

## Another Example: A Multiplication Algorithm

input(u,v)

$x \leftarrow u$

$y \leftarrow 0$

while  $x \neq 0$                       ( $x \neq 0 \equiv B$ )

$x \leftarrow x-1$

$y \leftarrow y+v$

end while

$$\left\{ \frac{xv + y = uv}{P} \wedge \frac{x = 0}{\neg B} \right\} \implies \{y = uv\}$$

## (Multiplication Algorithm)

input(u,v)

$x \leftarrow u$

$y \leftarrow 0$

while  $x \neq 0$  ( $x \neq 0 \equiv B$ )

$\{P'[x-1/x]\} = \{(x-1)v + y+v = uv\} = \{xv + y = uv\} \equiv P$

$x \leftarrow x-1$

$\{P[y+v/y]\} = \{xv + y = uv\}[y+v/y] = \{xv + y+v = uv\} \equiv P'$

$y \leftarrow y+v$

$\{xv + y = uv\} \equiv P$

end while

$$\left\{ \frac{xv + y = uv}{P} \wedge \frac{x = 0}{\neg B} \right\} \Rightarrow \{y = uv\}$$

$$\{P'[x-1/x]\} = \{(x-1)v + y+v = uv\} = \{xv + y = uv\} \equiv P$$

$$\{P[y+v/y]\} = \{xv + y = uv\}[y+v/y] = \{xv + y+v = uv\} \equiv P'$$

$$\{xv + y = uv\} \equiv P$$

## (Multiplication Algorithm)

input(u,v)

$$\{xv = uv[u/x]\} = \{uv = uv\} = \mathbf{true}$$

$x \leftarrow u$

$$\{P[0/y]\} = \{xv + y = uv\}[0/y] = \{xv = uv\}$$

$y \leftarrow 0$

$$\{xv + y = uv\} \equiv P$$

while  $x \neq 0$  ( $x \neq 0 \equiv B$ )

$$\{P'[x-1/x]\} = \{(x-1)v + y+v = uv\} = \{xv + y = uv\} \equiv P$$

$x \leftarrow x-1$

$$\{P[y+v/y]\} = \{xv + y = uv[y+v/y]\} = \{xv + y+v = uv\} \equiv P'$$

$y \leftarrow y+v$

$$\{xv + y = uv\} \equiv P$$

end while

$$\left\{ \frac{xv + y = uv}{P} \wedge \frac{x = 0}{\neg B} \right\} \Rightarrow \{y = uv\}$$

$$\{xv = uv[u/x]\} = \{uv = uv\} = \mathbf{true}$$

$$\{P[0/y]\} = \{xv + y = uv[0/y]\} = \{xv = uv\}$$

$$\{xv + y = uv\} \equiv P$$