of the Internet sector, for example, is the availability of historical stock price data going back at least 10 years.

Our descriptive statistics show that there is a strong correlation between growth rates in assets and growth rates in market value in that industry. The traditional finance literature posits that as firms' revenues grows so will its earnings thus driving up its market price per share.

**Table 6. Sensitivity Analysis of the Mean Errors and Mean Absolute Errors, for Biotech Firms with Respect to Changes in Model Parameters (Mean Return of Start-up Sample and Largest Firm Asset Size).**

|                           |         | One Standard Deviation Interval Around Estimated Filtered Start-up Mean Return |             |             |             |            |             |             |             |             |
|---------------------------|---------|--|-------------|-------------|-------------|------------|-------------|-------------|-------------|-------------|
| Largest Firm's Asset Size |         | 11%  | 15.35%      | 19.76%      | 24.16%      | 28.57%     | 32.98%      | 37.39%      | 41.79%      | 46.20%      |
| Hypothetical              | \$5,850 | <b>-92%</b>  | <b>-75%</b> | <b>-54%</b> | <b>-31%</b> | <b>-8%</b> | <b>14%</b>  | <b>36%</b>  | <b>55%</b>  | <b>73%</b>  |
|                           |         | <i>133%</i>  | <i>116%</i> | <i>100%</i> | <i>94%</i>  | <i>88%</i> | <i>88%</i>  | <i>91%</i>  | <i>100%</i> | <i>111%</i> |
| Hypothetical              | \$4,850 | <b>-84%</b>  | <b>-66%</b> | <b>-44%</b> | <b>-20%</b> | <b>3%</b>  | <b>25%</b>  | <b>45%</b>  | <b>63%</b>  | <b>80%</b>  |
|                           |         | <i>130%</i>  | <i>111%</i> | <i>97%</i>  | <i>92%</i>  | <i>87%</i> | <i>88%</i>  | <i>93%</i>  | <i>102%</i> | <i>113%</i> |
| Amgen                     | \$3,850 | <b>-75%</b>  | <b>-54%</b> | <b>-31%</b> | <b>-7%</b>  | <b>15%</b> | <b>37%</b>  | <b>56%</b>  | <b>73%</b>  | <b>89%</b>  |
|                           |         | <i>125%</i>  | <i>106%</i> | <i>95%</i>  | <i>90%</i>  | <i>88%</i> | <i>89%</i>  | <i>96%</i>  | <i>106%</i> | <i>116%</i> |
| Hypothetical              | \$2,850 | <b>-61%</b>  | <b>-38%</b> | <b>-14%</b> | <b>9%</b>   | <b>32%</b> | <b>52%</b>  | <b>70%</b>  | <b>85%</b>  | <b>99%</b>  |
|                           |         | <i>120%</i>  | <i>102%</i> | <i>95%</i>  | <i>90%</i>  | <i>90%</i> | <i>93%</i>  | <i>101%</i> | <i>111%</i> | <i>121%</i> |
| Hypothetical              | \$1,850 | <b>-41%</b>  | <b>-15%</b> | <b>10%</b>  | <b>33%</b>  | <b>54%</b> | <b>72%</b>  | <b>88%</b>  | <b>101%</b> | <b>113%</b> |
|                           |         | <i>111%</i>  | <i>100%</i> | <i>94%</i>  | <i>92%</i>  | <i>94%</i> | <i>101%</i> | <i>109%</i> | <i>119%</i> | <i>127%</i> |

Note: Mean errors are in bold and mean absolute errors in italics.

This implies that there should be a strong correlation between the growth in sales and market values. However, it appears that for start-up firms in high technology industries, growing their asset base through external financing is also a function of market acceptance and hence is more likely to lead to an appreciation in their market value. Interestingly, looking at the results for Established Growth firms, we see that their behavior is more consistent with the traditional view of valuation, as the correlation between market values and sales is now in the same range as that of market values and assets.

It is important to emphasize that our valuation formula is only appropriate for valuing firms with relatively small assets and sales. Our results indicate that the formula, on average, underestimates the actual market value of Start-Up firms by about 15%. To examine the sensitivity of the model to the choice of the period considered, we apply the model to four prior quarters. Based on the parameters, we find that for these quarters, the average error for start-up companies varies between 15% and 60%. It is worth noting that because we interpret our estimates in terms of percentage errors relative to current market values, it is quite possible that a significant portion of these errors could actually be the result of overpricing of biotechnology stocks in the first place.

Perhaps one of the more interesting insights gained from the sensitivity analysis is that we are able to empirically define the most reasonable ranges for both the return on assets and the growth in assets for the inflection firm in the biotechnology industry. The reasonable range of growth in assets for the average (inflection) firm is between 19.75% and 27.75%. On the other hand, the range for the return on assets for the average firm appears to be anywhere above the 13% range. Furthermore, the model appears to be robust and produce very small average errors, for a reasonable range of parameter values.

Even though business risk is tackled in our formula indirectly through the input of the equity rate of return and the growth rates in sales and assets, one possible extension of the model could be to incorporate risk in terms of survival probability of a start-up firm along its lifecycle. Another extension of the model might be to examine the effects of changing risk on the discount rate along the firm's lifecycle. A further extension might be to

model the evolution of dividend payout ratios along the industry lifecycle and compute the value of the firm as the present value of future dividends.

While we focus on financial data in this paper, we must acknowledge that non-financial data may also play an important role in the valuation of high technology firms. Amir and Lev (1996) examine the value relevance of financial and non-financial information for independent cellular telephone companies. They conclude that book values, earnings, and cash flows alone were largely irrelevant to these companies' stock prices. In the biotechnology field, announcements of discoveries, patents, and FDA approvals appear to be relevant in explaining some of our large discrepancies. Further research on the impact of these non-financial events on stock prices is certainly warranted.

## NOTES

<sup>1</sup> In the last few years, there has been a surge of accounting-based valuation models in which market value is modeled as a linear function of book equity and current or expected future cash flows. For instance, Ohlson (1995) and Barth, Beaver and Landsman (1998) use OLS to estimate linear models between market value and other accounting variables. In addition, several other studies such as Ye and Finn (1999) and Hand (2000) use log-linear regressions within accounting-based valuation frameworks. Ye and Finn arrive at a valuation formula for stock equity by assuming that the return on equity follows an AR (1) process.

<sup>2</sup> In a similar vein, Hand (2000) finds that book values seem to explain a large portion of market values for Internet firms, based on a log linear regression approach.

<sup>3</sup> Sales are a good measure of maturity since they are an indicator of market share, which typically is a major determinant of a firm's maturity within a given industry.

<sup>4</sup> It is important for our analysis to focus on an industry that it is going through one lifecycle only. One of the criticisms of the industry lifecycle theory is that industry growth does not always go through the S-shaped pattern at all. Sometimes industry growth revitalizes after a period of decline, as occurred in the motorcycle and bicycle industry and in the radio broadcasting industry (Porter, 1998).

<sup>5</sup> The process described in equation (2.1) could also be modeled as a stochastic process, where the evolution of the growth rate of assets could be affected by a white noise. While this might provide a more direct interpretation of risk, it is doubtful that the model would be empirically tractable.

<sup>6</sup> Note that this relationship is monotonic along the industry lifecycle. While in principle negative earnings could lead to lower asset values for a start-up firm, in practice this may not be the case mainly because this type of firm has the ability to raise external equity, as their actual and potential sales grow.

<sup>7</sup> This is the firm for which  $A_j = A_{max} (J)/e$ , where  $e$  is the exponential constant. This firm is located at the inflection point of the S-shaped curve that depicts the industry lifecycle. We normalize the sales variable so that the value of sales for the inflection firm is  $S_e = 1$ . This allows us to derive a relationship that is independent of the changing scale of these variables over time.

<sup>8</sup> In Porter (1998), each transition is determined by conditions on second derivatives. For example, the point of acceleration of the slope determines the transition between the first and second phase. In our case, and as we will explain later in the empirical section, our own classification overlaps with these three phases.

<sup>9</sup> Traditionally, the market value of a firm is computed as the present value of future expected dividends or free cash flows in the absence of dividends. Our model certainly applies to the case of using free cash flows instead of earnings. For the sake of convenience, it is worth noting that under reasonable assumptions, discounting earnings is equivalent to discounting free cash flows to equity to arrive at equity value. The relationship between earnings after tax (EAT), and free cash flows to equity (FCFE), can be expressed as:  $FCFE = EAT - (1 - \text{Debt Ratio}) (\text{Capital Expenditures} - \text{Depreciation} - \text{Change in Working Capital})$ . If we make the reasonable assumptions that Capital Expenditures are approximately equal to Depreciation and that Changes in Working Capital are negligible, the equivalency between earnings and free cash flows is established. For the biotechnology industry, we estimated the correlation between earnings after tax and free cash flows for the year 1999 data to be 95%. Furthermore, discounting earnings instead of dividends helped us avoid the problem of no-dividend payments typical of start-up firms or having to estimate terminal values. Why are we doing this analysis in the early stages? Because the lowest bound of integration in deriving our formula is a firm with positive assets and *zero* sales.

<sup>10</sup> As mentioned earlier, there is no reason to expect a simple one-to-one relationship between sales levels and time, as a company matures. A crucial interpretation of our normalization of the discount rate  $k$  is that for any firm  $j$ , the

ratio of sales divided by the growth rate the inflection firm's sales  $S_j/G_s$  measures the *average time* it took firm  $j$  to reach its current maturity stage in the industry lifecycle.

<sup>11</sup> It is important to note that this formula is derived under the approximation that  $A_{max}$  is "infinitely" larger than  $A_0$ . This tends to make the formula overstate the "true" market value of the firm. On the other hand, the formula will simultaneously tend to understate the true market value of the firm, as we begin at zero sales and many start-up firms have sales that are positive, thus exaggerating the effect of negative earnings. All in all, these two effects work to offset one another.

<sup>12</sup> Our characterization incorporates risk indirectly. Both the growth rates in sales and growth rates in assets, along with the disparity between a small firm and the largest firm in the industry, capture important risk metrics about the firm and the biotechnology industry.

<sup>13</sup> Note that the first term does not depend on the discount rate, which is the result of the hypothesized behavior of the return on assets along the lifecycle in equation (2.3).

<sup>14</sup> The comparative statics for the discount rate  $I$  is derived from numerical simulations.

<sup>15</sup> Identifying the largest firm as the upper bound in our model is not a significant limitation since, in practice, that upper bound is a moving target, as the size of the largest firm in the industry changes over time. Regarding the valuation implications of cash flows that could be received *after* the firm exceeds that upper bound, it is fair to assume that these cash flows would happen far enough in the future, so that their present value is negligible.

<sup>16</sup> Sales are normalized so that the inflection firm's sales  $S_e=1$  in the first quarter of 1999. Looking at the regression equation and noting that  $S_e=1$  implies that  $Ln(\beta) = g$ . The maximum asset size for the industry was chosen to be slightly higher (\$3,850 million) than that of the largest firm in the industry (Amgen = \$3,818.3 million).

<sup>17</sup> We also regressed straight lines through these samples as a benchmark comparison. The adjusted R-squares vary from 76% in Q1-1998, 85% in Q2-1998, 87% in Q3-1998, 85% in Q4-1998 and 84% in Q1-1999. We also tested the relationship between return on assets and normalized assets expressed in equation (2.4). Using data on return on assets for 96 companies for the first quarter of 1999, we find an adjusted R-squared of 28% with t-statistics of 6.2 for  $\alpha_1$  and 2.8 for  $\alpha_2$ . The same levels of significance are found for Q1, Q2, Q3 and Q4-1998.

<sup>18</sup> The reason that we use asset size instead of sales is that sales are very volatile in the short-run.

<sup>19</sup> These categories translate into *sales* categories based on the relationship we empirically established between assets and sales via the Gompertz curve.

<sup>20</sup> The cut-off level of sales can be derived based on the Gompertz curve to lead to a normalized *lower bound sales* =  $1 - \text{Ln}((3 + (5)^{0.5})/2)/g$ . For example, for Q1-1999, the actual lower bound sales is \$261 million, which is obtained by multiplying the normalized sales by the inflection firm's sales = \$740 million. Then, the cut-off value of assets is obtained by plugging the normalized sales in the Gompertz equation.

<sup>21</sup> The value of  $G_s$  is gotten by  $G_s = G_a/g$ , where  $g$  is the estimated growth rate of assets vs. sales at the inflection point on the Gompertz curve. The value of  $g$  is estimated to be equal to 1.54 in Q1-99.

<sup>22</sup> It is worth noting that while our model requires seven parameter values, the Schwartz-Moon (2000) model by contrast requires at least twenty parameters for its implementation, many of which require subjective judgment.

<sup>23</sup> For each firm the percentage error is calculated as  $\text{error} = (\text{Actual MV} - \text{Computed MV})/(\text{Actual} + \text{Computed})/2$ . An important step in the application of the model is filtering out negative mean returns that we obtained from individual companies' time series data. These estimates are clearly inappropriate to use as proxies for ex-ante required returns on equity. In addition, we also filtered out extreme mean historical returns. The extreme values are those that fell outside the range of one standard deviation around the sample mean estimated in Table 1. The procedure we followed in testing the model is to substitute these extreme values with the mean of the particular sample according to the phase of the lifecycle.

<sup>25</sup> Consistent with the model formulation, it is expected that the average error will grow with more mature phases. We report here that for Q1 99 the average error is 120% for the Emerging Growth sample and 133% for the Established Growth sample. Average absolute errors are larger and follow the same pattern. Another interesting observation is that positive and negative errors seem to be evenly distributed across the Start-Up sample.

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