

Exchange Rate Crises and Fiscal Solvency

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Abstract

This paper combines insights from generation-one currency crisis models and the Fiscal Theory of the Price Level (FTPL) to create a dynamic FTPL model of currency crises. The initial fixed-exchange-rate policy entails risks due to an upper bound on government debt and stochastic surplus shocks. Agents refuse to lend into a position for which the value of debt exceeds the present-value of expected future surpluses. Policy switching, usually combined with currency depreciation, restores fiscal solvency and lending. This model can explain a wide variety of crises, including those involving sovereign default. We illustrate by explaining the crisis in Argentina (2001).

Key Words: Currency Crises, Generation One Currency Crisis Models, Fiscal Theory of the Price Level, Policy Switching, Passive Fiscal Policy, Active Fiscal Policy, Sovereign Default

JEL Codes: F31, F33, E42, E44, E63

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1 Introduction

The generation-one model of exchange rate crises (Krugman 1979, Flood and Garber 1984) provided invaluable insights into the causes of exchange rate crises, offering an explanation for many of the crises of the 1980's, in which government budget deficits and declining reserves played prominent roles. Yet, the model failed to explain many crises after 1990. These failures led researchers to modify the generation-one model with specifics which would allow it to explain particular crises, and to develop new generations of exchange rate crisis models. New-generation models do not use fiscal solvency as a fundamental determinant of crises. In this paper, we argue that many exchange rate crises, including many of those which occurred after 1990, can be explained by a model which retains fiscal insolvency as the fundamental generating the crisis.

We combine insights from generation-one models and the Fiscal Theory of the Price Level (FTPL) to create a dynamic FTPL model of currency crises. In the original generation-one models (Krugman 1979, Flood and Garber 1984), fixed government spending is financed, initially by declining reserves, and subsequently by increased money growth. Since increased money growth is incompatible with a fixed exchange rate, an exchange rate collapse is inevitable. Burnside, Eichengreen, and Rebelo (2001, 2006) generalize this model to allow a shock to anticipated future government spending to be financed by an anticipated discrete increase in money and by increased money growth, generating inflation and currency depre-

ciation before the actual money growth begins, as in Sargent and Wallace (1981). They add nominal consols such that increases in the money supply generate traditional seigniorage revenue and reduce the real value of consols, creating debt devaluation. As in the original models, the fixed exchange rate must collapse because the monetary policy necessary to balance the government's budget is incompatible with a fixed exchange rate.

The FTPL requires nominal government debt¹ and active fiscal policy. Active fiscal policy is defined as policy in which the government's intertemporal budget is balanced only for a unique price level. When fiscal policy is active, the government does not take purposeful action, including raising tax or seigniorage revenue, to balance its budget. Instead, any shock to current or expected future primary surpluses is offset by a change in the real value of debt due to a price level jump, adjusting the real value of debt to the expected present value of primary surpluses. Under active fiscal policy, debt devaluation itself, and not as a consequence of budget-balancing expected future monetary policy, restores government's intertemporal budget balance. The monetary authority must accommodate the price level jump to allow an equilibrium; equivalently monetary policy must be passive (Leeper 1991).

Daniel (2001a, 2001b) presents an FTPL exchange rate crisis model, in which an unexpected reduction in the present value of primary surpluses requires a price level jump, effectively ending the fixed exchange rate. However, Daniel's model cannot explain exchange rate crises as a result of stochastic shocks to fiscal policy. Positive and negative shocks would require exchange rate jumps of both signs, effectively implying that fixed exchange rates are incompatible with a stochastic element in an active fiscal policy.²

Sims (1997) introduces policy-switching to create a dynamic FTPL model of an exchange rate crisis, which is compatible with stochastic shocks to fiscal policy. Initially, fiscal policy is passive, allowing the monetary authority to fix the exchange rate. However, there is a positive probability, which is increasing in government debt, that fiscal policy will switch from passive to active. An exchange rate crisis occurs because the possibility of switching increases the equilibrium interest rate, increasing equilibrium debt, thereby increasing the probability of switching. When the stochastic policy switch occurs, the fixed rate fails with the price level and exchange rate adjusting to balance the government's intertemporal budget.

The model presented in this paper combines Sims' idea of policy switching with the generation-one assumption that policy switching occurs only when the prevailing policy mix becomes infeasible. We follow Sims and assume that initially fiscal policy is passive, implying that the government's intertemporal budget is balanced for any initial price level. This gives the monetary authority the ability to fix the exchange rate. However, a series of negative shocks could require very large values for future primary surpluses to service the debt, and every government faces limits on its abilities to raise taxes. These limits imply an upper bound on the present value of future primary surpluses and, equivalently, on debt. When fiscal policy is subject to stochastic shocks and an upper bound on debt, there is risk that the initial policy mix is not viable over a particular horizon.

If a current or expected future fiscal shock creates the expectation that the government cannot service its desired debt at market interest rates, then agents refuse to lend. The sudden stop in lending prevents the government from borrowing to continue the initial fiscal

policy and defines the debt crisis. Policy-switching, conditional on being unable to borrow to continue passive fiscal policy, allows the exchange rate to jump, assuring intertemporal budget balance and generating the currency crisis. In contrast to Sims (1997), debt is initially driven towards its upper bound by negative fiscal shocks, in the spirit of generation-one models, not by the effect of stochastic switching on interest rates. Additionally, in this paper, policy-switching is an endogenous policy response to restore fiscal solvency. This contrasts with random or conditionally random policy-switching in models by Sims (1997), Davig and Leeper (2006), and Davig, Leeper, and Chung (2007).

Why is it important to allow fiscal policy to have a stochastic element? The presence of stochastic shocks allows consideration of a policy mix which is sustainable over a given horizon with a probability less than one. This contrasts with unsustainable policy in generation one models. And policy-makers do respond to shocks in the economy. Such behavior is exemplified most recently by the policy-response to the global recession of 2008-2009. A reasonable specification of fiscal policy must allow policy-makers to deviate from the baseline adjustment of the primary surplus to debt, required by passive fiscal policy. We model these deviations as stochastic shocks to the primary surplus. These shocks include fiscal reaction to the state of the economy as well as non-economic shocks due to politics or war.

The FTPL currency-crisis model presented here can be viewed as placing the static currency crisis model in Daniel (2001b) in a dynamic context with stochastic shocks to fiscal policy. Or alternatively, it modifies Sims' (1997) dynamic FTPL model to allow policy switching to be the endogenous resolution of a solvency crisis.

FTPL policy switching is not the only possible response to the sudden stop in lending. A government could devalue and repeg at a lower exchange rate, while maintaining the existing policy mix. We show, however, that such a policy implies a post-crisis period of instability with arbitrarily high interest rates and exchange rate depreciations. Outright default also restores solvency and is another option. Alternatively, a government could receive an IMF loan to replace private capital flows, conditional on policy change which increases the present value of future primary surpluses, thereby restoring solvency. However, raising surpluses, following shocks which have reduced them, could be politically and economically painful and might not be desirable when debt devaluation is available as a source of revenue.

This paper is organized as follows. The next section presents the model. Section 3 characterizes dynamics in the FTPL policy-switching model of exchange rate crises. Section 4 considers other policy reactions, and Section 5 applies the model to the 2001 crisis in Argentina. Section 6 contains conclusions.

2 Model

2.1 Overview

In this section, we set up a simple model of a small open economy with fiscal risk. The model contains four key assumptions. First, international creditors lend to a government only when they expect to receive the market rate of return. Second, the domestic government issues debt denominated in its own currency. Third, there is an upper bound on the value of government debt. Fourth, fiscal policy is subject to stochastic shocks. Together, the upper bound and stochastic shocks imply risk on government debt, reflecting the reality that a government's

commitment to raise taxes to finance expenditures cannot be totally unconditional.

2.2 Goods and Asset Markets

There is a single good in the world, implying that goods market equilibrium requires the law of one price. Normalizing the foreign price level at unity implies that the exchange rate, S_t , defined as the domestic-currency price of foreign currency, equals the domestic price level. The world interest rate (i) is constant. To keep the model simple, output growth is zero.³

The **first key assumption** is that international creditors are willing to buy government bonds as long as the domestic interest rate, i_t , satisfies interest rate parity. Interest rate parity can be derived, using the Euler equations for a representative world agent, when the covariance of the country's interest rate with world-agent consumption is zero, yielding

$$\frac{1}{1+i_t} = \left(\frac{1}{1+i} \right) E_t \frac{S_t}{S_{t+1}}, \quad (1)$$

where E_t denotes the expectation conditional on time t information.⁴ The domestic interest rate is increasing in expected depreciation.

2.3 Monetary Policy

Monetary policy is assumed to have a fixed exchange rate (price level) target. When there is no possibility of a change in the exchange rate in the next period, interest rate parity from equation (1) implies that the domestic interest rate equals the world rate.

2.4 Fiscal Policy

2.4.1 Government Flow Budget Constraint

The **second key assumption** is that government bonds are denominated in domestic currency.⁵ This assumption is based on work by Jeanne and Guscina (2006), who show that even in emerging markets, a substantial fraction of government debt is denominated in domestic currency. Additionally, Burnside, Eichenbaum, and Rebelo (2006) show that in several crises in the 1990's, debt devaluation was a larger source of government revenue than money growth.

Letting G_t and T_t denote nominal government spending and tax revenue, respectively, the government's nominal flow budget constraint is given by

$$B_t + M_t = (1 + i_{t-1}) B_{t-1} + M_{t-1} + G_t - T_t. \quad (2)$$

Defining real government debt (b_t) and the real primary surplus (s_t) as,

$$b_t = \frac{1}{S_t} \left[B_t + \frac{1}{1 + i_t} M_t \right],$$
$$s_t = \frac{1}{S_t} \left[T_t + \frac{i_t}{1 + i_t} M_t - G_t \right],$$

the government's flow budget constraint in real terms can be expressed as

$$b_t = (1 + i_{t-1}) \left(\frac{S_{t-1}}{S_t} \right) b_{t-1} - s_t. \quad (3)$$

Defining γ_t as debt devaluation due to currency depreciation,

$$\gamma_t = \left(1 - \frac{S_{t-1}}{S_t} \right) (1 + i_{t-1}) b_{t-1},$$

and imposing interest rate parity from equation (1) yields⁶

$$b_t = (1 + i) b_{t-1} - (\gamma_t - E_{t-1}\gamma_t) - s_t. \quad (4)$$

This reveals that debt accumulates in response to expectations of depreciation which are not realized. Expectations of depreciation raise the interest rate, and when the depreciation does not occur, debt accumulates in response to the higher interest rate.

Optimization by the representative agent, together with the assumption that governments do not allow their debt to become negative in the limit, implies a government intertemporal budget constraint given by⁷

$$\lim_{T \rightarrow \infty} E_t b_{t+T} \left(\frac{1}{1+i} \right)^T = (1+i) b_{t-1} - (\gamma_t - E_{t-1}\gamma_t) - E_t \sum_{h=0}^{\infty} s_{t+h} \left(\frac{1}{1+i} \right)^h = 0. \quad (5)$$

Note that surprise depreciation ($\gamma_t - E_{t-1}\gamma_t > 0$) is a source of government revenue. Anticipated depreciation is not because it creates an offsetting increase in the interest rate from interest rate parity.

2.4.2 Upper Bound

The **third key assumption** is that there is an upper bound on the present value of future primary surpluses, equivalently from equation (5), on the value of debt. We motivate this assumption with the realization that taxes are distortionary such that there exists an upper bound on the present value of taxes that the government can collect.

The upper bound rules out an explosive equilibrium, in which government debt can rise forever as long as its rate of increase is less than the interest rate. Although the government's intertemporal budget constraint (equation 5) can be satisfied with debt growing forever, the

upper bound cannot. Debt will eventually exceed any upper bound.⁸ When debt is subject to an upper bound, fiscal sustainability requires that the model in the primary surplus and debt be dynamically stable, allowing debt to attain a long-run equilibrium value below its upper bound.

2.4.3 Surplus Rule

Fiscal policy is defined by the behavior of the primary surplus, which we refer to simply as the surplus.⁹ To enable computation of the expected present value of future surpluses, we assume that the fiscal authority is able to commit to a rule. The rule we choose is simple and does not require specification of a fully general equilibrium model. However, any rule with fiscal risk could be used to complete the model.

The **fourth key assumption** is that fiscal policy is subject to stochastic shocks. We assume a baseline fiscal rule in which the surplus responds positively to lagged debt service by a magnitude sufficient to allow a long-run equilibrium, in which the surplus services debt at the world interest rate.¹⁰ The baseline fiscal policy is augmented by introducing stochastic shocks. Stochastic shocks, together with the upper bound, imply risk to current fiscal policy.

The surplus rule is given by

$$s_t - s_{t-1} = \alpha (ib_{t-1} - s_{t-1}) + \nu_t \quad \frac{i}{1+i} < \alpha < 1, \quad (6)$$

where ν_t is a bounded, mean-zero, stochastic disturbance ($-\bar{\nu} \leq \nu_t \leq \bar{\nu}$). Fiscal shocks (ν_t) contain all determinants of the surplus not explicitly included in the surplus rule, many of which would be explicit if the model were placed in a full general equilibrium context. The restrictions on α assure that one root of the dynamic system in debt and the surplus is less

than unity and imply persistence in the surplus ($0 < 1 - \alpha < 1$). Persistence smooths the effects of shocks over time and is consistent with empirical evidence.

Substituting equation (6) into (4) yields a dynamic equation in debt,

$$b_t - b_{t-1} = i(1 - \alpha)b_{t-1} - (1 - \alpha)s_{t-1} - \nu_t - \gamma_t + E_{t-1}\gamma_t. \quad (7)$$

The dynamic model is given by equations (6) and (7). One root of the model is unity, and the other is $(1 - \alpha)(1 + i)$, which is less than one. The model is stable around a long-run equilibrium, which has a unit root. As long as initial debt is not too high, debt is expected to reach a long-run equilibrium value less than its upper bound, satisfying the government's intertemporal budget constraint. Therefore, the fiscal rule, given by equation (6), is passive.

It is useful to compare this policy assumption to those in generation-one and two models. The initial fiscal policy in generation-one models, financing a constant primary deficit with declining reserves, is unsustainable with probability one. Fiscal solvency is restored with an increase in the primary surplus generated from an increase in money growth after reserves have been exhausted. In contrast, our baseline policy mix in the absence of stochastic shocks is completely sustainable, as long as initial debt is not too high. Fiscal shocks, together with the upper bound, introduce risk of unsustainability.¹¹

Fiscal shocks give fiscal policy similarities to that in generation-two models (Obstfeld 1994). In these models, exchange rate depreciation has a stabilizing role, and policy-makers can optimally choose to abandon the fixed rate in response to a stochastic rise in unemployment. In the FTPL crisis model, the stochastic fiscal shocks represent policy responses to stochastic economic or non-economic variables. The model does not preclude these being op-

timal responses. A government facing a recession might optimally choose to let tax revenue fall and spending rise, while a government facing a banking collapse might optimally choose to recapitalize banks, even though these responses could lead to insolvent fiscal positions, imminently or in the future. Additionally, a government might allow a series of small negative fiscal shocks to increase debt over time, with the expectation that the economic or political event creating the negative shocks would end before debt had accumulated sufficiently to raise crisis risk. Therefore, in both the generation-two model and the FTPL model, a crisis can be caused by the policy response, possibly optimal, to stochastic changes in the state of the economy. The models differ in the effect of exchange rate depreciation on the economy; macroeconomic stimulus compared with fiscal solvency restoration.

2.5 Stability and Dynamics in Equilibrium

2.5.1 Equilibrium under Fixed Exchange Rates: Initial Policy Mix

Definition 1 *For values of debt low enough that $E_{t-1}\gamma_t = 0$, constant values for the world interest rate and price level, together with the passive surplus rule from equation (6) and a monetary policy fixing the exchange rate such that $i_t = i$, an equilibrium is a set of time series processes for the surplus, debt, and debt devaluation, $\{b_t, s_t, \gamma_t\}_{t=0}^{\infty}$, such that the government's flow and intertemporal budget constraints, given by equations (5) and (7), hold, expectations are rational, and world agents expect to receive the return on assets determined by interest rate parity (equation 1).*

The phase diagram for equations (6) and (7), with ν_t and γ_t at their expected values of zero, is given in Figure 1. Note that the $\Delta b = 0$ and $\Delta s = 0$ schedules lie on top of each other with $ib_t = s_t$. The upper bound on debt service, given by $i\bar{b}$ at point L, implies an upper bound on the long-run value of the surplus, given by \bar{s} . Current fiscal shocks (ν_t) move the system away from the $\Delta s = \Delta b = 0$ locus, say to point K. As long as $E_{t-1}\gamma_t = \gamma_t = 0$,

equations (6) and (7) can be used to show that the expected relationship between debt and surpluses along an adjustment path like KF is given by

$$\frac{i(E_t b_{t+1} - b_t)}{E_t s_{t+1} - s_t} = \frac{i(1 - \alpha) - E_t \nu_{t+1}}{\alpha + E_t \nu_{t+1}}. \quad (8)$$

When the conditional mean of future fiscal shocks is zero ($E_t \nu_{t+1} = 0$), the slope of the adjustment path is constant, as drawn in Figure 1. Current fiscal shocks have long-run effects due to the unit root.

Expected future fiscal shocks ($E_t \nu_{t+1} = 0$) change the slope of the adjustment path such that a positive expected shock implies lower expected long-run values for the debt and surplus for given initial values. Expected future fiscal shocks do not affect the current equilibrium positions for the debt and the surplus as long as $E_{t-1} \gamma_t = 0$.

When the economy is on an adjustment path like KF, leading to a long-run equilibrium substantially below L, passive fiscal policy permits active monetary policy to fix the exchange rate such that $\gamma_t = 0$. However, $E_{t-1} \gamma_t = 0$ requires that there be no possibility of a one-period-ahead crisis, a topic to which we turn below. Rationally-determined expectations of depreciation increase as the economy moves onto adjustment paths toward long-run equilibria closer to L. Expectations change the adjustment path, as shown below.

The upper bound on the present value of surpluses and equivalently on debt implies that adjustment paths above HL cannot represent equilibrium paths. These paths require that the present value of future surpluses be larger than their upper bound in order to service debt. Rational agents would not embark on such paths because they know the present value of surpluses necessary to service debt along those paths is infeasible, implying that they cannot

expect market interest rates. The government must have plans to restore fiscal solvency in the event that shocks send it toward an infeasible path. We assume that agents know those plans and use them to form expectations.

The first plan we consider is a policy of switching, whereby the fiscal authority switches to active policy and the monetary to passive. Before considering the switching model, we present equilibrium under the post-crisis policy mix.

2.5.2 Equilibrium with Flexible Exchange Rates: Post-Crisis Policy Mix

In this section, we characterize equilibrium with the policy mix after switching, active fiscal policy and passive monetary policy. Under active fiscal policy, the surplus responds to a surplus target, defined as the value of the surplus in the long-run stationary equilibrium, instead of to lagged debt. The fiscal authority chooses the surplus target on the switching date, and we specify \hat{s} as the largest target they would tolerate. We show below that a government, which chooses to maintain the initial policy mix for as long as possible, will usually choose the target equal to \hat{s} . The active fiscal rule with a target of \hat{s} is given by

$$s_t - s_{t-1} = \alpha(\hat{s} - s_{t-1}) + \nu_t \quad \hat{s} < \bar{s} - \bar{\nu}. \quad (9)$$

The surplus target must be below the upper bound, and, depending on tolerance for taxes, the target could be substantially lower.¹² The additional restriction is made to assure that there is no possibility of hitting the upper bound when the surplus equals the target.

The evolution of debt can be computed using equations (4) and (9) to yield

$$\Delta b_t = b_t - b_{t-1} = ib_{t-1} - (1 - \alpha)s_{t-1} - \alpha\hat{s} - (\gamma_t - E_{t-1}\gamma_t) - \nu_t. \quad (10)$$

The passive monetary authority chooses expected inflation with its choice of the nominal interest rate, but it loses control over the actual price level and exchange rate. We assume that the inflation target is zero such that it chooses the interest rate to be the world value. When the surplus is low enough that there is no possibility of debt crossing the upper bound in the next period, the zero inflation target implies that $E_{t-1}\gamma_t = 0$.

Definition 2 *For values of the surplus low enough that $E_{t-1}\gamma_t = 0$, constant values for the world interest rate and price level, together with a surplus rule from equation (9) and a monetary policy setting $i_t = i$, an equilibrium is a set of time series processes for the surplus, debt, and debt devaluation, $\{b_t, s_t, \gamma_t\}_{t=0}^{\infty}$, such that the government's flow and intertemporal budget constraints, given by equations (5) and (7), hold, expectations are rational, and world agents expect to receive the return on assets determined by interest rate parity (equation 1).*

The model with active fiscal policy and passive monetary policy is given by equations (9) and (10). The phase diagram is given in Figure 2. This is a saddlepath-stable model in which there are debt-surplus pairs for which the present-value of debt explodes in the limit, violating the upper bound on debt.¹³ To assure equilibrium, there must be one jumping variable to keep the system on the saddlepath, labeled SP, leading to long-run values for debt and the surplus at point F. The monetary authority's inflation target restricts $E_{t-1}\gamma_t = 0$, but places no restrictions on γ_t . Therefore, γ_t jumps, implying jumps in b_t from equation (4), to keep the system on the saddlepath. Stochastic and symmetric surprise appreciations and depreciations ($\gamma_t - E_{t-1}\gamma_t$), finance positive and negative stochastic surplus shocks. This is the mechanism in the FTPL. Debt devaluations and revaluations are symmetric in response to symmetric fiscal shocks, implying that policy shocks do not generate systematic revenue in the post-crisis equilibrium (Daniel 2007). This contrasts with post-crisis policy in the original generation-one model in which systematic money growth generates seigniorage.

Denoting the present-value of expected future surplus shocks by $V_t = E_t \sum_{h=t}^n \frac{\nu_h}{(1+i)^{h-t}}$, the relationship between debt and the surplus along the saddlepath can be expressed as

$$b_t = \left(\frac{1}{\alpha + i} \right) \left[\frac{1+i}{i} \alpha \hat{s} + (1-\alpha) s_t + (1+i) V_t \right], \quad (11)$$

where $V_t = 0$ along SP in Figure 2. The larger the surplus target, the higher is the value of debt along the saddlepath.

From equation (11), an increase in expected future government spending, denoted by $V_t < 0$, reduces the equilibrium value of debt. Assume that the system is at point A along SP in Figure 2 at time t , when agents begin to expect an increase in spending at time $h > t$. Therefore, $E_t \nu_h < 0$. This expected future spending shock requires additional revenue to assure intertemporal budget balance. Price surprises generate revenue, whereas anticipated price changes do not, implying that a price increase on the date that expenditures rise cannot generate the necessary revenue. Therefore, the price must jump on the date on which news about future spending arrives. The jump reduces real debt from point A to point B in Figure 2. Debt and the surplus then follow the unstable arrows of motion, with debt falling and the surplus increasing to reach point C on date h . The increase in spending and the associated increase in debt on date h return the system to SP at point D.

The upper bound on debt implies that the post-crisis policy mix is not sustainable with probability one. Positive surplus shocks could send debt along the saddlepath above its upper bound. This would require a plan for reverse-switching, not explicitly considered here. We do assume in equation (9) that $\hat{s} \leq \bar{s} - \bar{\nu}$ to assure that reverse switching cannot occur within one period after reaching \hat{s} .¹⁴

3 Exchange Rate Crisis with Policy Switching

In this section we consider an equilibrium in which fiscal policy is initially passive and monetary policy is active (Regime 1) with plans to switch to active fiscal policy and passive monetary policy (Regime 2) once equilibrium in Regime 1 is no longer feasible. We assume that the government maintains its commitments to the fixed exchange rate and the passive fiscal rule until agents refuse to lend. This assumption is not subject to the criticism of generation-one models by Rebelo and Vegh (2002), who argue that a government should optimally abandon the fixed exchange rate regime as soon as failure becomes inevitable. In our model, lending stops, precipitating the crisis, as soon as a crisis becomes inevitable. Moreover, a country, which continues its initial policy mix when crisis probability becomes positive, could receive favorable shocks and avoid the need to abandon the exchange rate.

3.1 Equilibrium with Switching

Definition 3 *Given constant values for the world interest rate and price level, an upper bound on the long-run value of debt, a policy mix, defined by a surplus rule from equation (6) and a monetary policy fixing the exchange rate, which the government will maintain as long as possible, and policy-switching in the event that the initial policy mix becomes infeasible, an equilibrium is a set of time series processes for the surplus, debt, and debt devaluation, $\{b_t, s_t, \gamma_t\}_{t=0}^{\infty}$, such that the government's flow and intertemporal budget constraints, given by equations (7) and (5), hold, expectations are rational, debt does not exceed its upper bound, and world agents expect to receive the return on assets determined by interest rate parity, equation (1).*

To allow Regime 1 to be initially viable, but subject to risk, we assume that the initial value of the surplus (s_0) is below the target surplus (\hat{s}), and that the initial value of debt service (ib_0) is below the saddlepath.

3.2 Fiscal Choice of Post-Crisis Surplus Target

The post-crisis system is saddlepath stable. Therefore, debt must be on the saddlepath immediately after the policy switch, to enable it to reach a long-run equilibrium value.

The value of the target surplus determines the position of the saddlepath. The position of debt under passive fiscal policy, relative to the saddlepath, determines the value of exchange rate depreciation on the crisis date. We assume that **the fiscal authority chooses the surplus target as the largest tolerable long-run surplus, subject to the constraint that the exchange rate be allowed to depreciate, but never to appreciate.**

On the crisis date, if the value of debt under passive fiscal policy is above the \hat{s} -saddlepath, the fiscal authority chooses \hat{s} as the surplus target. The equilibrium exchange rate depreciates, reducing the real value of debt onto the \hat{s} -saddlepath. If debt is below the \hat{s} -saddlepath, the fiscal authority avoids the reduction in government revenue, which would be associated with appreciation, by allowing the surplus target to fall to $\hat{s}' < \hat{s}$. This shifts the saddlepath downward such that debt is on the \hat{s}' -saddlepath without exchange rate change.

3.3 Exchange Rate Depreciation and Expectations

Conditional on policy-switching, exchange rate depreciation could be necessary to place the system on the saddlepath. To solve for exchange rate expectations, assume that agents know the policy response to a sudden stop in lending. Since post-crisis debt must be on the saddlepath, the exchange rate must depreciate when debt under prevailing passive fiscal policy, given by b_t in equation (7), is above the saddlepath to \hat{s} . Using equations (11) and (7), the distance between the \hat{s} -saddlepath value of debt and its time t value can be expressed

as

$$\Omega_t = (\gamma_t - E_{t-1}\gamma_t) + \frac{1+i}{\alpha+i} [\delta_{t-1} + \nu_t], \quad (12)$$

where δ_{t-1} is the state variable determining this distance at time t and is given by

$$\delta_{t-1} = \frac{\alpha}{i} (\hat{s} - ib_{t-1}) + (1 - \alpha) (s_{t-1} - ib_{t-1}). \quad (13)$$

The state variable determining the time t distance is known at time $t - 1$, and therefore receives a $t - 1$ subscript. It is increasing in the values for \hat{s} and α .¹⁵

We define a shadow value of depreciation, analogous to the shadow value of the exchange rate in generation-one currency crisis models (Flood and Garber 1984). The shadow value of depreciation is the reduction in the value of debt needed for the economy to reach the saddlepath to \hat{s} , equivalently to set $\Omega_t = 0$. The shadow value is positive when debt generated by passive fiscal policy is above the saddlepath and negative when it is below.

Definition 4 *The **shadow value** of depreciation at time t , $\tilde{\gamma}_t$, is defined as the value of γ_t for which $\Omega_t = 0$.*

Setting $\Omega_t = 0$ in equation (12) and solving yields

$$\tilde{\gamma}_t = E_{t-1}\gamma_t - \frac{1+i}{\alpha+i} (\delta_{t-1} + \nu_t). \quad (14)$$

Expectations of exchange rate depreciation raise the interest rate, increasing debt, increasing the value of actual depreciation needed to set the real value of debt on the saddlepath.

To solve for expected depreciation, **assume that agents believe that a lending crisis will occur if $\tilde{\gamma}_t \geq 0$** .¹⁶ We prove that this assumption is consistent with a rational expectations equilibrium below. The government responds to the crisis with policy-switching, and

when $\tilde{\gamma}_t > 0$, currency depreciation places the system on the saddlepath to \hat{s} . When $\tilde{\gamma}_t = 0$, the system is on the saddlepath without depreciation. This implies that the equilibrium value for depreciation in period t is given by

$$\gamma_t = \max \{ \tilde{\gamma}_t, 0 \} = \max \left\{ E_{t-1}\gamma_t - \frac{1+i}{\alpha+i} (\delta_{t-1} + \nu_t), 0 \right\}, \quad (15)$$

where we have used equation (14) to substitute for $\tilde{\gamma}_t$. To solve for γ_t , we must first solve for $E_{t-1}\gamma_t$.

Proposition 1 *Under the initial policy with plans for switching, an equilibrium solution for expected depreciation ($E_{t-1}\gamma_t$) exists iff the state variable determining the distance to the saddlepath at time t is greater than or equal to zero ($\delta_{t-1} \geq 0$).*

The proof is contained in the appendix. The value for δ_{t-1} measures the amount by which b_t is below the saddlepath at time t , before accounting for expectations ($E_{t-1}\gamma_t$), current fiscal shocks (ν_t), and actual depreciation (γ_t) (equation 12). Intuitively, the proposition implies that when the distance between the saddlepath and this ex ante value for b_t is negative, there is no value for the exchange rate, conditional on policy switching, which can both restore fiscal solvency and provide the market rate of return to international creditors.

Figure 3 superimposes the $\Delta s = 0$ curve and the saddlepath for the active-fiscal-policy system on the phase diagram for the passive-fiscal-policy system. When the system is far below SP, say at point A with $\delta_{t-1} > \bar{\nu}$, no shock could send the system above SP, and the arrows of motion for the passive-fiscal-policy system govern. Consider the feasibility of a position like C , where we assume $0 < \delta_{t-1} < \bar{\nu}$. When ex ante debt is near the saddlepath, δ_{t-1} is less than the upper bound on fiscal shocks ($\bar{\nu}$), and the market begins to anticipate depreciation.¹⁷ This anticipation raises the interest rate from equation (1) for interest rate

parity. The monetary authority allows the interest rate to rise to keep the exchange rate fixed. Therefore, debt is expected to increase more quickly than implied by the locus CD, reaching SP at a point like E . Once expectations of depreciation become positive, realizations of fiscal shocks must be more favorable than average to keep debt from rising above the saddlepath. Equivalently, the probability of a near-term crisis rises above fifty percent. Even so, the probability of avoiding a near-term crisis is positive, and sufficiently favorable shocks could sustain the initial policy mix.

Once ex ante debt has risen so much that it lies on the saddlepath ($\delta_{t-1} = 0$), expectations of depreciation are so high that one-period-ahead depreciation could be avoided only for the most favorable fiscal shock. Using equation (15) to solve for depreciation when $\delta_t = 0$ yields

$$\gamma_t = \tilde{\gamma}_t = E_{t-1}\gamma_t - \left(\frac{1+i}{\alpha+i}\right)\nu_t \geq 0. \quad (16)$$

The sign restriction is required since depreciation must be greater than or equal to zero for any realization of ν_t , including its upper bound value of $\bar{\nu}$. This yields $E_{t-1}\gamma_t \geq \left(\frac{1+i}{\alpha+i}\right)\bar{\nu}$. Therefore, when ex ante debt is on the saddlepath, there are multiple equilibria with policy-switching in which expectations of depreciation and actual depreciation must be positive and can be arbitrarily large. To verify, take the expectation of equation (16) to yield an identity in the expectation.

A value of $\delta_{t-1} < 0$ would imply that ex ante debt is above the saddlepath. All fiscal shocks, including the most favorable, send the system above the saddlepath such that the probability of depreciation is unity. However, taking expectations of equation (15), when

the probability of depreciation is unity, yields

$$E_{t-1}\gamma_t = E_{t-1}\gamma_t - \frac{1+i}{\alpha+i}\delta_{t-1}. \quad (17)$$

With $\delta_{t-1} < 0$, there is no solution for $E_{t-1}\gamma_t$ which satisfies equation (17). Rationally-anticipated policy switching cannot restore fiscal solvency because expected depreciation cannot equal itself plus a gap. Therefore, in equilibrium, the dynamics must bound the system away from positions for which $\delta_{t-1} < 0$. This criterion determines crisis timing.

Proposition 2 *A crisis occurs in the first period for which $\delta_t < 0$. Policy-switching restores equilibrium and allows government borrowing.*

The proof is contained in the appendix. To understand the dynamics, assume that $0 < \delta_{t-1} < \bar{\nu}$, such that in period $t-1$, there is an equilibrium with lending under passive fiscal policy, but $E_{t-1}\gamma_t > 0$. Consider whether there will be a passive fiscal-policy equilibrium in period t , where debt in period t is given by equation (7).

The value for $E_{t-1}\gamma_t$ together with the realization of the fiscal shock (ν_t) determine $\tilde{\gamma}_t$ from equation (14). In turn, the value for $\tilde{\gamma}_t$, together with passive fiscal policy dynamics, determines the evolution of δ_t as¹⁸

$$\delta_t = -(\alpha+i)\tilde{\gamma}_t - \alpha(\hat{s} - ib_t). \quad (18)$$

Since $\hat{s} - ib_t > 0$ in the relevant region, a value for $\tilde{\gamma}_t > 0$ is sufficient to imply $\delta_t < 0$. Therefore, a positive shadow value of debt devaluation is sufficient to assure that period- t debt would be above the saddlepath in the absence of depreciation. Agents refuse to lend into this position, requiring policy-switching and depreciation to restore lending.

A positive shadow value of depreciation is sufficient but not necessary for a crisis. The dynamics for the surplus and debt under passive policy could imply that without regime switch, debt would travel above the \hat{s} -saddlepath in period $t + 1$ with probability one, such that $\delta_t < 0$, even though period- t debt is on or below the saddlepath with $\tilde{\gamma}_t \leq 0$. This is possible since the slope of the adjustment path (from equation 8) is greater than the slope of the saddlepath (using equation 11).¹⁹ The fiscal authority does not allow exchange rate appreciation to reach the \hat{s} -saddlepath. Instead, it chooses the target surplus $\hat{s}' < \hat{s}$ to set the distance between debt and its value along the \hat{s}' -saddlepath to zero, such that

$$\Omega'_t = -E_{t-1}\gamma_t + \frac{1+i}{\alpha+i} \left[\frac{\alpha}{i} (\hat{s}' - ib_{t-1}) + (1-\alpha)(s_{t-1} - ib_{t-1}) + \nu_t \right]. \quad (19)$$

The lower target surplus shifts the saddlepath downward such that desired debt at the fixed exchange rate is on the saddlepath. Policy switching replaces the level of debt service in the fiscal rule with the larger value for the target surplus, thereby increasing near-term surpluses. For this case, the increase in the expected present value of surpluses is sufficient to restore fiscal solvency without currency depreciation.

Therefore, Proposition (2) states that as long as $\delta_t > 0$, the probability of receiving near-term shocks, favorable enough to avoid a crisis, is positive, and no crisis occurs. Once $\delta_t < 0$, there is no possibility of receiving near-term shocks favorable enough to avoid the crisis, and the crisis occurs immediately. If $\delta_t = 0$, then the one-period-ahead probability of a crisis is unity, but this is a probability zero event when a crisis is due to fiscal shocks.

Since δ_{t-1} is increasing in the value of the surplus target from equation (13), a fiscal authority, who wants to maintain the initial policy mix as long as possible, will choose the

surplus target equal to \hat{s} for all crises with depreciation. A smaller value would reduce the distance to the post-crisis saddlepath, increasing the probability of a crisis each period.

After the regime switch, capital gains and losses on debt due to exchange rate changes are symmetric, implying that expectations of inflation and exchange rate changes return to their original values of zero. Therefore, in contrast to the post-crisis equilibrium in generation one models, real money demand is unchanged, implying that any effects of the currency crisis, which tend to reduce the money supply, must be sterilized.²⁰

3.4 Shocks Other Than Current Fiscal Shocks

The shadow value of depreciation, $\tilde{\gamma}_t$, is affected by anything which changes the post-crisis position of the saddlepath value of debt relative to its current value. An increase in $\tilde{\gamma}_t$ can cause a crisis if the increase is large enough that $\tilde{\gamma}_t$ becomes positive, or can increase the probability of one. Assume that the economy is in Regime 1 with $s_0 < \hat{s}$ and ib_0 below the saddlepath.

3.4.1 Expected future fiscal shocks

Consider the effect of an increase in expected future government spending.²¹ From equation (11), this reduces the equilibrium value of debt under post-crisis policy, modifying the expression for the distance to

$$\Omega_t = (\gamma_t - E_{t-1}\gamma_t) + \frac{1+i}{\alpha+i} [\delta_{t-1} + \nu_t + V_t].$$

An increase in expected future government spending is represented by negative expected present-value surplus shocks ($V_t < 0$). Distance (Ω_t) falls, raising $\tilde{\gamma}_t$.

3.4.2 Confidence and the parameters of the surplus rule

We have assumed that the parameters governing fiscal policy are known. A reduction in \hat{s} shifts $\Delta s = 0$ down, shifting the saddlepath down. A reduction in α increases the slope of the saddlepath without increasing long-run equilibrium values. Both reduce δ_{t-1} from equation (13), thereby raising $\tilde{\gamma}_t$ from equation (14).

However, going forward, these parameters are not known. Agents form expectations based on current and past government behavior, and use these expectations as measures of the parameter values. An economic or political crisis could reduce confidence in the government's ability to raise taxes in response to an increase in debt, reducing α , or to generate taxes necessary to service as high a level of debt as before, reducing \hat{s} . Therefore, a reduction in confidence could create a solvency crisis.

4 Alternative Policy Responses to a Crisis

4.1 Devalue and Repeg without Policy-Switching

The government could respond to a lending crisis with a devaluation, repegging the exchange rate at a lower value to reach the adjustment path toward its surplus target, without any fiscal policy change. The target could be lower than the upper bound, say at \hat{s} in Figure 1. Adjustment paths above those leading to \hat{s} cannot be equilibrium paths because positions along them would imply a negative ex ante distance to the adjustment path. This implies that the relevant adjustment path becomes KF, leading to the surplus target \hat{s} .

Let δ_{t-1} be redefined as the state variable determining the distance between the target

value for debt, given by \hat{s}/i , and the current expectation of its long-run value under passive fiscal policy from equations (6) and (7). With the monetary authority maintaining the fixed exchange rate for as long as possible, the interest rate rises as this distance shrinks, assuring interest rate parity from equation (1).

Proposition 3 *A policy in which the government devalues to place the system on the adjustment path toward \hat{s} and repeg at the lower rate without fiscal reform will fail next period with probability one.*

The proof is in the appendix. Since the policy response sets $\delta_t = 0$, there are multiple equilibria with arbitrarily high expectations of devaluation and accompanying high interest rates. Additional devaluation is needed each period to set $\delta_{t+i} = 0$, implying that markets remain turbulent. Given sustained post-crisis turbulence, it would be difficult to make a case that this policy represents an optimal response.

4.2 Default

The government could plan to respond to a crisis by reneging on its no-default commitment. Both default and devaluation reduce the real value of outstanding debt, moving the system toward the \hat{s} -saddlepath. Since default solves the same fiscal solvency problem as devaluation, default and devaluation can occur together.²²

4.3 IMF Loan

Assume that the country plans to resolve the crisis by securing an IMF loan to replace the private market source of loans it loses in a crisis. To simplify the presentation and contrast this policy with those preceding it, we assume there is no accompanying debt devaluation

either through depreciation or default.

The IMF is willing to make the loan when the private market is not because the IMF can mandate fiscal policy change as a condition for receiving the loan. IMF programs for countries with fiscal problems usually require an increase in the value of the government surplus. We model this as an increase in the mean of ν_t for a fixed number of periods. In Figure 1, this flattens the adjustment path, leading to a lower expected value for the long-run surplus. Fiscal solvency is restored because the present value of future surpluses rises, not because the real value of debt falls.

However, success requires confidence in the stronger fiscal policy. Past failures to comply with IMF mandates could weaken confidence in the government's ability to deliver the present-value surpluses necessary to service debt. Additionally, governments might not be willing to restore fiscal solvency solely through increased present-value surpluses when other methods of raising revenue, including depreciation and default, are available. This could explain why IMF loans are often combined with exchange rate depreciation, allowing domestic-currency debt devaluation.

5 Model Applied to Argentina 2001

The dynamic FTPL currency crisis model claims that a crisis will occur once the government's debt under the passive fiscal rule becomes so large that one-period ahead exchange rate depreciation, conditional on policy switching, could not both assure fiscal solvency and compensate creditors for expectations of exchange rate depreciation. According to the IMF's

assessment of the crisis in Argentina,

"eventually the required primary surplus became implausibly large, particularly in relation to the political system's ability to deliver....By 2001, almost no strategy would have succeeded without a sovereign debt restructuring that reduced the present value of Argentina's public debt burden." (IMF 2003, p. 67)

A sudden stop in lending at the end of November 2001 defined the debt crisis and forced a policy response to restore solvency.²³ The response included elimination of the currency board to allow currency depreciation, pesoization of government debt, and ultimately, outright default on 75 percent of remaining government dollar-denominated liabilities (IMF 2005, p. 13).

How did Argentina's debt grow so large, relative to its ability to service it? Figure 4 shows the behavior of the primary surplus relative to GDP and debt relative to GDP over the decade of the 1990's.²⁴ After the introduction of the currency board in 1991, strong fiscal surpluses brought debt down. However, from 1993, debt has an upward trend, which becomes steeper in 1998, following the recession which began with the Russian default. Interest rates²⁵, shown in Figure 5, were relatively stable from 1993, with the exception of the 1995 Tequilla crisis, until they began to anticipate the crisis at the end of 2001. Figure 6 shows that the change in the surplus does respond to debt service at world interest rates relative to the surplus, as required for passive fiscal policy, with that response seeming to weaken after 1998.

We simulate the FTPL policy-switching model to understand how the crisis erupted. The simulation algorithm is given in Table 1. According to the model, the crisis could have been caused by: 1) bad luck, reflected in a string of negative shocks to the fiscal surplus; 2) a loss of confidence in the government's ability to raise taxes, creating a change in the perceived parameters of the fiscal rule (α, \hat{s}) ; or 3) both negative fiscal shocks and a loss of confidence. It is reasonable to argue that the 1998 recession, together with persistent failures to meet surplus targets (IMF 2003), weakened both the fiscal stance with negative shocks as well as confidence in fiscal policy, changing agents' perceptions of fiscal policy parameters. For the FTPL model to successfully explain the crisis, the probability of a crisis should be reasonably high under at least one of the three hypotheses.

For the simulations, we assume that the values for the primary surplus and debt in the model represent ratios to GDP, implying that the interest rate variable is the growth-adjusted real interest rate. We use a value of 2%, based on real interest rates of about 5-6% and reasonable expected growth rates of 3-3.5% (IMF 2003, p.12). We let the long-run value for the surplus target be 1.5% of GDP, larger than any surplus achieved in the 1990's, although not larger than the IMF surplus targets for the short run. At this interest rate, $\hat{b} = 75\%$ of GDP.²⁶ We assume that fiscal shocks have a uniform distribution and let the upper bound on fiscal shocks be 2%.²⁷ To characterize initial policy, we estimate equation (6), using OLS for the period 1991-1998, and obtain a statistically significant value for α of 0.61.²⁸ The significantly positive value is consistent with the hypothesis that fiscal policy was initially sustainable and passive.

To obtain a baseline and determine the probability of an exchange rate crisis due to fiscal shocks under initial policy, we simulate the model using the above parameter values for the fiscal rule together with 1991 values for debt/GDP and the primary surplus/GDP. The simulations are presented on the first line of Table 2. They reveal that Argentina's initial fiscal policy was safe, with no probability of a currency crisis in twenty years.

The first hypothesis is that negative fiscal shocks, which drove debt up to 50.9 percent of GDP in 2000, were responsible for the crisis. To test this hypothesis, we set debt and the surplus at their 2000 values, while leaving the parameters of the fiscal rule unchanged. Line 2 in Table 2 shows that the probability of a crisis, under the hypothesis of negative fiscal shocks alone, is only 0.6% over twenty years.

To test the second hypothesis of whether changes in the perceived parameters of the fiscal rule could have caused the crisis, we simulate the model with different values for α and \hat{s} . Particularly in 1999 and 2001, the surplus failed to respond as much to debt service relative to the surplus, as in earlier periods. Therefore, we reestimate α . Fiscal policy after 1998 is more consistent with a value of $\alpha = 0.13$.²⁹ Additionally, a crisis of confidence is likely to have lowered the public's expectation of the largest surplus which the government could sustain, so we assume that \hat{s} falls from 1.5% to 1.25%, implying a fall in \hat{b} from 75% to 62.5% of GDP.³⁰ We simulate the model with 1998 values for the debt and surplus and the parameter values associated with a weaker fiscal stance. The probability of a crisis under the hypothesis that a loss of confidence changed the perceived parameters of the fiscal rule, presented on line 3 of Table 2, is substantially higher at 22.4% over a three-year period.

The probability of a crisis under the hypothesis that fiscal policy was hit by both bad luck and a crisis of confidence is determined by setting debt and the surplus at their 2000 values and using the parameters of the weakened fiscal stance. The probability of a crisis in three years rises to 62.5% with the mean time to a crisis being a little less than one year, as shown on line 4 of Table 2. Therefore, negative fiscal shocks, which increased the level of debt, together with a weakened fiscal rule, can predict the crisis which occurred.

6 Conclusions

This paper provides a dynamic FTPL model of currency crises. The model is the product of insights from the original generation-one model, which highlights fiscal solvency as a key fundamental in exchange rate crises, and the FTPL, which allows capital gains and losses on debt to maintain fiscal solvency. In the FTPL currency-crisis model, a crisis can be caused by current fiscal shocks, which reduce the current surplus and raise debt under passive fiscal policy, by expectations of new future fiscal commitments, which reduce the expected present-value of future surpluses, and/or by a change in the current fiscal rule or the expectation of such a change. Agents will not lend when there is no value for the one-period-ahead exchange rate which could both restore fiscal solvency and yield market interest rates. Equilibrium with lending cannot be restored until policy responds to restore expectations of fiscal solvency. One possibility is a regime switch, in which the fiscal rule becomes active and monetary policy becomes passive, as in the FTPL. This allows future price level (and exchange rate) surprises to offset stochastic surplus shocks and usually requires exchange rate depreciation

in the crisis period to reduce the outstanding value of debt. We label the model with policy switching a dynamic FTPL model of currency crises.

Alternatively, a policy of devaluation to restore fiscal solvency, without a change in the fiscal rule, will restore equilibrium, but at the cost of arbitrarily high expected future devaluation and interest rates. This result highlights the importance of fiscal reform in restoring orderly markets after a crisis. Solvency can also be restored through sovereign default. Since both default and currency depreciation reduce the value of government debt, thereby contributing to the restoration of solvency, they can occur together.

The FTPL model of currency crises can explain a wider variety of crises than the original generation-one model, including those in the 1980's, explained well by generation-one models, and many after 1990, which are not explained by the original generation-one model. Generation-one models explain the crises of the 1980's as due to a policy of financing government spending with reserves until they are exhausted, and then switching to money finance. Since increased money growth is incompatible with a fixed exchange rate regime, its collapse is inevitable. Based on this model, a currency board should eliminate currency crises since a country cannot finance spending using reserves. However, a currency-board country can finance spending with debt, and the FTPL model demonstrates that this behavior can also lead to a currency crisis. In the FTPL model, a fiscal policy, in which net debt is accumulating due to stochastic shocks to the passive fiscal rule, has risk of a currency crisis, whether the financing is in the form of reserve loss or actual debt increase. Therefore, the FTPL model has the potential to explain the crises of the 1980's, in which debt accumulation was

in the form of a reduction of reserves, as well as explain the Argentine crisis in 2001, in which debt increased directly.

The FTPL model can also explain currency crises due to an increase in expected future government spending. Burnside, Eichenbaum, and Rebelo (2001) extend the original generation-one model to explain the Asian currency crises by an increase in expected future government spending. Their explanation requires that at least part of this increase in spending be financed by expected future money growth, placing their model in the generation-one classification. The FTPL currency crisis model does not require a particular assumption about future money growth to explain the crisis. The increase in expected future spending reduces the equilibrium value of debt along the post-crisis saddlepath, raising the probability of a crisis.

The FTPL currency crisis model can also explain crises due to a political or economic event, which affects agents' confidence in the ability of the government to raise taxes to finance expenditure. A change in the perceived parameters of the fiscal rule, reflecting a loss of confidence, can create a crisis. This implies that a political event alone can create a crisis, reflecting the importance of expectations in the model.

7 Appendix: Proofs

7.1 Proof of Proposition 1

Let $f(\nu_t)$ be a bounded, symmetric, mean-zero distribution for ν_t , with bounds $\pm\bar{\nu}$. Define a critical value for ν_t , given by ν_t^* , such that for $\nu_t < \nu_t^*$, $\gamma_t > 0$, and for $\nu_t \geq \nu_t^*$, $\gamma_t = 0$.

When such a critical value exists, taking the expectation of equation (15) yields

$$E_{t-1}\gamma_t = \int_{-\bar{\nu}}^{\nu_t^*} \gamma_t f(\nu_t) d\nu_t = \int_{-\bar{\nu}}^{\nu_t^*} \left[-\frac{1+i}{\alpha+i} [\delta_{t-1} + \nu_t] + E_{t-1}\gamma_t \right] f(\nu_t) d\nu_t. \quad (20)$$

Defining $F(\nu_t^*)$ as the cumulative at ν_t^* and collecting terms on the expectation yields

$$[1 - F(\nu_t^*)] E_{t-1}\gamma_t = - \left(\frac{1+i}{\alpha+i} \right) \left[\delta_{t-1} F(\nu_t^*) + \int_{-\bar{\nu}}^{\nu_t^*} \nu_t f(\nu_t) d\nu_t \right]. \quad (21)$$

Substituting into equation (15), yields an expression for γ_t as

$$[1 - F(\nu_t^*)] \gamma_t = \max \left\{ - \left(\frac{1+i}{\alpha+i} \right) \left[\delta_{t-1} + \int_{-\bar{\nu}}^{\nu_t^*} \nu_t f(\nu_t) d\nu_t + [1 - F(\nu_t^*)] \nu_t \right], 0 \right\}. \quad (22)$$

To solve for $E_{t-1}\gamma_t$, first solve for ν_t^* . Define $\chi_t = \int_{-\bar{\nu}}^{\nu_t^*} \nu_t f(\nu_t) d\nu_t + [1 - F(\nu_t^*)] \nu_t^*$. A solution for ν_t^* exists iff there exists a value ν_t^* , satisfying $-\bar{\nu} \leq \nu_t^* \leq \bar{\nu}$, such that $\delta_{t-1} + \chi_t = 0$.

First, prove that $\chi_t \leq 0$ for all feasible values for ν_t^* . Let ν_t^* take on its smallest possible value of $-\bar{\nu}$, implying that $\chi_t = -\bar{\nu} < 0$. The derivative of χ_t with respect to ν_t^* is given by $1 - F(\nu_t^*)$. For $\nu_t^* < \bar{\nu}$, this is positive. Therefore, as ν_t^* rises, χ_t rises monotonically. Once ν_t^* takes on its largest possible value, given by $\bar{\nu}$, $1 - F(\bar{\nu}) = 0$, and χ_t takes on its maximum value of zero. Therefore, $\chi_t \leq 0$ for all feasible values of ν_t^* . Since $\chi_t \leq 0$, a necessary and sufficient condition for $\delta_{t-1} + \chi_t = 0$ is that $\delta_{t-1} \geq 0$. Given a value for ν_t^* , equation (20) yields a solution for $E_{t-1}\gamma_t$.

When $\delta_{t-1} < 0$ there is no solution for ν_t^* , and the expectation in equation (20) is taken over the full range $\{-\bar{\nu}, \bar{\nu}\}$ to yield

$$E_{t-1}\gamma_t = -\frac{1+i}{\alpha+i}\delta_{t-1} + E_{t-1}\gamma_t,$$

an impossibility. Therefore, there is no solution for $E_{t-1}\gamma_t$ when $\delta_{t-1} < 0$.

7.1.1 Proof of Proposition 2

From Proposition 1, when $\delta_t < 0$, there is no equilibrium. Therefore, equilibrium dynamics must be restricted to assure $\delta_t \geq 0$.

Policy switching when $\delta_t < 0$ restores equilibrium with fiscal solvency by setting $\Omega_t = 0$, placing the system on the saddlepath associated with the post-crisis policy mix. If $\tilde{\gamma}_t \geq 0$, then $\gamma_t = \tilde{\gamma}_t$, sets $\Omega_t = 0$. If $\tilde{\gamma}_t < 0$, but $\delta_t < 0$, then $\hat{s}' < \hat{s}$, sets $\Omega_t = 0$ in equation (19).

7.1.2 Proof of Proposition 3

The state variable determining the distance to the adjustment path leading to \hat{s} can be expressed as $\delta_{t-1} = (1-\alpha)s_{t-1} - \alpha b_{t-1} + \frac{1-(1-\alpha)(1+i)}{i}\hat{s}$, and the distance is given by $\Omega_t = \frac{\delta_{t-1} + \nu_t + \alpha(E_{t-1}\gamma_t - \gamma_t)}{1-(1-\alpha)(1+i)}$ (Daniel and Shiamptanis 2010). After devaluation to place the system on the \hat{s} -adjustment path, $\delta_t = 0$, and the probability of devaluation next period is unity by Proposition 2. Expectations of devaluation ($E_t\gamma_{t+1}$) are at least as high as $\frac{\bar{\nu}}{\alpha}$, implying high interest rates. This is because the shadow rate of devaluation is given by $\tilde{\gamma}_{t+1} = E_t\gamma_{t+1} + \frac{1}{\alpha}[\delta_t + \nu_t]$. With $\delta_t = 0$, $E_t\gamma_{t+1} \leq -\frac{\bar{\nu}}{\alpha}$. With probability one, the fixed rate fails in every subsequent period with devaluation.

Notes

¹Jeanne and Guscina (2006) document that a large fraction of emerging market debt is denominated in domestic currency in contrast to the "original sin" hypothesis.

²Uribe (2006) presents a fiscal theory model in which the role of devaluation is elimination of hyperinflation, not restoration of fiscal solvency. Cochrane (2003, 2005) notes that the FTPL can explain a currency crisis.

³The model is equivalent to one specified in terms of values as a fraction of GDP when the real interest rate is interpreted as the growth-adjusted real interest rate. We make this modification in the section where we apply the model to explain the Argentine 2001 crisis.

⁴Letting real foreign consumption be denoted by c_t^* , the Euler equations, using world bonds and domestic bonds, respectively, are $U'(c_t^*) = \beta E_t [(1+i) U'(c_{t+1}^*)]$ and $U'(c_t^*) = \beta E_t [(1+i_t) (S_t/S_{t+1}) U'(c_{t+1}^*)]$.

⁵We could allow some government bonds to be denominated in foreign currency with no substantive change to the analysis, as long as some bonds are denominated in domestic currency. Magnitudes would change with larger depreciation needed the smaller the fraction of domestic-currency debt in total debt.

⁶First, substitute for γ_t in equation (3) yielding $b_t = (1+i_{t-1})b_{t-1} - \gamma_t - s_t$. Then use interest rate parity to yield $(1+i_{t-1})b_{t-1} = [(1+i)b_{t-1}]/E_{t-1}(S_{t-1}/S_t)$, which implies $(1+i_{t-1})b_{t-1}E_{t-1}(S_{t-1}/S_t) = (1+i)b_{t-1}$. Noting that $(1+i_{t-1})b_{t-1}E_{t-1}(S_{t-1}/S_t) = E_{t-1}[(1+i_{t-1})b_{t-1}(S_{t-1}/S_t)] = (1+i_{t-1})b_{t-1} - E_{t-1}\gamma_t$, and substituting, the equation becomes $(1+i_{t-1})b_{t-1} - E_{t-1}\gamma_t = (1+i)b_{t-1}$. Solving for $(1+i_{t-1})b_{t-1}$ and substituting into

the first equation above yields the expression in the text.

⁷Woodford (1994) derives the government intertemporal budget constraint as an equilibrium condition for a closed economy.

⁸With output growth, these restrictions can be expressed with variables defined as a fraction of output and with the interest rate defined as the real growth-adjusted interest rate.

⁹By treating the surplus as determined by equation (6), we are ignoring the effect of capital gains or losses on seigniorage revenue $[(M_t/P_t)(i_t/(1+i_t))]$ under the assumption that the fiscal authority can adjust the surplus to offset these. We are also assuming that the government chooses real expenditures and taxes.

¹⁰This requires that the change in the surplus respond to debt service with the negative of its response to the lagged surplus, such that the surplus is no longer changing when the surplus equals debt service.

¹¹The unit root in debt implies that although the probability of unsustainability is less than one in finite time, it equals one in infinite time. Therefore, all discussion about the probability of fiscal unsustainability being less than one should be interpreted as within finite time.

¹²A government could feasibly raise more taxes, but they might choose not to do so.

¹³One root of the dynamic model is $1+i$ and the other is $1-\alpha$.

¹⁴This assures that $E_{t-1}\gamma_t = 0$ along the saddlepath as s approaches \hat{s} from the left.

¹⁵From equation (11), the increase in α reduces the slope of the saddlepath and increases

the intercept under that assumption that $\hat{s} > 0$. Under the assumption that the system begins to the left of \hat{s} below the saddlepath, this reduces the vertical distance between the saddlepath and the value of b_{t-1} .

¹⁶This does not rule out a crisis with $\tilde{\gamma}_t < 0$. Since such a crisis would not entail depreciation, equation (15) below is accurate.

¹⁷See equation (21) in the appendix.

¹⁸Use equations (4), (12), and (13), with $\gamma_t = 0$ to yield

$$\delta_t = -(\alpha + i) E_{t-1} \gamma_t + (1 + i) (\delta_{t-1} + \nu_t) - \alpha (\hat{s} - ib_t).$$

Then substitute equation (14) for $\tilde{\gamma}_t$.

¹⁹However, since the slopes are not very different for reasonable values of i and α , the region in which a crisis without depreciation could occur is small. In simulations, depreciation is required over 95% of the time.

²⁰It is possible to consider alternative post-crisis inflation targets with different implications for sterilization as in Daniel (2001b).

²¹In the model, the current increase in expected future spending is totally unanticipated.

²²For an analysis of default as a response to the crisis, see Daniel and Shiamptanis (2010). The interest rate parity equation must be modified such that the world interest rate is equated with the expected return on the domestic asset, conditional on the possibility of default.

²³The event was a run on private sector deposits of more than US\$ 3.6 billion (6 percent of the deposit base) during November 28-30 (IMF 2003, p. 61).

²⁴Data is from Krueger (2002, pp.2-3) for 1991 and from IMF (2003, p.9) for other years.

²⁵Data is from IFS.

²⁶Krueger (2002) argues that the upper bound on debt for a country like Argentina, which has difficulty raising taxes, is much lower than the upper bound for an industrialized country.

²⁷We use the residuals from the estimate of equation (6) to calibrate the upper bound on fiscal shocks, $\bar{\nu}$. When we estimate the fiscal rule, using only the last three years of data, the residual in 2001 is -0.0188.

²⁸The equation is estimated without a constant, as required by equation (6), when the mean of surplus shocks is zero. The estimate for α is statistically significant at the 1% level, and the R^2 is .39, not too bad for an equation in first differences with 7 observations! The addition of a constant raises the standard error of the equation, and the constant is not significant at any conventional level.

²⁹This is the OLS regression estimate for α in equation (6) using three years of data from 1999-2001. The argument for a fall in α is further supported by an estimate of 0.48 over the full sample (1991-2001), compared with 0.61, for the first eight years (1991-1998).

³⁰The IMF's post-crisis assessment (2003) shows reduced confidence in the government's ability to generate taxes.

Figure 1: Passive Fiscal Policy

Figure 2: Active Fiscal Policy

Figure 3: Switching Regime

Figure 4: Primary Surplus and Debt

Figure 5: Prime Lending Rate

Figure 6: Primary Surplus Adjustment

Figure 1

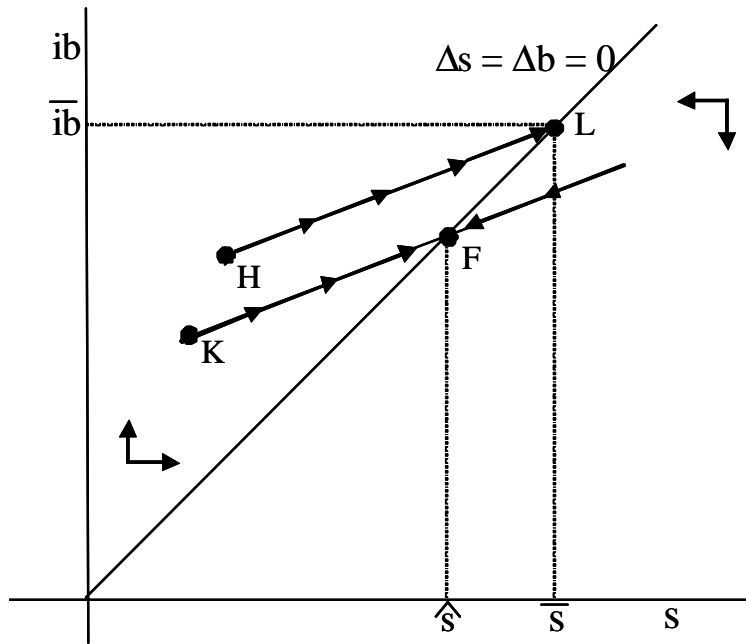


Figure 2

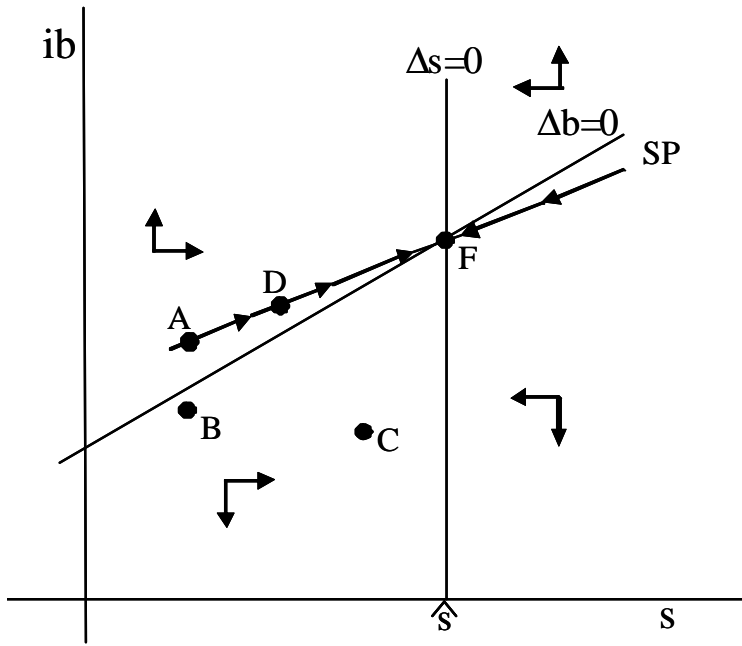


Figure 3

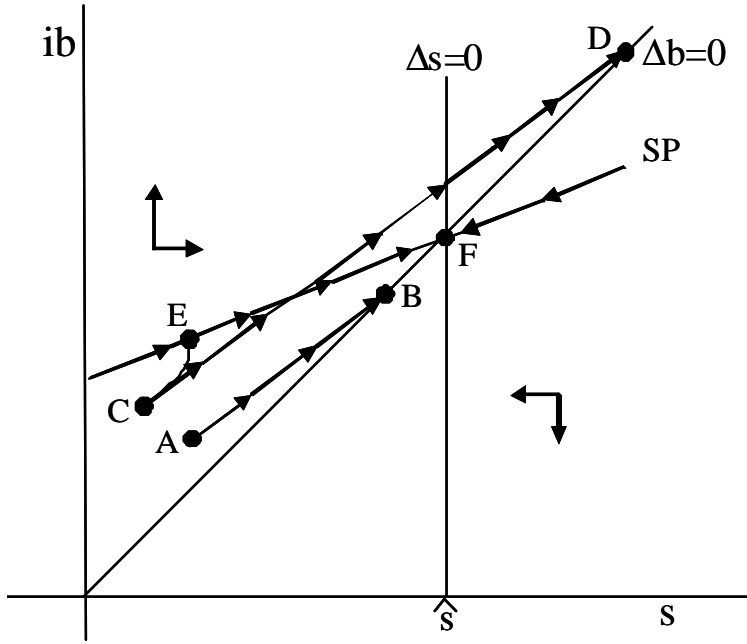


Figure 4

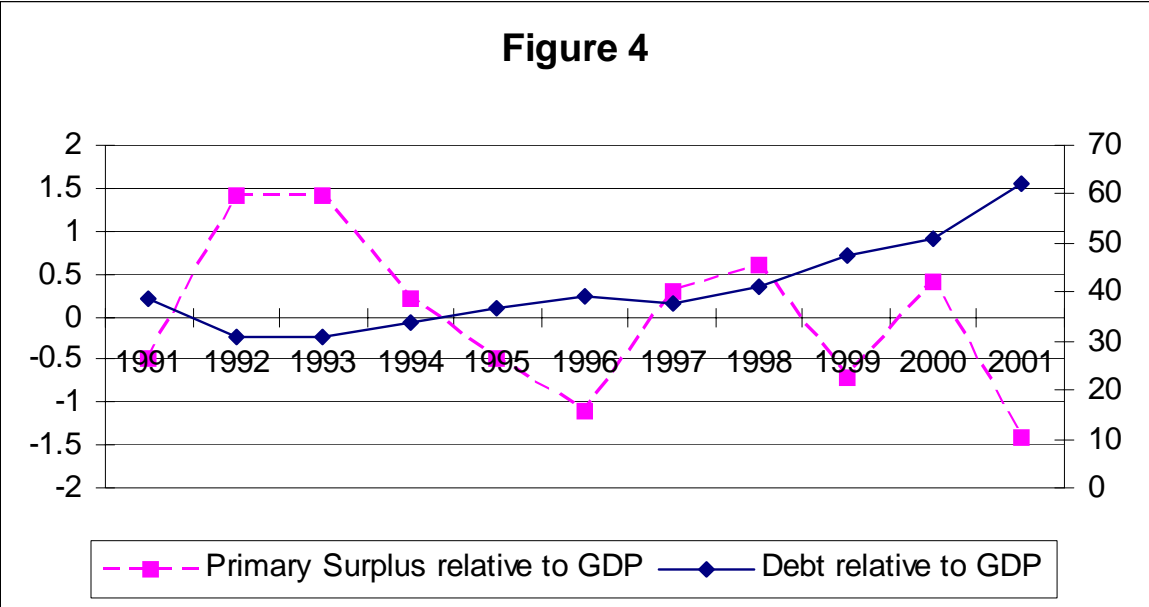


Figure 5

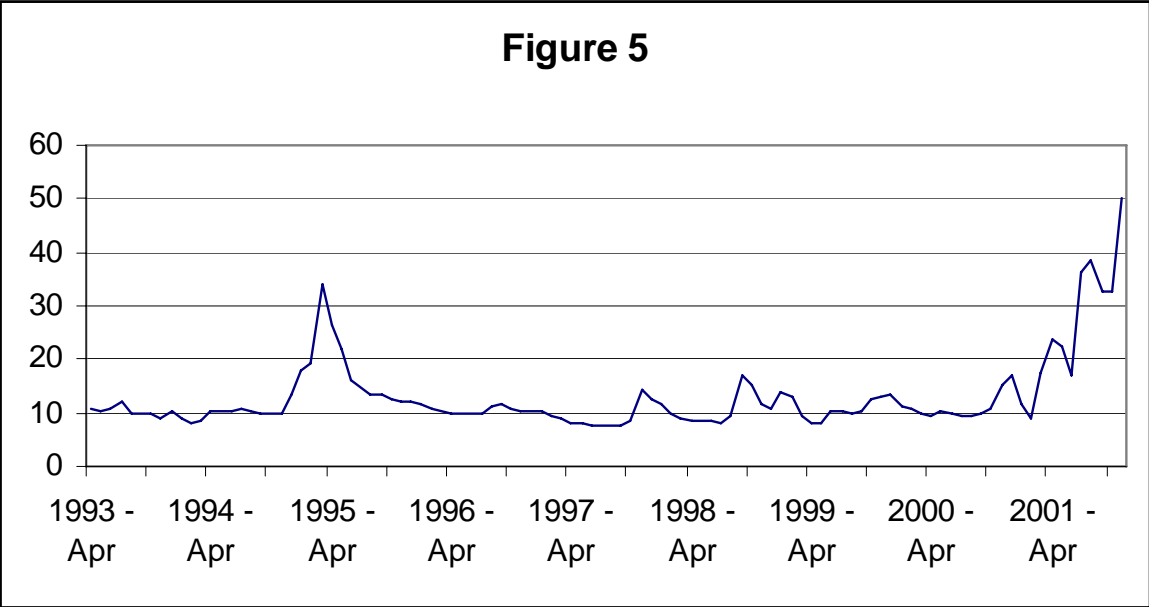


Figure 6

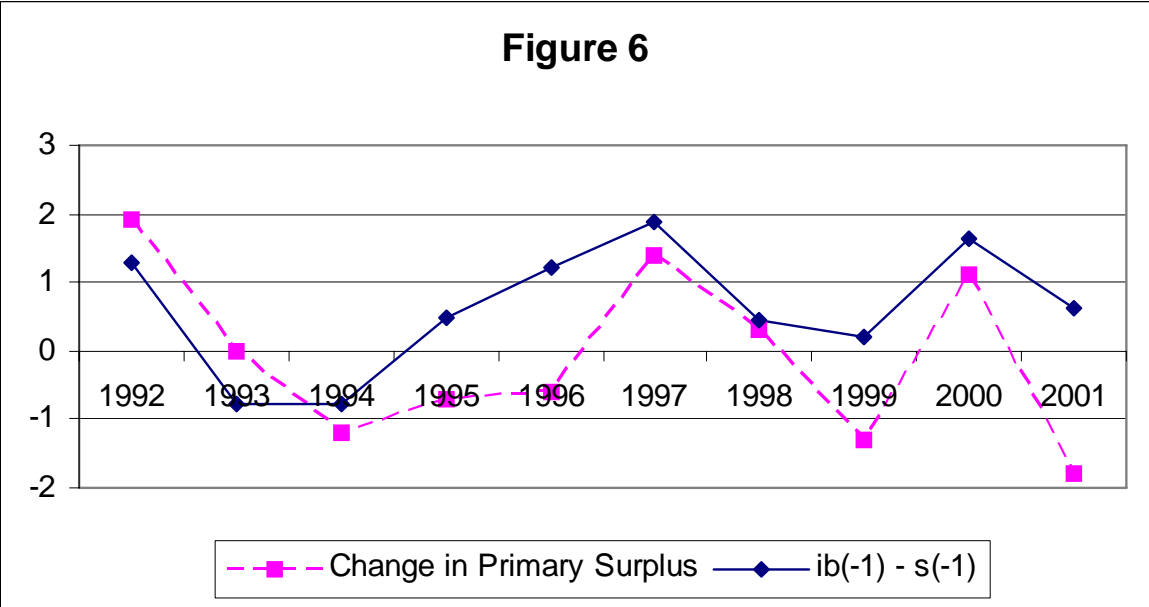


Table 1

Simulation Algorithm: Probability of a Crisis in Twenty Years

Compute δ_{t-1} using initial values for the debt and surplus and equation (13).
Compute $E_{t-1}\gamma_t$, based on equation (21) in the appendix.
Draw a fiscal shock, ν_t , from the uniform distribution.
Calculate γ_t using equation (15).
If $\gamma_t \leq 0$, update the surplus and debt using equations (6) and (4), and update δ_t .
If $\gamma_t > 0$ or $\delta_t < 0$ a crisis is called and the simulation ends.
If not, repeat up to twenty periods.
Repeat the twenty-year simulation 5000 times.

Table 2

Crisis Simulations: Argentina

		b	s	α	\hat{b}	Pr Crisis (#yrs)	Mean time to Crisis
Baseline	1991	38.5	-0.5	0.61	75.0	0 (20)	–
Negative shocks	2000	50.9	0.4	0.61	75.0	0.6% (20)	17.1
Confidence	1998	40.9	0.6	0.13	62.5	22.4% (3)	2.0
Both	2000	50.9	0.4	0.13	62.5	62.5% (3)	0.9

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