Public Default Dynamics *

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Abstract

This paper provides a dynamic model of a public debt crisis in which fiscal insolvency is the cause. The insolvency is created by a combination of a passive fiscal policy with an upper bound on the long-run budget surplus, yielding risky government debt, and a string of bad luck. As debt rises and a crisis approaches, the interest rate rises, due to a risk premium on debt in anticipation of possible default. A position, in which the contractual value of debt under passive fiscal policy exceeds the maximum expected present-value surplus, is infeasible. Should passive fiscal policy require such a point, a crisis occurs. Crisis resolution includes a promise of fiscal reform, with a switch to active fiscal policy as in the Fiscal Theory of the Price Level (FTPL). The switch creates larger near-term surpluses, which serve to raise the expected present-value surplus, and greater exchange rate flexibility, which eliminates the default premium on debt. If the present-value surplus is still too low relative to debt after the policy switch, default is necessary.

Key Words: *Default, Financial Crisis, Currency Crisis, Debt Crisis, Regime Switching, Fiscal Theory of the Price Level*

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1 Introduction

What causes a national government to default on its liabilities? How are the causes of public default related to the causes of an exchange rate crisis? Default and devaluation often occur together (Reinhart 2002); both can reduce government liabilities. Outright default reduces government interest-bearing liabilities, while devaluation reduces the real value of liabilities denominated in domestic currency.[1] Could default and devaluation share a common cause related to the need to restore fiscal solvency through an equilibrium reduction in government liabilities? In this paper, we show how an initially sustainable fiscal policy with stochastic shocks could reach a path in which agents expect that the government cannot service its debt. Agents refuse to lend, creating a public debt crisis. Fiscal insolvency is the cause of the crisis, and crisis resolution must restore solvency. Currency devalution and/or outright default can be necessary to restore solvency.

To address the relationship between currency and debt crises, it is useful to draw parallels between the literatures on public default and currency crises. The first models of public default focused on a sovereign's default to foreign agents and asked why a sovereign would ever choose to repay. In this literature, surveyed by Eaton and Fernandez (1995), a sovereign compares the benefits of default with the punishments; the sovereign chooses default whenever debt becomes large enough such that benefits exceed the costs. The parallel currency crisis model is the second generation model, introduced by Obstfeld (1994). In this model, the government compares the stimulatory benefits of devaluation with the reputational costs

[1] Devaluation is accompanied by higher domestic prices under purchasing power parity.

and chooses devaluation only when the benefits dominate. Since a sovereign would choose default only when fundamentals become weak, these crises are anticipated and they can be self-fulfilling. For this set of models, the benefits of default and devaluation are inherently different, implying that default and devaluation do not share a common cause. The benefits of default are a reduction in the government's external liabilities, while the benefits of devaluation are modeled as the demand stimulus created by a reduction in the relative price of domestic goods. Additionally, both models face empirical challenges. The default models imply a debt ceiling relative to GDP, supported by the punishments to default, which is empirically too small to support the large quantity of debt owed by many sovereigns.[2] The devaluation model is consistent with the European currency crises in the early 1990's, but in more recent crises, devaluation does not always appear to be expansionary, as required by the model.

Another set of default models includes those with multiple equilibria in which crises are the consequence of illiquidity. Default is a response to sudden stops of credit inflows based on expectations about the probability of future credit inflows. Cole and Kehoe (1996, 2000) develop a sudden stop model in which agents, expecting others not to lend in the future, choose not to lend today, creating capital flight and a self-fulfilling default.[3] On the currency crisis side, these models are labeled financial fragility or third-generation models. A private-sector bank run can create a run on the central bank's reserves when the bank's deposits are denominated in foreign currency and the central bank serves as a lender of last resort (Chang and Velasco 1999). Crises develop suddenly and are basically unanticipated. These models

[2] See, for example, Arellano and Heathcote (2004), who find that credit market exclusion can sustain debt ceilings of between one and three percent of GDP.

[3] Chamon (2005) shows that it is possible to design contracts to eliminate these multiple equilibrium when the debtor is the sovereign.

are offered as explanations for crises like those in Southeast Asia in 1997, which developed suddenly. Devaluation and/or default in these models is the response to illiquidity in one of the multiple equilibria and not to a fundamentals-based insolvency.

The most influential currency crisis model is the first-generation model, introduced by Krugman (1979) and extended by Flood and Garber (1984). In this seminal model, a central bank gives up its exchange rate peg only once reserves are depleted such that it has no alternative. This is a powerful and simple model. The government finances a fiscal deficit using money. Coupled with a monetary policy of fixing the exchange rate and an equation for asset market equilibrium, this policy yields a negative trend for reserves, culminating in a speculative attack in which reserves hit a floor. These crises are due to the reduction in reserves over time as a result of a fundamentally unsustainable policy mix, and they are therefore anticipated. The focus of the model is on the timing and nature of the speculative attack in which reserves are exhausted. However, the crisis which occurs at the reserve floor is a solvency crisis. That is, the government cannot[4] reduce its need for financing and cannot borrow to replace reserves. Although the fiscal solvency issues are not explicitly worked out in the original models, adopting a flexible exchange rate allows additional seigniorage revenue, and a devaluation, if there is one, reduces the value of the government's outstanding liabilities.[5] Therefore, generation-one currency crises are due to sustained fiscal solvency problems, which the abandonment of the fixed exchange rate resolves.

Fiscal insolvency could also create a public debt crisis. As a reflection of this, empirical

[4] Perhaps this should be "has chosen not to", but the process of fiscal choice is not modeled.
[5] Daniel (2001) works out the solvency requirements for alternative choices by the monetary authority of the timing of the exchange rate collapse. Burnside, Eichenbaum, and Rebelo (2006) argue that currency devaluation is accompanied by devaluation-related sources of revenue including debt-devaluation and an increase in the real value of the government surplus when expenditures are concentrated on tradeables and taxes on non-tradeables with sticky prices.

assessment of the probability of a debt crisis is often solvency based.[6] However, straightforward generalization of first generation currency crisis models to debt crisis models raises interesting questions. Consider the following scenario. Assume the small open economy begins in a long-run equilibrium in which tax plus seigniorage revenue equals government expenditure plus interest on debt, so that debt is not growing. An exogenous and persistent negative surplus shock triggers the growth of debt. The surplus remains at its lower level and debt accumulates until debt reaches an upper bound, analogous to the lower bound on reserves in the currency crisis model.[7] Once debt reaches its upper bound, the primary surplus increases, returning the actual surplus to zero, such that tax plus seigniorage revenue equals government spending plus interest on the debt. In the currency crisis literature, the government budget is rebalanced completely with an increase in seigniorage revenue. What happens next? What if the economy is stochastic and additional shocks occur after the resolution of the initial crisis? Government debt is at its maximum level, determined, presumably, by the ability to raise distortionary taxes and seigniorage revenue to service that debt. How does the government respond to a negative surplus shock when it cannot raise the revenue to service additional debt necessitated by the subsequent negative surplus shock?

Generation-one currency crisis models assume that fiscal policy is passive, that is, for any level of outstanding debt, the government will eventually raise enough seigniorage revenue to balance the intertemporal budget, even if this additional revenue must be postponed by the

[6] For example, Garcia and Rigobon (2004) assume that a debt crisis will be triggered once debt reaches a critical level. They use this assumption to assess the probability of a debt crisis in Brazil by estimating and simulating whether debt will breech a ceiling, determined by solvency criteria.

[7] The lower bound on reserves is binding only if there is an upper bound on debt preventing the government from borrowing to replinish reserves.

commitment to maintain a fixed exchange rate as long as possible. This is fundamentally a one shock model where the only shock occurs in the initial period. The increase in the surplus, created by the increase in seigniorage revenue on the date of the crisis, is just sufficient to service the maximum level of debt, solving the solvency problem for this shock. However, when we generalize the model and allow the possibility of a negative surplus shock after debt has reached its maximum, we must recognize that the government cannot continue its policy of adjusting the surplus whenever debt rises to its upper bound. This is because when debt is at its upper bound, the surplus adjustments necessary to service debt beyond this bound are infeasible. Fiscal policy cannot be unconditionally passive in the face of stochastic shocks. And in equilibrium, the resolution of the crisis created by the first shock must anticipate the possibility of future shocks and the government's response to them.

This paper considers the creation and resolution of a public debt crisis in a stochastic environment when the government chooses to follow a passive fiscal policy for as long as feasible. A government chooses passive fiscal policy when choosing a fixed exchange rate or when choosing to let the price level be determined by monetary policy instead of by fiscal policy (active monetary policy and passive fiscal policy). This choice could be based on the reduction in transactions costs associated with fixed exchange rates or on a comparison of the benefits of the more stable price level, when price stability is the responsibility of the monetary authority, compared with the costs associated with financing stochastic government expenditure using distortionary taxes. However, when taxes are distortionary, there will be an upper bound on the size of the surplus.[8] Debt cannot grow so large that the

[8] The upper bound could be a maximum feasible value determined by something like a Laffer curve or could be determined by optimal tax and spending considerations.

surplus necessary to service it exceeds this upper bound. A debt crisis occurs when agents refuse to lend to the government, conditional on the current value of outstanding debt and the maintenance of current policy. The dynamics and timing of the crisis will depend on expectations about how the government plans to resolve the crisis.

The paper is organized as follows. Section 2 considers alternative government responses to a financial crisis in which it cannot borrow to continue with the current passive fiscal policy. Section 3 characterizes the behavior of fiscal and monetary policy in the small-open economy. Section 4 describes dynamics under pure passive and active fiscal policies without an upper bound. Section 5 presents the regime switching model of default, in which fiscal reform is triggered by expectations of fiscal insolvency. Simulations of the model with alternative values for the policy and other parameters demonstrate how policy can be designed to yield default with greater or lesser frequency, and show how the magnitude of default is related to policy choices. Section 6 contains conclusion.

2 Fiscal Response to a Crisis

The purpose of this section is to consider alternative fiscal responses to a public debt crisis, thereby motivating the particular fiscal response to a financial crisis that we model. Consider the government's options when faced with the inability to borrow to increase the level of debt under current policy. One option is default. For purposes of this paper, we define default broadly as reneging on contractual obligations, so that debt restructuring to reduce the value of outstanding liabilities and devaluation of a fixed exchange rate are both forms of default; depreciation of a flexible exchange rate is not. Another option is fiscal reform whereby the surplus and/or the fiscal policy rule is adjusted.

To consider crisis dynamics, assume that the government begins in a long-run equilibrium in which the debt and surplus are well below the upper bound. A negative surplus stock stimulates the growth of debt. There is an upper bound on the size of the long-run value of the surplus and therefore on the level of debt which the surplus can service. We assume that the government can commit to its plans so that agents know how the government plans to behave in the event of a crisis.

A straightforward extension of generation-one currency crisis models offers a particular fiscal reform as the response to a crisis. The fiscal authority allows the negative surplus shock to remain in place until debt hits a maximum, and then raises the surplus to its maximum level, yielding seigniorage revenue to service the debt. This policy is consistent with equilibrium when there is no possibility of another fiscal shock, as in the deterministic generation-one currency crisis model. And it predicts that a crisis will occur with debt at its maximum, while actual crises seems to occur with debt much less than its sustainable maximum.[9]

When fiscal policy is stochastic, a negative surplus shock could occur in the period after the crisis. If the country experiences a negative surplus shock with debt at its maximum, determined by the maximum surplus, then the surplus cannot be raised further. Continuing truly passive policy is impossible. Intertemporal government budget balance can be restored only with a reduction in the real value of debt, that is, with default. Agents know this and they anticipate default in the post-crisis period, requiring an interest rate premium. Therefore, the equilibrium upper bound on debt will be lower to allow the maximum surplus

[9] In answer to the question regarding why financial crises occur at levels of debt deemed sustainable, Reinhart et al (2004, 2005) dispute the notion that relatively low levels of debt are sustainable. They note that countries which default repeatedly have demonstrated that they cannot sustain high levels of debt. At the end of the paper, we offer an alternative explanation for repeated default.

to service debt inclusive of the interest rate premium, implying that a crisis occurs at a level of debt which would be sustainable in non-crisis times. Additionally, the equilibrium probability of a crisis in the next period equals the probability of a negative surplus shock. And if the government plans to set the magnitude of default, conditional on a negative surplus shock, high enough to substantially reduce debt in order to avoid a succession of near-term crises, then expected default is large and the interest premium is high.

Therefore, this particular fiscal reform allows the government to avoid default in the period in which the crisis breaks, but at the cost of a continuing high probability of large default and a high interest premium after the initial crisis. And the gereration-one-type fiscal policy raises the issue of why the government would wait until a crisis to make the fiscal adjustment promised under passive policy (Rebelo and Vegh 2002). In contrast, passive fiscal policy rules are usually specified to allow the surplus to adjust continuously to the debt instead of requiring that debt hit some threshold before triggering adjustment. This suggests that generation-one type fiscal reform might not be the best response to a public debt crisis.

Empirically, promises of fiscal reform are often part of crisis resolution,[10] implying that it is interesting to explore alternative fiscal reform. Assume that the government initially follows a passive fiscal policy in which it adjusts the surplus to debt over time without waiting for debt to hit a threshold. Even with some adjustment, a string of negative surplus shocks could send debt toward its upper bound. Assume that the government's reaction to a crisis is a promise to switch from passive fiscal policy to active fiscal policy, under

[10]For example, Argentina (2001) and Brazil (2002) both promised fiscal reform and adopted surplus targets. The IMF often advises fiscal reform as part of crisis resolution.

which stochastic surplus shocks cause price level jumps, which rebalance the government's intertemporal budget constraint by revaluing outstanding nominal debt.[11] Since the price level must be free to adjust to surplus shocks, the policy switch also requires adopting flexible exchange rates as in the generation-one currency crisis model.[12] The policy switch implies that after crisis resolution, the expectation of future default and of capital gains and losses on government debt is zero, eliminating the interest rate premium. Therefore, this type of fiscal reform requires that the government accept letting fiscal shocks affect the price level and exchange rate, but it eliminates the interest rate premium together with the possibility of future default. Additionally, when the fiscal surplus has persistence, as required to match the data, the policy switch raises the present value of expected future surpluses, possibly allowing the government to avoid default as a resolution to the current crisis. And when default is necessary to restore an equilibrium after a crisis, its magnitude is smaller than it would be in the absence of a regime switch.[13]

Another option for crisis resolution is default without fiscal reform. Conditional on a debt crisis, the government could plan to reduce its debt obligations through default, thereby restoring intertemporal budget balance. To avoid a succession of near-term crises, the default must be large enough to reduce the value of debt so much that the probability of future near-term crises under current policy is low. We show below that not only does this policy response require large defaults, compared with the switch to active policy, but it also limits the ability of the government to maintain passive policy over time.

[11] Capital losses on government debt under flexible exchange rates do not involve broken contracts and are not examples of default.

[12] However, using unexpected capital gains and losses on government debt to balance the government's intertemporal budget is different from using an increase in systematic seigniorage revenue as in the generation-one currency crisis model.

[13] This is partly because to avoid a succession of near-term crises under passive policy, default must achieve a substantial reduction in debt.

In the model that follows, we assume that the government initially follows passive fiscal policy adjusting the surplus to debt, and that the surplus is subject to stochastic shocks. Monetary policy is active, fixing the exchange rate. In the event of a debt crisis, in which the government finds itself unable to borrow to continue the current passive policy, the government promises fiscal reform in the form of a switch from passive to active fiscal policy. The expectation of this response has significant implications for the dynamics leading up to the crisis.

Although the paper is not set up to evaluate the optimality of policy, it is useful to consider whether a government might reasonably choose such a policy. First, it requires that the government prefer passive fiscal policy, at least while debt is low, and that it not wait for a crisis to make surplus adjustments to shocks which affect the level of debt. The controversy over the "Fiscal Theory of the Price Level", in which many question not only the optimality of active fiscal policy but also its feasibility implies that passive policy is a standard choice. Chari, Christiano, and Kehoe (1991) report that governments typically adjust distortionary taxes to spending shocks, as required by passive policy. Additionally, passive fiscal policy is necessary if exchange rates are fixed. Second, although the government is assumed to adjust surpluses without waiting for a crisis, it does wait for a crisis to institute fiscal reform. It seems reasonable that political support for major fiscal reform might require a crisis. Additionally, the cost of active policy in terms of price stability are smaller, once debt has become large, as long as a significant fraction of debt is in nominal terms.[14] This implies that the government would become less adverse to active fiscal policy as debt grows. Finally, the government must prefer fiscal reform with the accompanying minimization of

[14] Kumhoff (2004) shows that a significant fraction of government debt is denominated in domestic-currency.

the size of the default to no fiscal reform with a much larger default. Governments do appear to be adverse to default. The promise of fiscal reform allows the government to maintain passive fiscal policy longer and to avoid the large interest premium associated with the lack of reform, but does imply active fiscal policy following a crisis. Empirically, fiscal reform and exchange rate flexibility are often advocated by the IMF, most recently in Argentina and Brazil, as appropriate response to a public debt crisis. Thus, the policy we suggest does seem plausibly desirable, even though we cannot assure that it is optimal.[15]

3 Small Open Economy Model

The setting for the crisis is a small open-economy model. We assume both purchasing power parity with one good and interest rate parity, allowing the equilibrium prices for domestic goods and assets to be determined on world markets. We assume that the rest of the world operates a passive fiscal policy and that monetary policy fixes the world price level. The foreign nominal interest rate is also fixed. These simplifications eliminate the need to present a full general equilibrium model of an economy.[16] To characterize crisis dynamics in a rational expectations equilibrium, we need a policy equation for the fiscal surplus, conditional on values for debt, on the willingness of agents to lend to the government, and on past policy. The fiscal policy determining the surplus, together with equilibrium interest rates determined

[15]The optimality of active versus passive fiscal policy in the absence of regime switching is a subject for debate. Woodford (1998b) and Sims (1994) show that debt revaluation through unexpected price level changes can be part of an optimal tax program. Chari, Christiano and Kehoe (1991) agree that an optimal tax plan includes stochastic real returns, generated by price level surprises, as part of an optimal tax plan. Kumhof (2004) and Kumhof and Tanner (2005) argue that this ignores the effects of debt devaulation on the banking sector, giving unanticipated inflation costs. In spite of much literature favoring price surprises as a way to finance stochastic government spending, Chari et al (1991) report that distortionary tax adjustment is usually the chosen method.

[16]The model of fiscal and monetary policy could be embedded in a fully-specified general equilibrium model with no significant changes.

by interest rate parity and monetary policy, govern the evolution of government debt.

3.1 Fiscal Surplus

Following a substantial body of work on monetary and fiscal policy, we assume that the fiscal authority can commit to a policy feedback-rule which is subject to stochastic shocks.[17] In the literature, a typical surplus rule includes business cycle indicators like the output gap to allow counter-cyclical fiscal policy, measures of transitory changes in spending to allow Barro-type optimal accumulation of debt (1979), lagged debt, and, when empirical relevance matters, a lagged value of the surplus to allow persistence. For purposes of this paper, we need a surplus rule, which nests active and passive fiscal policy based on parameter values,[18] and is characterized by persistence and stochastic shocks. Since we are not specifying a full general equilibrium model of the economy, other terms are relegated to the error. Therefore, the error term includes both politically-determined shocks to taxes or government spending and responses, perhaps countercyclical or optimal tax, of the fiscal authority to the fundamental shocks that face the economy. We assume that the fiscal authority cannot do anything to offset the effect of current fiscal shocks on the current value of the surplus. However, the fiscal policy rule is designed to adjust the surplus over time.

A fiscal rule with surplus persistence, which nests both active and passive fiscal policy, is given by:

$$s_t = (1-\alpha)\, s_{t-1} + \alpha \left[(1-\lambda)\, \hat{s} + \lambda \frac{i}{1+i} d_{t-1} \right] + \eta_t \qquad \frac{i}{1+i} < \alpha < 1 \qquad 0 \leq \lambda, \qquad 0 < \hat{s} \leq \bar{s} \tag{1}$$

[17]This work includes papers on fiscal sustainability (Bohn 2004), work determining the nature of fiscal policy (Canzoneri, Cumby, and Diba (2001), work on the implications of policy-switching (Davig, Leeper, and Chung 2003), as well as a very large literature on monetary policy rules including the Taylor rule.
[18]For examples, see Leeper (1991), Canzoneri, Cumby and Diba (2001), and Davig, Leeper, and Chung (2003).

where s_t is the value of the real primary surplus, $\hat{s}$ is the value for the target surplus, where i denotes the world's risk-free interest rate, d_t is the real value of debt at the end of the period inclusive of interest, and η_t is a bounded, zero-mean, independently and identically distributed, stochastic disturbance representing fiscal shocks. Note that the surplus rule could be stated in a more general way with the surplus depending on a constant, its on lagged value, and lagged debt. The expression in (1) reveals the nature of a stationary long-run equilibrium, which, for any admissible values of α and λ requires that the surplus exactly pay interest on debt [19] $\left(s = \frac{i}{1+i}d\right)$, and for $\lambda \neq 1$, that the surplus equal its target value $(s = \hat{s})$.

The upper bound on a target for the surplus $(\bar{s})$ is based on the assumption that the surplus cannot be indefinitely large over a long period of time. We assume explicitly that $\bar{s}$ is the upper bound on the expected long-run value for the surplus. The dependence of the surplus on its on lag could reflect either persistence in policy, whereby surplus changes are gradual, or autocorrelated fiscal shocks. Catao and Terrones (2005) find substantial persistence in the surplus for a large sample of advanced and emerging-market countries. A small value for α reflects a large degree of persistence in the surplus. Values for λ and $\hat{s}$ are policy choices, and they can differ across regimes. The surplus is assumed to adjust to a target variable given by a linear combination of $\hat{s}$ and the previous period's level of debt service, with the weights determined by λ. The value for λ determines whether the fiscal rule is active or passive, as shown below.

[19] Recall that d is interest inclusive such that $\frac{d}{1+i}$ is interest-exclusive debt.

3.2 Government Debt and Interest Rate Parity

Equation (1) is a dynamic equation for the surplus in the lagged surplus and the level of debt. To complete the model, we need an equation describing the equilibrium evolution of government debt. In order to make the point that default can occur as currency devaluation or as outright default on interest-bearing debt, we include currency (m_t), as well as one-period domestic-currency bonds (b_t) and foreign-currency bonds (b_t^*), as government debt instruments. Under this assumption, the real value of interest-inclusive debt is defined as:[20]

$$d_t = (1 + i_t)\, b_t + (1 + i_t^*)\, b_t^* + m_t.$$

We assume that the domestic country is a small open economy in which equilibrium values of both interest rates are determined in world markets according to interest rate parity. Under the assumption that the foreign risk-free interest rate (i) is constant, the equilibrium interest rate on domestic-currency government bonds is determined by interest rate parity as

$$\frac{1}{1+i_t} = \left(\frac{1}{1+i}\right) E_t \left[\frac{\delta_{t+1}^h}{1+\pi_{t+1}}\right], \tag{2}$$

where δ_{t+1}^h is the fraction of the domestic debt in nominal terms actually repaid in period $t+1$, and $1 + \pi_{t+1} = \frac{P_{t+1}}{P_t}$, where P_t is the domestic price of the single world good. With purchasing power parity and the assumption that the world price is fixed at unity, P_t is also the exchange rate. Note that the equilibrium interest rate must rise above the risk-free rate whenever there is any possibility that the government might default on its debt, either by refusing to honor its face value $\left(\delta_{t+1}^h < 1\right)$ or by devaluing and allowing inflation

[20] The term structure of debt is not important in this analysis since we determine overall default, and not its allocation among assets. We could include longer-term debt leaving fundamental points unchanged. Long-term debt would affect the timing of the crisis as in Cochrane (2001) and Daniel (2001b).

$\left(\frac{1}{1+\pi_{t+1}} < 1\right)$.

The contractual interest rate on foreign-currency debt, determined by interest rate parity, is given by

$$\frac{1}{1+i_t^*} = \left(\frac{1}{1+i}\right) E_t\left[\delta_{t+1}^f\right],$$

when foreign-currency debt is positive, where δ_{t+1}^f is the fraction of foreign-currency debt repaid, and by $i_t^* = i$ when foreign-currency debt is negative.

Default on can occur with exchange rate devaluation ($\pi_{t+1} > 0$), which, given PPP, reduces the real value of money and domestic-currency bonds,[21] or with debt restructuring (outright default with $\delta_{t+1}^h < 1$, and/or $\delta_{t+1}^f < 1$) whereby the contractual values of interest-bearing obligations are reduced. Under interest rate parity, expectations of default raise equilibrium interest rates.

To determine how to apportion default over various instruments, the fiscal authority would compare costs versus benefits across instruments. When net benefits are relatively similar, but rising in the magnitude of default, the fiscal authority would choose similar default rates across all instruments, implying that debt restructuring and devaluation would occur together. Alternatively, if net benefits for a particular instrument were low, then the model could yield either a pure debt crisis or a pure currency crisis. A sovereign debt crisis would occur when net benefits from defaulting on foreign-currency (often foreign-owned) debt is high. This model focuses on the determinants of the need for default and not on how liability reduction is spread across assets. Therefore, given an assumption below regarding

[21] It is not conventional to define a reduction in the real value of government liabilities due to a price level increase as default. However, given the definition of default here, as reneging on a contractual agreement, devaluation of a fixed exchange rate with the accompanying reduction in the real value of domestic-currency liabilities under PPP, is a default.

how default is expected to be apportioned across debt instruments, there is no further role for the multiple-dimensional nature of debt in this paper.[22]

We assume that the government can borrow any amount it wants on international markets as long as it is fiscally solvent, a term defined below. Therefore, government debt evolves as:

$$d_t = (1+i_t)\left[\frac{\delta_t}{1+\pi_t}d_{t-1} - s_t\right], \tag{3}$$

where δ_t is the fraction of nominal debt that will be repaid.[23] Define

$$\gamma_t = \left(\frac{\delta_t}{1+\pi_t} - 1\right) d_{t-1},$$

such that $\gamma_t < 0$ denotes a capital loss on the outstanding stock of debt.

Substituting into equation (3), assuming that any capital gain or loss is expected to be apportioned equally across assets,[24] and imposing interest rate parity from equation (2) yields:

$$d_t = -E_t\gamma_{t+1} + (1+i)\left[d_{t-1} + \gamma_t - s_t\right] \tag{4}$$

as a linear dynamic equation for real end-of-period debt. Note that debt is increasing in the expectation of capital loss due to the interest risk-premium created by the expectation $\left(-E_t\gamma_{t+1}\right)$.

[22]Sturzenegger and Zettlemeyer (2005) report that among public debt restructurings occuring between 1998 and 2005, that debt instruments are not necessarily treated equally, but that there is no tendency to favor those owned by domestic residents.

[23]Given the definition of debt above, this requires that the surplus include $\frac{i_t}{1+i_t}m_t + \frac{i_t - i_t^*}{1+i_t}b_t^*$, that is, interest savings (or cost) when debt is not held as domestic one-period debt. The term with money is standard and is often called seigniorage. The term with foreign bonds is required because the interest rates on domestic and foreign bonds could differ. Additionally, $\frac{\delta_t}{1+\pi_t}$ is defined as a linear combination of capital losses on each type of asset and is given by

$$\frac{\delta_t}{1+\pi_t} = \frac{\delta_t^h\left(\frac{1+i_{t-1}}{1+\pi}\right)b_{t-1} + \delta_t^f\left(1+i_{t-1}^*\right)b_{t-1}^* + \frac{m_{t-1}}{1+\pi}}{d_{t-1}}.$$

[24]Some assumption about how capital gains and losses are expected to be distributed across assets is necessary in order to compare the expected capital loss (or gain) on domestic-currency debt with the expected capital loss on total government debt.

Solving forward, and imposing $\lim_{k\to\infty}\left(\frac{d_{t+k}-E_{t+k-1}\gamma_{t+k}}{(1+i)^k}\right) = 0$, yields the government's intertemporal budget constraint as:

$$d_{t-1} + \gamma_t = s_t + \sum_{j=1}^{\infty} \left(s_{t+j} + E_{t+j-1}\gamma_{t+j} - \gamma_{t+j}\right)\left(\frac{1}{1+i}\right)^j. \tag{5}$$

The intertemporal budget constraint states that the outstanding stock of debt, inclusive of current capital gains and losses, must equal the present value of government surpluses plus the present-value of revenue generated from future surprise revaluations and devaluations of government debt.[25] The FTPL with active fiscal policy views equation (5) as an equilibrium condition determining γ_t with $\delta_t = 1$ and the present-value surplus as exogenous, while a model with passive fiscal policy views it as a budget constraint, determining the present value-surplus with γ_t exogenously determined by monetary policy. Either way, the equation must hold in equilibrium. Therefore, we can take expectations and view the current value of outstanding debt, inclusive of current capital gains and losses $(d_{t-1} + \gamma_t)$, as equal to the expected present value of future surpluses. The government is fiscally solvent under passive fiscal policy, when the expectational equation holds with γ_t exogenously chosen by the monetary authority.

Finally, substituting for the current surplus, from equation (1) into equation (4), yields an expression for debt as a function of expected capital loss together with lagged debt and

[25] Since this equation must also hold in expectation, surprise changes in the value of government debt will occur simultaneously with surprise changes in the present-value surplus.

the lagged surplus according to:[26]

$$d_t = -E_t\gamma_{t+1} + (1+i)\left[\left(1-\alpha\lambda\left(\frac{i}{1+i}\right)\right)d_{t-1} + \gamma_t - (1-\alpha)\,s_{t-1} - \alpha\,(1-\lambda)\,\hat{s} - \eta_t\right]. \tag{6}$$

3.3 Monetary Policy

We assume that monetary and fiscal policy are coordinated, assuring an equilibrium. When fiscal policy is passive, an active monetary policy pegs the nominal exchange rate, thereby fixing the price level and setting $\gamma_t = 0$. And when the passive fiscal policy is permanent, $E_t\gamma_{t+1} = 0$ as well. When fiscal policy is active, a passive monetary policy pegs the nominal interest rate at the fixed international rate of i. Under the assumption of no international inflation, this assures that $E_t\gamma_{t+1} = 0$, but allows $\gamma_t \lesseqgtr 0$.

4 Stability and Dynamics in Equilibrium

The dynamic model for the equilibrium values of the real surplus and real debt is given by equations (1) and (6), together with the assumption that monetary policy is compatible, allowing equilibrium determination of γ_t. Expectations of γ_t are rational. The model allowing default is a switching model, in which fiscal policy is initially passive, switching to active in the event of a crisis. To understand the switching model, it is first necessary to understand the dynamics for the pure passive-fiscal-policy and active-fiscal-policy models without switching. We present each pure model first, under the assumption that there is no upper bound on the long-run value for the surplus ($\bar{s} \to \infty$), and show that there is no role for default in either

[26]It should be noted that this formulation ignores debt shocks, whereby internal or external shocks (along with surpluses or defaults) change the value of debt. Calvo, Izquierdo, and Talvi (2003) consider terms of trade shocks as a debt shock, whereby a deterioration in the terms of trade reduces the real value of debt. Explicit consideration of this would require that we switch from the one-good model. This is left for future research.

pure model.

In either pure model, expected capital gains and losses on debt are zero $\left(E_t\gamma_{t+1} = 0\right)$.[27] We show below that the characterization of fiscal policy as active or passive depends on the value for λ. When $\lambda \neq 1$, this system has a stationary long-run equilibrium determined by $\hat{s}$. When $\lambda = 1$, the long-run equilibrium is non-stationary, reflecting a unit root. Letting ψ represent eigenvalues, the characteristic equation for the system is given by:

$$(1-\alpha)(1+i) - \psi\left[1 + i(1-\alpha\lambda) + 1 - \alpha\right] + \psi^2 = 0.$$

Consider the dynamic stability of the model for different values of λ when there is no upper bound on the long-run surplus. A passive fiscal policy is defined as a surplus rule with fixed coefficients such that the government's intertemporal budget constraint, given by equation (5), holds for any stochastic process for γ_t. This requires that the dynamic model in debt and the surplus be globally stable. This frees an active monetary policy to chose the initial position by its choice of a fixed value for the exchange rate. In contrast, an active fiscal policy is defined as a surplus rule with fixed coefficients such that the government's intertemporal budget constraint, given by equation (5), does not hold for any stochastic process for γ_t. This implies that under active fiscal policy the model in debt and the surplus is saddlepath stable. Debt must be a jumping variable,[28] keeping the system on the saddlepath, implying that monetary policy must be passive to allow the real value of debt to jump with a jump in the exchange rate (and equivalently the price level).

[27] With passive fiscal policy, the active monetary authority sets $\gamma_t = 0$ every period. With active fiscal policy, expected capital gains and losses are symmetric so that the expectation of the capital loss (gain) is zero.
[28] From equation (6), debt jumps because γ_t jumps.

4.1 Passive Fiscal Policy

The passive fiscal regime we solve is one with $\lambda = 1$ in which one root is given by $(1-\alpha)(1+i)$, which is positive and less than unity under the parameterization assumed for α, and the other root is unity. Monetary policy is active and fixes the exchange rate, setting $E_t\gamma_{t+1} = \gamma_t = 0$. Assuming an initial equilibrium, in which the surplus equals interest on the debt,

$$s_{-1} = \left(\frac{i}{1+i}\right) d_{-1},$$

equations (1) and (6) with $\lambda = 1$, $\gamma_t = E_t\gamma_{t+1} = 0$, and $\bar{s} \to \infty$, can be solved to express the time path for debt and surpluses as a function of initial values and fiscal shocks yielding:

$$s_t = s_{-1} - \sum_{j=0}^{t} \left[\frac{\frac{i}{1+i} - \alpha\left[(1-\alpha)(1+i)\right]^{t-j}}{1-(1-\alpha)(1+i)} \right] (1+i)\,\eta_j, \tag{7}$$

$$d_t = d_{-1} - \sum_{j=0}^{t} \left[\frac{1-\left[(1-\alpha)(1+i)\right]^{t+1-j}}{1-(1-\alpha)(1+i)} \right] (1+i)\,\eta_j. \tag{8}$$

To facilitate understanding and motivate the solution method for the switching model, it is useful to represent the dynamics of each system using phase diagrams. We can construct the phase diagram for the system by subtracting lagged values of the surplus and debt, respectively from equations (1) and (6) with $\lambda = 1$ and $\gamma_t = E_t\gamma_{t+1} = 0$, to yield:

$$s_t - s_{t-1} = \alpha \left[\left(\frac{i}{1+i}\right) d_{t-1} - s_{t-1} \right] + \eta_t, \tag{9}$$

$$d_t - d_{t-1} = (1+i)(1-\alpha) \left[\left(\frac{i}{1+i}\right) d_{t-1} - s_{t-1} \right] - (1+i)\,\eta_t. \tag{10}$$

The phase diagram is given in Figure 1.

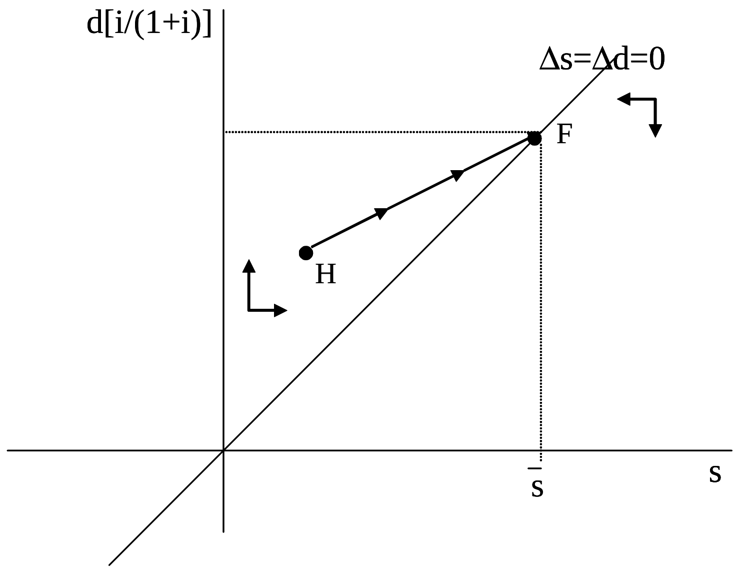

Figure 1

Passive Fiscal Policy

Note that the $\Delta d = 0$ and $\Delta s = 0$ schedules lie on top of each other with $s_t = \left(\frac{i}{1+i}\right) d_t$. Fiscal shocks move the system away from the $\Delta s = \Delta d = 0$ locus, but since the model is globally stable, the surplus and debt are always expected to return to some point on the locus. Since the model is expected to reach the $\Delta s = \Delta d = 0$ locus for any initial position, the fiscal rule with $\lambda = 1$ is passive. Active monetary policy assures $\gamma_t = 0$, leaving no role for default. When fiscal shocks move the system away from the $\Delta s = \Delta d = 0$ locus, say to point H, either equations (7) and (8) or equations (9) and (10) can be used to show that the relationship between debt and surpluses along the adjustment path HF is given by[29]:

$$\frac{E_t d_{t+1} - d_t}{E_t s_{t+1} - s_t} = \frac{(1-\alpha)(1+i)}{\alpha}. \tag{11}$$

The expected long-run effect of a shock on the debt and surplus can be computed by letting $t \to \infty$ in equations (7) and (8), to yield a non-zero long-run effect of η_0 on the debt

[29] The slope of HF is given by $\frac{(1-\alpha)i}{\alpha}$ which is less than unity under the parameterization assumed for α.

as $\frac{-(1+i)}{1-(1-\alpha)(1+i)}$ and on the surplus as $\frac{-i}{1-(1-\alpha)(1+i)}$, reflecting the unit root.[30] Note that the greater the surplus persistence (smaller value for α), the larger is the long-run effect of a fiscal shock.[31] For purposes of comparison with what follows, point F has been drawn as equal to the upper bound for the long-run surplus, given by $\bar{s}$.

4.2 Active Fiscal Policy

A reduction in λ from unity increases the size of the larger root and decreases the smaller root, such that a regime with $0 < \lambda < 1$ has one unstable and one stable root. The active fiscal policy system we consider is comprised of equations (1) and (6) with $\lambda = 0$, in which the roots are $1+i$ and $1-\alpha$. Monetary policy must be passive allowing the value for γ_t to set the coefficient on the explosive root to zero.[32] The solutions for the surplus and real debt are given by:

$$s_t = \hat{s} + \left[(1-\alpha)(s_{-1} - \hat{s}) + \sum_{j=0}^{t} \left(\frac{1}{1-\alpha} \right)^j \eta_j \right] (1-\alpha)^t . \tag{12}$$

$$d_t = \left(\frac{1+i}{i} \right) \hat{s} + \left(\frac{1+i}{\alpha+i} \right) \left[(1-\alpha)(s_{-1} - \hat{s}) + \sum_{j=0}^{t} \left(\frac{1}{1-\alpha} \right)^j \eta_j \right] (1-\alpha)^{t+1} , \tag{13}$$

Since the solutions contain only the stable root, there is no long-run effect of shocks; the expected value of the surplus infinitely far into the future is always $\hat{s}$. These equations can

[30] Fiscal policy remains passive with a small increase in λ from unity. This reduces the size of the larger root and increases the size of the smaller root. Both roots are positive and less than unity, and the system is globally stable around a stationary long-run value for the surplus of $\hat{s}$. Note that the rise in λ pivots the the $\Delta d = 0$ curve counter-clockwise around F and pivots the $\Delta s = 0$ curve clockwise. A value for $\lambda \geq 1$ implies passive fiscal policy, since the system is globally stable.

[31] In the absence of surplus persistence ($\alpha = 1$), debt would be a random walk and the adjustment path would be a horizontal line along which the surplus adjusts to debt.

[32] The requirement that the coefficient on the explosive root be zero implies:

$$d_{-1} - \left(\frac{1+i}{\alpha+i} \right) \left[\left(\frac{1+i}{i} \right) \alpha \hat{s} + (1-\alpha) s_{-1} \right] + \sum_{j=0}^{t} \left(\frac{1}{1+i} \right)^j \left[\gamma_j - \left(\frac{1+i}{\alpha+i} \right) \eta_j \right] = 0.$$

This equality holds only if each term in the sum equals zero and the remaining term equals zero.

be used to express the saddlepath relationship between debt and the surplus as:

$$d_t = \left(\frac{1+i}{\alpha+i}\right)\left[\frac{1+i}{i}\alpha\hat{s} + (1-\alpha)\, s_t\right]. \tag{14}$$

When there are stochastic shocks to the surplus, the real value of debt must jump to keep the system on the saddlepath, as in the FTPL.

The equations for the phase diagram with a surplus target of $\bar{s}$ can be computed by subtracting lagged values of the surplus and debt, respectively from equations (1) and (6) with $\lambda = 0$, $E_t\gamma_{t+1} = 0$, and $\hat{s} = \bar{s}$, to yield:

$$s_t - s_{t-1} = \alpha\left[\bar{s} - s_{t-1}\right] + \eta_t,$$

$$d_t - d_{t-1} = (1+i)\left[\left(\frac{i}{1+i}\right) d_{t-1} - (1-\alpha)\, s_{t-1} - \alpha\bar{s} + \gamma_t - \eta_t\right].$$

The phase diagram is given in Figure 2. Compared with passive fiscal policy in Figure 1, the fall in λ from unity pivots the $\Delta d = 0$ curve clockwise around the long-run equilibrium value of F and pivots the $\Delta s = 0$ curve counter-clockwise. The saddlepath is labeled SP.

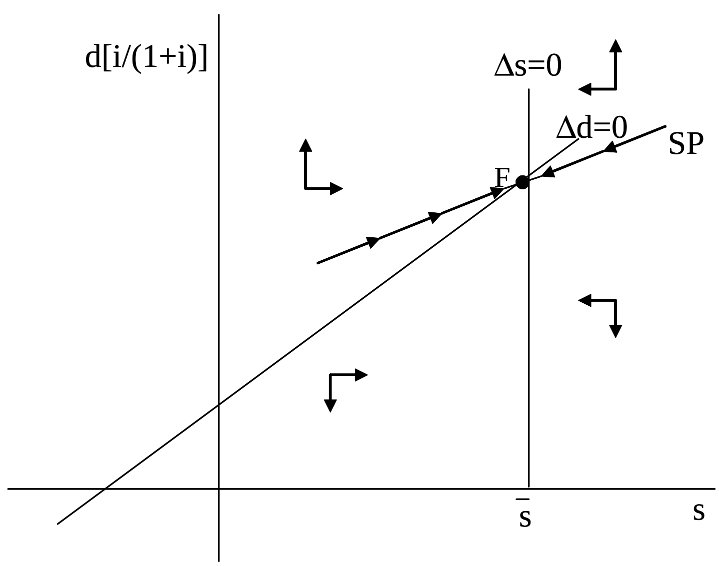

Figure 2
Active Fiscal Policy

Since the system does not reach an equilibrium for arbitrary starting values, this is an active fiscal rule. There must be one jumping variable to assure the system is on the saddlepath. From equation (6), d_t jumps with each jump in γ_t, allowing the system to remain on the saddlepath. Since jumps occur with unanticipated shocks, they are symmetric and $E_t\gamma_{t+1} = 0$. The existence of stochastic surplus shocks implies that a commitment to a fixed exchange rate is impossible with this rule since γ_t must be allowed to adjust both positively and negatively in response to shocks to keep the system on the saddlepath.[33] Therefore, for an equilibrium to exist, monetary policy must be passive, as assumed. Default plays no role because there is no promise to set $\gamma_t = 0$.[34]

4.3 An Upper Bound

Now, consider the implications of an upper bound on the value for the long-run surplus, given by $\bar{s} < \infty$, for the sustainability of alternative fiscal regimes. Consider active fiscal policy first. The active fiscal regime must always be along the saddlepath. As long as $\hat{s} \leq \bar{s}$ and the variance of fiscal shocks is not too large, the purely-active fiscal policy regime will almost surely not spend indefinitely long periods with the surplus very much above the upper bound. This is because the dynamics assure that the surplus is always expected to return to $\hat{s}$. We assume that $\bar{s}$ is defined such that with $\hat{s} \leq \bar{s}$ the active fiscal policy is sustainable indefinitely, implying fiscal solvency. The largest feasible value for the expected present-value surplus under active fiscal policy, given s_t, is determined by the value of debt along the saddlepath with a target surplus of $\bar{s}$. Note additionally from equation (14), that

[33] Without persistence $(\alpha = 1)$, the saddlepath is horizontal, fixing debt at its long-run equilibrium value while the surplus deviates temporarily from its long-run value due to stochastic shocks.

[34] Uribe (2002) develops a fiscal theory of default using active fiscal policy and the assumption that the magnitude of the possible change in the flexible exchange rate is limited. Under these assumptions, outright default (debt revaluation) is necessary to restore solvency for a large negative (positive) shock.

a reduction in $\hat{s}$ from $\bar{s}$ shifts the saddlepath downward.

In contrast, the purely-passive fiscal regime could spend indefinitely large amounts of time above $\bar{s}$ since there is nothing in the dynamics to systematically return the system to a particular value for the surplus. However, we have assumed that long-run surpluses above $\bar{s}$ are infeasible. Therefore, under passive fiscal policy, paths converging to a position above $\bar{s}$ are infeasible. This limits feasible positions for the debt and surplus under passive fiscal policy to positions in Figure 1 on or below the adjustment path which converges to $\bar{s}$, labeled HF. The adjustment path converging to $\bar{s}$ can be viewed as determining the maximum expected present-value surplus, given s_t, under passive fiscal policy. A government which initially commits to passive fiscal policy could find itself in a position in which desired debt exceeds the maximum possible present-value surplus under passive policy, represented by a position above HF. In the absence of expectations of future fiscal reform, agents would refuse to lend, precipitating a financial crisis.

Figure 3 superimposes the saddlepath for an active policy system with a target surplus of $\bar{s}$, such that $\hat{s} = \bar{s}$, hereafter called the $\bar{s}$-saddlepath, on the passive policy system.

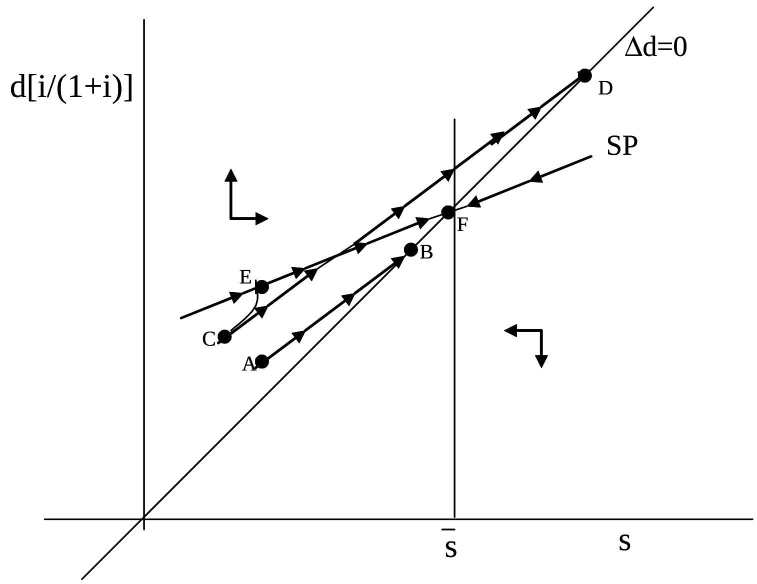

Figure 3
Switching Regime

Recall that the $\bar{s}$-saddlepath determines the largest possible value for the present-value surplus under active fiscal policy. The adjustment path converging to F (not shown) determines the largest present-value surplus under passive policy. From equations (11) and (14), the slope of the saddlepath is less than the slope of the expected adjustment path under passive fiscal policy. Therefore, below $\bar{s}$, the maximum present-value surplus under passive policy for each value of s_t is less than the maximum present-value surplus under active policy. This implies that a switch to active fiscal policy raises the maximum present-value for $s_t < \bar{s}$. From equation (1), this occurs because the switch increases near-term surpluses when $\hat{s} = \bar{s}$ and debt is below the long-run value associated with $\bar{s}$. Fiscal reform raises the maximum expected present-value surplus, giving it a positive role in crisis resolution.[35]

[35]Surplus persistence is important here. Were $\alpha = 1$, then the maximum possible present-value surplus under both policy regimes would be given by a horizontal line at F, implying that regime switch could not raise the maximum present-value surplus.

5 A Regime-Switching Model of Default

5.1 Endogenous Regime Switching

To develop a regime-switching model of default, assume that there is a maximum long-run surplus given by $\bar{s}$, and that the government chooses the value for λ, together with the compatible fixed exchange rate monetary policy, prior to the realization of the fiscal shock. The surplus has persistence ($\alpha < 1$). Fiscal policy is initially passive ($\lambda = 1$) and monetary policy is active $(\gamma_t = 0)$. The government retains this policy regime as long as feasible.[36] The initial position is along the $\Delta d = \Delta s = 0$ schedule well below $\bar{s}$.[37]

We define a crisis period as a period in which the government can no longer borrow to continue with the passive fiscal rule. This is a crisis because without some fundamental change by the government, equilibrium is not possible. We assume that to resolve the crisis and establish equilibrium, the government promises to switch in the next period to an active rule with flexible exchange rates and $\hat{s} \leq \bar{s}$. The target surplus ($\hat{s}$) takes on the smallest value consistent with minimum default in the crisis period, where default has a lower bound of zero. If this is sufficient to restore equilibrium, resolving the crisis, then $\gamma_t = 0$ as promised, and there is no default. If not, then γ_t falls below zero, to place the system on the saddlepath with $\hat{s} = \bar{s}$.[38] This is default, since the passive-fiscal, active-monetary regime had promised

[36] It is interesting to note that this reverses Woodford's (1998b) concern that the active policy regime might breech the upper bound, requiring a switch to a passive fiscal regime.

[37] We assume an initial long-run equilibrium position which is attainable under the switching model from some other point. This rules out a position along $\Delta d = \Delta s = 0$ with $s_{-1} > \bar{s}$ in addition to positions above the saddlepath beyond $\bar{s}$.

[38] Davig et al (2003) derive interesting results about the effects of fiscal shocks when regime switching is purely stochastic and when there is no surplus persistence. Here regime switching is triggered by insolvency. And the assumption that the regime change is permanent is stronger than necessary. This is considered at the end of the paper. Note also that the alternative of a regime change which raises λ is not likely to be feasible. This would reduce near-term surpluses and raise surpluses further into the future. The required large future surpluses could be infeasible for the time duration necessary, even though the long-run surplus is not above the maximum.

$\gamma_t = 0$.

Consider the dynamics leading up to a crisis, using the phase diagram in Figure 3. Assume that the system begins in a regime with passive fiscal policy, and that the debt and surplus initially lie well below $\bar{s}$ along the $\Delta d = 0$ schedule, which gives the locus of long-run equilibrium values for debt and the surplus under the passive fiscal rule. Consider the effect of a shock which moves the system away from the $\Delta d = 0$ curve and places the system at point A. In the absence of additional shocks, debt and the surplus will travel along the locus AB, according to the arrows of motion of the passive-fiscal-policy system, eventually reaching point B. This realization of a shock does not create a crisis, and there is no switch in regimes. Indeed, the system could spend a long time in this regime with a passive fiscal rule, at points well below the saddlepath, realizing positive and negative surplus shocks, and following the arrows of motion back to points on the $\Delta d = 0$ curve below F. Solutions for the debt and surplus, at positions well below the saddlepath on either side of $\bar{s}$, are given by equations (7) and (8).

Consider, alternatively, a shock which places the system at point C. If the system were to follow the arrows of motion for the passive fiscal regime from point C, then, in the absence of other shocks, it would eventually reach $\Delta d = 0$ at point D, implying an expected long-run surplus greater than $\bar{s}$. The market knows that point D is infeasible; therefore it will not allow the government to continue with a passive fiscal rule indefinitely to reach point D. In the absence of an expected future policy switch, point C would not be attainable because debt is higher than the largest possible expected present-value surplus under passive fiscal policy.[39] Consider the effect of an expected policy switch on the feasibility of attaining point

[39]The maximum expected present-value surplus under passive fiscal policy is given by the value of the debt

C.

Since the saddlepath for the active fiscal regime with a target surplus of $\bar{s}$ represents the upper bound on the expected present-value surplus for $s_t \leq \bar{s}$, the market will not allow the system to travel above SP from point C. Debt would exceed the maximum present-value surplus, and no one would lend to allow this. At point C, the upper bound on the expected present-value of surpluses, conditional on a switch to active fiscal policy next period, lies on the saddlepath vertically above point C and exceeds the current value of debt. This implies that the expectation of a regime switch in the more distant future is sufficient to make the expected value of the present-value surplus equal to debt, making point C feasible in a switching regime. The expectation of a future regime switch allows the value of debt to increase above its value at C. In general, the expectation that a crisis will be resolved with fiscal reform permits the initial regime to remain viable over a larger range of values for the debt and surplus.

Point C is attainable with the expectation of future policy switch, but this expectation affects dynamics, including the timing of the forced regime switch. Consider the possibility that from point C, the debt and surplus travel along the locus CD until they reach the saddlepath, from which point they travel along the saddlepath to point F. This implies that the market allows passive fiscal policy until the passive-fiscal-policy adjustment path intersects the active-fiscal-policy saddlepath, at which point, it forces a switch to active fiscal policy as a condition for lending. In a certainty world, with the initial point C, this would be the adjustment path.[40] The policy switch cannot be postponed beyond the period in which

along the path converging to F. This is not drawn to simplify the graph.

[40]To the extent that the economy does not land exactly on the saddlepath due to discrete time, the target surplus drops below $\bar{s}$, shifting the saddlepath down. Therefore, as assumed, the long-run surplus (debt) is the smallest value consistent with minimizing the value of default.

the expected present-value of future surpluses reaches its upper bound under active policy.

However, when shocks can occur each period, the adjustment path necessarily changes in the neighborhood of the saddlepath due to the effect of expectations on the evolution of real debt. Specifically, as the passive-fiscal system approaches the saddlepath for the active-fiscal system, a negative shock could push it over. Since agents will not lend to allow a point above the saddlepath, a negative shock, which would push the passive-fiscal system above the saddlepath, precipitates a crisis. The government's first response to the crisis is to promise fiscal reform, raising the expected present-value surplus and eliminating the interest rate premium. If the reduction in interest-inclusive debt due to the reduction in the interest rate premium is not sufficient to place the system on or below the saddlepath, then default is necessary. Therefore, as the system approaches the saddlepath, the market begins to anticipate default.[41] This anticipation forces the interest rate under the passive fiscal rule to increase to incorporate a default premium from equation (2). This implies that in a world with surplus shocks, debt is expected to increase more quickly than implied by the locus CD, reaching the saddlepath at a point like E. Regime switch and possibly default is necessary to keep the system from going above the $\bar{s}$-saddlepath. The key to solving the switching model is solving for rational expectations of default given by γ_t.

5.2 Values for Default and Expectations

Agents begin to expect default when the system gets close enough to the saddlepath that a fiscal shock could push it over. However, we show below that expectations of default alone cannot create default, implying that this model does not have multiple equilibria.

[41]Since the probability of default is less than one, when a shock occurs requiring default, its magnitude is greater than expected allowing d to jump downwards.

The equation for the saddlepath value of debt is given by (14). Letting d_t^s represents an upper bound on debt for a given value for the surplus, it is useful to write an expression for the tightness of this bound by subtracting equation (6) with $\lambda = 1$ from equation (14) with $\hat{s} = \bar{s}$, to obtain:

$$d_t^s - d_t = E_t\gamma_{t+1} + (1+i)\left\{\left(\frac{1+i}{\alpha+i}\right)\left[\frac{\alpha}{i}\left(\bar{s} - \frac{i}{1+i}d_{t-1}\right) + (1-\alpha)\left(s_{t-1} - \frac{i}{1+i}d_{t-1}\right) + \eta_t\right] - \gamma_t\right\}. \tag{15}$$

When this difference is positive with $\gamma_t = 0$, there is no crisis and policy continues to follow the passive rule with $\lambda_t = 1$ and $\gamma_t = 0$. A crisis occurs only when this difference becomes zero or negative.

Note that the distance depends on the expectation of future default, which could accompany a regime switch. This is because this expectation increases the default premium on the interest rate, increasing the interest-inclusive value of debt. Therefore, while the passive fiscal rule prevails, in the neighborhood of the saddlepath, $E_t\gamma_{t+1} \leq 0$ due to the possibility of a regime switch with default. The expectation must be computed as the rational expectation of γ_{t+1}. On the first date $t = T$, whereby $d_T^s - d_T < 0$, the expectation of regime switch in period $T+1$ sets $\lambda_{T+1} = 0$, implying that $E_T\gamma_{T+1} = 0$. If regime switch, setting $E_T\gamma_{T+1} = 0$ and reducing d_t, through the reduction in the interest-rate premium, makes the distance to the $\bar{s}$-saddlepath positive or zero, then default does not occur.[42] However, if the distance remains negative, then γ_T falls to equate it to zero.

Note that expectations of future default $\left(E_t\gamma_{t+1}\right)$ alone cannot cause current default. If the only reason for $d_T^s - d_T < 0$ was $E_t\gamma_{t+1} < 0$, then the fully-credible regime switch,

[42] The target surplus ($\hat{s}$) adjusts such that the initial point after regime switch with $\gamma_t = 0$ is on the saddlepath.

setting $E_t\gamma_{t+1} = 0$ would be sufficient to resolve the crisis without default. The expectation of future default would cause a crisis with a regime switch, but not a default, implying that it could not be self-fulfilling.

To compute the expectation of γ_t, define $\tilde{x}_{t-1}$ as the state variable, reflecting tightness of the upper bound on debt at the end of period $t-1$ prior to the arrival of shocks at time t, as:

$$\tilde{x}_{t-1} = \frac{\alpha}{i}\left(\bar{s} - \frac{i}{1+i}d_{t-1}\right) + (1-\alpha)\left(s_{t-1} - \frac{i}{1+i}d_{t-1}\right). \tag{16}$$

From equations (15) and (16), γ_t is given by:[43]

$$\gamma_t = \min\left\{0, \left(\frac{1+i}{\alpha+i}\right)(\tilde{x}_{t-1} + \eta_t)\right\}.$$

Before solving for the expectation of γ_t, note from equation (6), that d_{t-1} and $E_{t-1}\gamma_t$ are simultaneously determined. Prior expectations of default raise the risk premium raising the interest-inclusive value of debt. Substituting backwards one period for debt and the surplus using equations (1) and (6) allows expression of the tightness variable as:

$$\tilde{x}_{t-1} = x_{t-1} + \frac{\alpha + i(1-\alpha)}{1+i}E_{t-1}\gamma_t,$$

where

$$x_{t-1} = \frac{\alpha}{i}\left(\bar{s} - \frac{i}{1+i}d_{t-2}\right) + (1-\alpha)\left[1 + i(1-\alpha)\right]\left(s_{t-2} - \frac{i}{1+i}d_{t-2}\right) + \left[\alpha + i(1-\alpha)\right]\eta_{t-1}. \tag{17}$$

This allows the expression of the distance between the saddlepath value of debt and its current value to be expressed as:

$$d_t^s - d_t = E_t\gamma_{t+1} + (1+i)\left\{\left(\frac{1+i}{\alpha+i}\right)\left[x_{t-1} + \eta_t + \frac{\alpha + i(1-\alpha)}{1+i}E_{t-1}\gamma_t\right] - \gamma_t\right\}. \tag{18}$$

[43] Recall that if $\gamma_t < 0$, then $E_t\gamma_{t+1} = 0$.

When $d_t^s - d_t < 0$ with $\gamma_t = 0$, a financial crisis occurs. The first step in resolving the crisis is to promise fully credible regime switch next period.[44] This sets $E_t\gamma_{t+1} = 0$.[45] If this is not sufficient to eliminate the negative distance to the saddlepath, then capital loss on debt at time t sets the distance to the saddlepath at zero yielding the magnitude of default as:

$$\gamma_t = \min\left\{0, \left(\frac{1+i}{\alpha+i}\right)\left[x_{t-1} + \eta_t + \frac{\alpha + i\,(1-\alpha)}{1+i}E_{t-1}\gamma_t\right]\right\}. \tag{19}$$

The expression for actual default in equation (19) can be used to determine the rational expectation of default, once we specify a distribution for the surplus shock. Since γ_t depends on the state and an error as well as on its own expectation, arbitrary values for expectations cannot be self-fulfilling. To compute the fundamentals-based expectation for γ_t, assume that the distribution for η_t is uniform on the interval $\{-\bar{\eta}, ..., 0, ...\bar{\eta}\}$.[46] Since the distribution has finite support, it is necessary to compute expectations for values of x_{t-1} in three ranges, one in which no values of η_t would cause a crisis, another in which some values would cause a crisis, and a final in which all values would cause a crisis.

Define ρ_t as the probability of default in period t. Consider, first, large values of x_{t-1} such that the system is far from the saddlepath. We assumed initially that point A in Figure 3 was such a position. If $x_{t-1} \geq \bar{\eta}$, then there is no value of η_t which could cause $\gamma_t < 0$. Therefore, when $x_{t-1} \geq \bar{\eta}$, $\rho_t = 0$, $\gamma_t = 0$, and

$$E_{t-1}\left(\gamma_t \mid \rho_t = 0\right) = 0.$$

Now, consider small values of x_{t-1}. There will be a value of x_{t-1} so small such that all

[44] The assumption that regime switch is credible is essential here. Perhaps that is one role of IMF conditionality. The study of imperfect credibility is left to future work.
[45] Therefore, γ_t does not depend on $E_t\gamma_{t+1}$.
[46] The assumption simplifies solution and presentation of the model.

values of η_t cause collapse, yielding $\rho_t = 1$. This value will be larger than $-\bar{\eta}$ due to the expectation in equation (19). To calculate this lower value for x_{t-1}, set $\rho_t = 1$, and compute the expectation of γ_t, conditional on $\rho_t = 1$, using equation (19) to yield

$$E_{t-1}\left(\gamma_t \mid \rho_t = 1\right) = \frac{1+i}{\alpha i} x_{t-1}. \tag{20}$$

The largest value of x_{t-1} such that even $\eta_t = \bar{\eta}$ would yield default can be calculated by substituting the expectation into equation (19), setting $\eta_t = \bar{\eta}$, and solving for the value of x_{t-1} which sets the equation to zero. This yields the largest value for x_{t-1} for which the probability of default is unity as $\frac{-\alpha i}{\alpha+i}\bar{\eta}$. Therefore, when $x_{t-1} \leq \frac{-\alpha i}{\alpha+i}\bar{\eta}$, $\rho_t = 1$, and the expectation is given above. The magnitude of default depends on the realization of η_t and is given by:

$$\gamma_t = \frac{\alpha+i}{\alpha i} x_{t-1} + \frac{1+i}{\alpha+i}\eta_t \quad \text{for } \rho_t = 1. \tag{21}$$

Note that since $x_{t-1} < 0$, default occurs even with a small positive surplus shock.

Now, it is necessary to calculate the expectation for values of x_{t-1} such that $0 < \rho_t < 1$, that is for $\frac{-\alpha i}{\alpha+i}\bar{\eta} < x_{t-1} < \bar{\eta}$. Define η_t^* as the smallest value of η_t for which $\gamma_t = 0$. Any value of $\eta_t < \eta_t^*$ triggers default. Recall that when $\gamma_t < 0$, a regime switch occurs, such that capital gains and losses in the future occur with equal probability, setting $E_t\gamma_{t+1} = 0$. With a uniform distribution for η_t, the probability of getting an $\eta_t \leq \eta_t^*$, is $\frac{\eta_t^*+\bar{\eta}}{2\bar{\eta}}$. The mean of η_t, conditional on $\eta_t \leq \eta_t^*$ is given by $\frac{\eta_t^*-\bar{\eta}}{2}$. Taking the expectation of equation (19) yields:

$$E_{t-1}\left(\gamma_t\right) = \left(\frac{1+i}{\alpha+i}\right)\left[x_{t-1} + \frac{\eta_t^*-\bar{\eta}}{2} + \frac{\alpha+i\left(1-\alpha\right)}{1+i}E_{t-1}\left(\gamma_t \mid x_{t-1} < \bar{\eta}\right)\right]\left(\frac{\eta_t^*+\bar{\eta}}{2\bar{\eta}}\right).$$

Solving for the expectation yields:

$$E_{t-1}\left(\gamma_t\right) = \left(1+i\right)\left\{\frac{2x_{t-1}\left(\eta_t^*+\bar{\eta}\right)+\left(\eta_t^*-\bar{\eta}\right)\left(\eta_t^*+\bar{\eta}\right)}{2\left[-\left(\alpha+i\right)\left(\eta_t^*-\bar{\eta}\right)+\alpha i\left(\eta_t^*+\bar{\eta}\right)\right]}\right\}. \tag{22}$$

Substituting into the expression for γ_t implies:

$$\gamma_t = \min\left\{0, \left(\frac{1+i}{\alpha+i}\right)\left[x_{t-1} + \eta_t + [\alpha + i(1-\alpha)]\left\{\frac{2x_{t-1}(\eta_t^* + \bar{\eta}) + (\eta_t^* - \bar{\eta})(\eta_t^* + \bar{\eta})}{2[-(\alpha+i)(\eta_t^* - \bar{\eta}) + \alpha i(\eta_t^* + \bar{\eta})]}\right\}\right]\right\}. \tag{23}$$

It remains necessary to solve for η_t^* for $\frac{-\alpha i}{\alpha+i}\bar{\eta} \leq x_{t-1} \leq \bar{\eta}$. Recall that η_t^* is the smallest value of η_t for which $\gamma_t = 0$.[47] Therefore, setting $\eta_t = \eta_t^*$ in equation (23) and solving for $\eta_t^* \in \{-\bar{\eta}, ..., 0, ...\bar{\eta}\}$ yields:[48]

$$\eta_t^* = \frac{\bar{\eta}(\alpha + i + \alpha i) - 2\sqrt{(\alpha+i)^2\bar{\eta}x_{t-1} + (\alpha+i)\alpha i\bar{\eta}(\bar{\eta} - x_{t-1})}}{\alpha + i - \alpha i} \quad \text{for } 0 < \rho_t < 1. \tag{24}$$

Substituting for η_t^* in equation (23) for γ_t yields an expression for actual default when $0 < \rho_t < 1$, as:

$$\gamma_t = \min\left\{0, \left(\frac{1+i}{\alpha+i}\right)\left[\eta_t - \frac{\bar{\eta}(\alpha + i + \alpha i) - 2\sqrt{(\alpha+i)^2\bar{\eta}x_{t-1} + (\alpha+i)\alpha i\bar{\eta}(\bar{\eta} - x_{t-1})}}{\alpha + i - \alpha i}\right]\right\} \tag{25}$$

Using equations (24) and (22), the value for expected default is given by:

$$\begin{aligned} &E_{t-1}(\gamma_t \mid 0 < \rho_t < 1) \qquad (26) \\ = &\left(\frac{-(1+i)}{\alpha + i - \alpha i}\right)\left[x_{t-1} + \frac{\bar{\eta}(\alpha + i + \alpha i) - 2\sqrt{(\alpha+i)^2\bar{\eta}x_{t-1} + (\alpha+i)\alpha i\bar{\eta}(\bar{\eta} - x_{t-1})}}{\alpha + i - \alpha i}\right]. \end{aligned}$$

Applying the definition of η_t^* more generally in all ranges, note that for $x_{t-1} \geq \bar{\eta}$, $\eta_t^* = -\bar{\eta}$ and for $x_{t-1} \leq \frac{-\alpha i}{\alpha+i}\bar{\eta}$, $\eta_t^* = \bar{\eta}$. Therefore, as x_{t-1} rises from $\frac{-\alpha i}{\alpha+i}\bar{\eta}$, η_t^* falls from $\bar{\eta}$, passing through values given by (24), ultimately reaching -$\bar{\eta}$, as x_{t-1} reaches $\bar{\eta}$.

5.3 Numerical Solution of the Switching Model

Equations (7) and (8) provide a solution for the system as long as it remains well below the saddlepath. The state variable representing tightness of the upper bound can be expressed

[47] Note that we have already calculated values for η_t^* when $\rho_t = 0$ and when $\rho_t = 1$ as $\bar{\eta}$ and $-\bar{\eta}$, respectively.
[48] The positive square root yields a value outside the bounded interval.

as:

$$x_{t-1} = \frac{\alpha}{i}\left(\bar{s} - s_{-1}\right) + \frac{\sum_{j=0}^{t-1}\left[\alpha - i\left(1-\alpha\right)\left[\left(1-\alpha\right)\left(1+i\right)\right]^{t-j}\right]\eta_j}{1-\left(1-\alpha\right)\left(1+i\right)}. \tag{27}$$

Once $x_{t-1} < \bar{\eta}$, the nature of the solution changes since $E_{t-1}\gamma_t < 0$. The expectation affects both the evolution of debt and the probability of a financial crisis. The solution can be computed numerically.

Given s_{t-1} and d_{t-1}, the largest value of η_t, which would cause default (η_t^*), and the expected magnitude of default can be computed using equation(s) (20), or (24) and (26), depending on the value of x_{t-1}. The arrival of period t beings new information with a new value for the surplus shock. If the value of the surplus shock is less than the critical value, then a financial crisis with default ensues. The magnitude of default is given by either equation (21) or (25). Regime switch occurs next period, with the system in an initial position on the active-policy saddlepath.

Alternatively, if the value of the surplus shock is larger than the critical value, then there is no default this period. However, the absence of default does not necessarily imply the absence of a financial crisis. Even though the distance to the saddlepath is positive with the expectation of default next period set at zero (the criterion to determine whether or not default occurs), the distance could be negative when the expectation takes on its value in the passive-fiscal-policy regime. This is because the default premium on debt itself raises the value of interest-inclusive debt. Therefore, it remains necessary to calculate the expectation for next period's default to determine whether the default premium itself creates a financial crisis. The expectation can be calculated by updating the tightness variable x_t, using equation (17) and computing $E_t\gamma_{t+1}$ based on a comparison of the value of x_t with

$\bar{\eta}$. The distance to the saddlepath can be computed, from equation (18). If this distance is positive with $\gamma_t = 0$, the system continues in the passive-policy regime next period with a new value for the surplus shock. However, if this distance is negative, a crisis occurs and the fiscal authority promises to switch to an active fiscal rule next period. When the expectation of regime switch next period sends the system to a point below the $\bar{s}$-saddlepath, then the target surplus is reduced such that the system begins on the saddlepath with $\gamma_T = E_T \gamma_{T+1} = 0$.

Simulation of the model allows us to generate magnitudes for default frequency, size of debt at a crisis, default, length of time for which crises are anticipated, and the frequency of false signals, which are defined as episodes in which crises, which never occur, are anticipated. We are guided in our choice of parameter values by recent experiences in Argentina and Brazil. For a baseline model, we set the upper bound on the surplus at 2.5% of GDP. This is probably more reasonable for Argentina than for Brazil as primary surpluses of 3% were assumed to be unattainable in Argentina, while Brazil achieved surpluses larger than 3%. We set the real interest rate at the standard value of $i = .04$. Together these parameter values imply an upper bound on debt of 65% of GDP. We set the upper bound on surplus shocks ($\bar{\eta}$) as equal to 1% of GDP. We assume a value of $\alpha = .2$, allowing a substantial degree of persistence in fiscal policy. This is consistent with estimates by Catao and Terrones (2005) of the autoregressive parameter in equations for the surplus relative to GDP for high inflation Latin American countries, including Argentina and Brazil.[49] We assume a long-run initial equilibrium, in which surpluses are just large enough to service debt and in which debt begins relatively low at 26% of GDP. We perform the simulations using the baseline parameters, and using alternative values. The baseline case is very safe, implying that a very large number

[49]They obtain .223 for Argentina and .233 for Brazil.

of iterations is necessary to generate a reasonably-sized sample of crises. Therefore, for the baseline case, we use 25,000 iterations, while for other cases 2,500 provides a large sample of devaluations.

Table 1 contains simulation results relating to the risk of crises and default. It includes the probability of a fiscal crisis within the next twenty years, the percentage of crises resolved with default, the maximum and mean magnitudes of default as a percentage of outstanding debt for crises which included default, and the range for the values of debt as a percent of GDP at which the crisis occurs.

Table 1: Risk of Crisis and Default

	Pr crisis	default crises	max default	mean default	max debt	mean debt	min debt
baseline	1.0%	59%	14.0%	2.5%	60%	54%	48%
$\alpha = .1$	21.6%	61%	36.5%	7.7%	50%	40%	30%
$i = .08$	57.3%	65%	25.6%	5.0%	30%	28%	22%
$d_{-1} = 50$	29.5%	64%	11.2%	2.5%	61%	57%	52%
$\bar{\eta} = 2\%$	19.4%	61%	27.0%	6.0%	56%	48%	38%
$\bar{s} = 2\%$	94%	65%	13.0%	3.5%	47%	42%	38%

First, note that since the upper bound on debt is 65% when $i = .04$ and $\bar{s} = 2.5\%$ (33.75% when $i = .08$, and 52% when $\bar{s} = 2\%$), debt is always below its maximum when a crisis occurs. For the baseline case, the mean value for debt in a crisis is 83% of its maximum (54/65). Crises occur before debt actually hits its upper bound because agents refuse to lend once they expect the government's demand for borrowing, conditional on the passive fiscal policy, to place it on an unsustainable path. When the surplus has persistence, the saddlepath is upward sloping, and the system crosses the saddlepath before debt actually hits its upper bound.

Since surplus shocks have an upper bound of 1% of GDP, small surplus shocks can produce relatively large defaults, with the largest defaults being substantially higher than their means. Empirically, the percentage reduction in the real value of affected assets in either a currency crisis or a debt crisis tends to be larger than the simulated mean rates (but not than the simulated maximum rates). Burnside, Eichenbaum, and Rebelo's (2006) estimates[50] can be used to calculate the percentage reduction in the real value of domestic debt in the currency crises in Korea (1997), Mexico (1994) and Turkey (2000) as 21%, 33%, and 22% respectively. This is in response to shocks to the present value of expected government surpluses, estimated respectively as 24%, 15%, and 18.2% of GDP. Sturzenegger and Zettelmeyer (2005) estimate the size of investor losses in six debt restructurings between 1998 and 2005 and find most losses between 25-35%. They do not estimate the corresponding magnitude of surplus shocks. To be comparable to the numbers in the simulation, both sets of estimates would have to be recomputed allowing the base level debt to be all debt, not just the debt which suffers a reduction in value. This would reduce the magnitudes. However, Burnside et al (2006) stress the importance of shocks that are expected to persist into the future. This implies that the expected present-value surplus shocks are larger than the independently-distributed surplus shocks in this model. To compare the predictions of this model for default magnitudes more carefully with the data, we would need to allow surplus shocks to be autocorrelated in order to have reasonably-sized current surplus shocks generate

[50] They compute debt reduction relative to GDP and debt relative to GDP.

much larger present-value surplus shocks.

The simulations also reveal that only between 59% and 65% of crises require any form of default. Fiscal reform is sufficient to resolve the crisis in the remaining cases. These are cases in which fundamentals are weak enough that the interest rate premium on debt, created by the expectation of default, is actually large enough to push the economy into crisis. Its elimination with the switch in policy ends the crisis.

The simulations can also be used to assess the effect of different initial conditions (value of outstanding debt) and different parameter values in the fiscal rule for the risk of a crisis over the next twenty years. The baseline model is very safe with the probability of a crisis over twenty years around 1%. The probability of a crisis increases with a fall in α or $\bar{s}$, or with an increase in the real interest rate, initial debt, or the bounds on the size of the surplus shocks ($\bar{\eta}$). With a fall of α to .1, the probability of a crisis increases substantially to 21%; an increase in initial debt (and the surplus) to 50 raises the probability of a crisis to 27%; larger surplus shocks raise the probability to 19.4%. The real interest rate can have a large impact on sustainability. Doubling the interest rate to .08 raises the probability of a crisis to 57%, consistent with the large number of debt crises in the early 1980's when world interest rates rose.

We also compute the length of time that crises are anticipated before they occur as well as the ratio of the number of episodes[51] in which default is expected relative to the number of crises. Due to the assumption that fiscal shocks are bounded, the government must be in a position in which a negative shock could send it to or above the saddlepath for a crisis

[51] An episode is defined as a string of consecutive periods, greater than or equal to one, in which default is expected.

to occur. Therefore, crises require weak fundamentals and must be anticipated at least one period in advance. Simulation results show that crises can develop very suddenly, being anticipated only in the period immediately preceding the crisis; alternatively, crises can develop slowly, taking as many as nine periods to develop in the baseline case. Additionally, governments manage to escape many anticipated crises. There are between 3.77 and 2.61 times as many episodes in which crises are anticipated, as episodes which culminate in a crisis. The signals to crises ratio generally increases as the probability of a crisis falls. Therefore, this model can be used to explain both crises which take time to develop, as in generation-one currency crisis models, as well as crises which develop suddenly, as in the financial fragility models. And it can explain expectations of crises which never develop.

5.4 Post-Crisis Policy Options

The switch to an active fiscal rule, when faced with a financial crisis, implies a switch to flexible exchange rates unless the authorities can completely eliminate fiscal shocks, a possibility not explicitly allowed here. This is because capital gains and losses on government debt, created by flexible prices and exchange rates, are necessary to keep the system on the saddlepath. These price and exchange rate changes are not defaults because the government has no promise to fix them. However, the government is not necessarily locked into flexible exchange rates forever. If the government wants to return to fixed exchange rates, it is necessary first to maneuver the economy to a position substantially below the saddlepath. It could use the opportunity of a strong positive fiscal shock,[52] sending the system in a southeast direction from the saddlepath to return to a policy of fixed exchange rates (active

[52] This would be even easier is these shocks were autocorrelated.

monetary policy) and passive fiscal policy.[53]

If the government returns to its initial policy, then is again faced with the same switching model as described above, implying some probability of future default. It could retain the same value for α or raise it, thereby reducing the probability of a financial crisis. The choice for the values of λ and α is a choice about the use of default to offset a string of bad luck. Presumably, the choice will depend on the punishments for default imposed by the financial markets compared with the economy's costs of generating surpluses. It is therefore not surprising that over time we see many of the same countries in default, as presented in Reinhart and Rogoff (2004). A country will choose policy allowing default until the circumstances affecting the costs and benefits of default change.

6 Conclusions

Debt crises and currency crises often occur together, suggesting that they could have the same cause. Both default and devaluation reduce government liabilities, implying that both could be caused by fiscal insolvency which generates the need for such a reduction. Generation-one currency crisis models have fiscal insolvency as the cause. This paper provides a public debt crisis model, with fiscal insolvency as the cause, analogous to the generation-one currency crisis model. We define default broadly as reneging on contractual agreements, so that devaluation of a fixed exchange rate and debt restructuring to reduce the contractual value of liabilities are both examples of default. The government can decide how to apportion the reduction in the value of liabilities over various instruments based on costs and benefits. When the calculation favors letting domestic-currency liabilities bear the full costs, the model

[53]For the solution of the model to hold with $E_t \gamma_{t+1} = 0$ after the policy switch, the prospects of returning to a fixed exchange rate must be far enough in the future to avoid affecting this expectation.

becomes a currency crisis model. As such, it generalizes the generation-one currency crisis model by addressing the fiscal solvency problem beyond the crisis date. More generally, the model highlights the role of fiscal insolvency in generating financial crises which are resolved with fiscal reform and possibly also with default.

We assume that fiscal policy follows a passive rule in which the surplus adjusts gradually to debt over time, subject to stochastic shocks. Additionally, we assume that there is an upper bound on the size of the long-run surplus, implying an upper bound of the value of debt. This implies that fiscal policy cannot be unconditionally passive. A string of negative shocks could push debt so high that the surplus could not adjust. We assume that the government continues with passive policy until a crisis forces some change. A public debt crisis occurs when agents refuse to lend to let the government continue with current policy. The timing of this crisis depends on expectations of the government's response to the crisis. We assume that the government reacts with fiscal reform, switching from passive to active policy. Fiscal reform reduces the interest premium, reducing the interest-inclusive value of debt and raises the present-value of expected future surpluses, possibly restoring fiscal solvency without default. If not, default occurs. Default can take the form of exchange rate devaluation or debt restructuring or both.

The debt crisis is preceded by an increase in the interest rate, reflecting the probability of default. Crises can develop suddenly or be expected for long periods of time. Many more crises are anticipated than actually occur. The expectation of fiscal reform, conditional on a crisis, enlarges the space for the debt and surpluses over which passive fiscal policy is feasible. When a crisis does occur, debt is well below its maximum sustainable level. Fiscal reform is sufficient to resolve many crises, but default, broadly defined, is often necessary.

The fiscal reform need not require an increase in seigniorage revenue, as in generation-one currency crisis models,[54] but it will require a shift to flexible exchange rates. These aspects of crisis resolution are consistent with IMF recommendations of fiscal reform and greater exchange rate flexibility. When default is necessary and the costs of a reduction in the value of a particular liability are increasing in the size of the reduction, devaluation and debt restructuring will occur together. Additionally, default magnitudes can be large.

This model should be viewed as initial step in using a regime-switching model, based on insights from the FTPL, to study a public debt crisis. The small-open economy, the simplifications imposed to model the distribution of the surplus shock, which ignore any dynamics associated with explicit cyclical and optimal-tax shocks or with empirical autocorrelation of surplus shocks, are clearly too strong to describe actual economies. However, we should be able to use the framework developed here to consider default in a more complicated, but more realistic, model. Receipt of new information on actual values of debt or on fiscal entitlements affecting the present-value of future surpluses independently of current surplus shocks, could have the effect of moving the system to a point at which debt and the surplus could be above values implied by the active-policy saddlepath, necessitating regime change and possibly default. Imperfectly credible fiscal policy is likely to have implications for the magnitude and frequency of default and for expected default. Furthermore, understanding the positive implications of a policy with a risk of default permits consideration of the optimality of alternative policies in the context of the optimal tax literature and the literature on fixed vs. flexible exchange rates. These explorations are left for future work.

[54]Burnside, Eichenbaum, and Rebelo (2006) make the point that seigniorage revenue does not typically increase after an exchange rate crisis.

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