Homework 3: Chapter 6

Answers

2. Hare: $5000 × $(1.03)^{50}$ = $21,919.50

   Tortoise: $5000 × $(1.01)^{50}$ = $8,223.15

3.

	20 Years Ago	Today	Percent Change
Y	1000	1300	30%
K	2500	3250	30%
N	500	575	15%

(a) $\Delta A/A = \Delta Y/Y - a_K\, \Delta K/K - a_N\, \Delta N/N$

$= 30\% - (0.3 \times 30\%) - 0.7 \times 15\%$

$= 30\% - 9\% - 10.5\%$

$= 10.5\%$

Capital growth contributed 9% ($a_K\, \Delta K/K$), labor growth contributed 10.5% ($a_N\, \Delta N/N$), productivity growth was 10.5%.

(b) $\Delta A/A = 30\% - (0.5 \times 30\%) - (0.5 \times 15\%)$

$= 30\% - 15\% - 7.5\%$

$= 7.5\%$

Capital growth contributed 15% ($a_K\, \Delta K/K$), labor growth contributed 7.5% ($a_N\, \Delta N/N$), productivity growth was 7.5%.

4. (a) $sf(k) = (n + d)k$

$0.3 \times 3k^{.5} = (0.05 + 0.1)k$

$0.9k^{.5} = 0.15k$

$0.9 / 0.15 = k / k^{.5}$

$6 = k^{.5}$

$k = 6^2 = 36$

$y = 3k^{.5} = 3 \times 6 = 18$

$c = y - (n + d)k = 18 - (0.15 \times 36) = 12.6$

(b) $sf(k) = (n + d)k$

$0.4 \times 3k^{.5} = (0.05 + 0.1)k$

$1.2k^{.5} = 0.15k$

$1.2 / 0.15 = k / k^{.5}$

$8 = k^{.5}$

$k = 8^2 = 64$

$y = 3k^{.5} = 3 \times 8 = 24$

$c = y - (n + d)k = 24 - (0.15 \times 64) = 14.4$

(c) $sf(k) = (n + d)k$

$0.3 \times 3k^{..5} = (0.08 + 0.1)k$

$0.9k^{..5} = 0.18k$

$0.9 / 0.18 = k / k^{..5}$

$5 = k^{..5}$

$k = 5^2 = 25$

$y = 3k^{..5} = 3 \times 5 = 15$

$c = y - (n + d)k = 15 - (0.18 \times 25) = 10.5$

(d) $sf(k) = (n + d)k$

$0.3 \times 4k^{..5} = (0.05 + 0.1)k$

$1.2k^{..5} = 0.15k$

$1.2 / 0.15 = k / k^{..5}$

$8 = k^{..5}$

$k = 8^2 = 64$

$y = 4k^{.5} = 4 \times 8 = 32$

$c = y - (n + d)k = 32 - (0.15 \times 64) = 22.4$

5. (a) The destruction of some of a country's capital stock in a war would have no effect on the steady state, because there has been no change in *s*, *f*, *n*, or *d*. Instead, *k* is reduced temporarily, but equilibrium forces eventually drive *k* to the same steady-state value as before.

(b) Immigration raises *n* from n^1 to n^2 in Figure 6.3. The rise in *n* lowers steady-state *k*, leading to a lower steady-state consumption per worker.

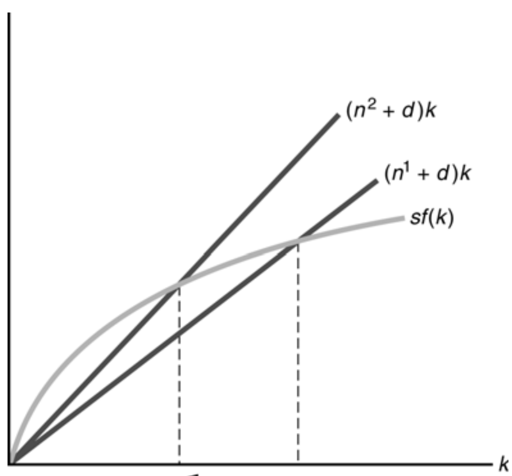

Figure 6.3

(c) The rise in energy prices reduces the productivity of capital per worker. This causes $sf(k)$ to shift down from $sf^1(k)$ to $sf^2(k)$ in Figure 6.4. The result is a decline in steady-state k. Steady-state consumption per worker falls for two reasons: (1) Each unit of capital has a lower productivity, and (2) steady-state k is reduced.

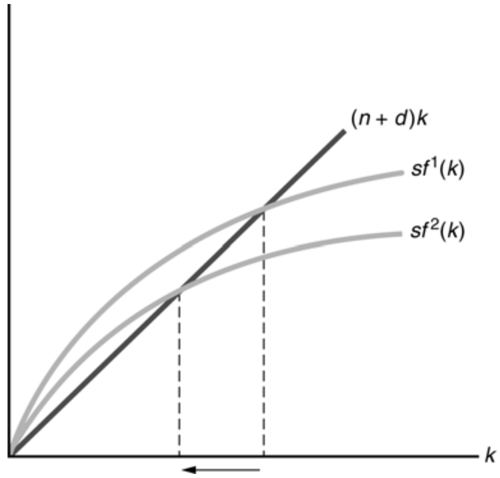

Figure 6.4

(d) A temporary rise in s has no effect on the steady-state equilibrium.

(e) The increase in the size of the labor force does not affect the growth rate of the labor force, so there is no impact on the steady-state capital-labor ratio or on consumption *per worker*. However, because a larger fraction of the population is working, consumption *per person* increases.

6. (a) Solow model

The rise in capital depreciation shifts up the $(n + d)k$ line from $(n + d^1)k$ to $(n + d^2)k$, as shown in Figure 6.5. The equilibrium steady-state capital-labor ratio declines. With a lower capital-labor ratio, output per worker is lower, so consumption per worker is lower (using the assumption that the capital-labor ratio is not so high that an increase in k will reduce consumption per worker). There is no effect on the long-run growth rate of the total capital stock, because in the long run the capital stock must grow at the same rate (n) as the labor force grows, so that the capital-labor ratio is constant.

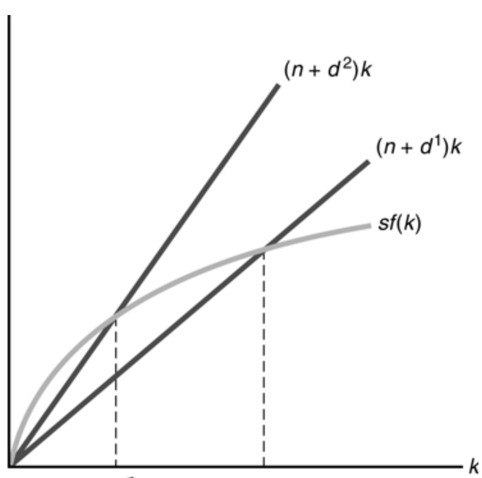

Figure 6.5

(b) Endogenous growth model

In an endogenous growth model, the growth rate of output is $\Delta Y/Y = sA - d$, so the rise in the depreciation rate reduces the economy's growth rate. Similarly, the growth rate of capital equals $\Delta K/K = sA - d$, which also declines when the depreciation rate rises. Since consumption is a constant fraction of output, its growth rate declines as well. So the increase in the depreciation rate reduces the long-run growth rate of the capital stock, as well as long-run capital, output, and consumption per worker.