Monetary Policy in a New Keyneisan Model
Walsh Chapter 8 (cont)
1 New Keynesian Model

• Demand is an Euler equation

\[ x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_t \pi_{t+1}) + u_t \]

• Supply is New Keynesian Phillips Curve

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t \]

– \( x_t \) is output relative to equilibrium level under flexible prices

– \( i_t \) is percent deviation of nominal interest rate relative to long-run equilibrium

– \( \pi_t \) is inflation (long-run equilibrium inflation is zero)
2 Policy Objectives

- Derive loss function from utility of representative agent

\[ V_t = U (Y_t, z_t) - \int_0^1 v (y_t (i), z_t) \]

- \( v (y_t (i), z_t) \) represents disutility of producing good \((i)\)

- \( z_t \) represents preference shocks

- \( Y_t \) is a composite good

\[ Y_t = C_t = \left[ \int_0^1 y_t (i)^{\frac{\theta - 1}{\theta}} \, di \right]^{\frac{\theta}{1-\theta}} \]
- take expected value of second-order approximation of utility about steady state values

\[
E_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\Omega E_t \sum_{i=0}^{\infty} \beta^i \left\{ [\pi_{t+i}]^2 + \lambda (x_{t+i} - x^*)^2 \right\} + \text{t.i.p.}
\]

* t.i.p. represents terms independent of policy, and \( \Omega \) and \( \lambda \) represent combinations of parameters

* \( x^* \) is the steady-state efficient level of the output gap (in absence of distortions from monopolistic competition)

* inflation implies price dispersion
- agents buy more cheap goods and fewer expensive goods

- due to diminishing marginal utility, additional utility from more cheap goods is less than loss in utility from fewer expensive ones, implying reduced welfare

* deviations of output from its non-distorted equilibrium also imply loss

- set $x^* = 0$ by assuming a government subsidy offsets monopoly distortion so that output can be at its optimal level in the absence of distortions

- eliminates inflation bias
Policy conflict between inflation stabilization and output stabilization

- No conflict when supply shock \( e_t \) is zero
  * Assume errors are iid, eliminating dynamics such that expectations of \( \pi_{t+1} \) and \( x_{t+1} \) are both zero
    \[
    x_t = - \left( \frac{1}{\sigma} \right) i_t + u_t.
    \]
  * Set
    \[
    i_t = -\sigma u_t
    \]
  * yielding
    \[
    \pi_t = \kappa x_t = 0
    \]
– Conflict when supply shock ($e_t$) is not zero: Let $e_t > 0$

* Leave $i_t$ unchanged
  
  · $x_t$ is unchanged

  · However, $\pi_t > 0$

* Raise $i_t$
  
  · $x_t < 0$

  · But with right magnitudes $\pi_t$ unchanged

* Trade-off between output gap and inflation
– Add sticky wages and allow a shock to marginal product of labor
  * Conflict because marginal product shock requires real wage adjustment to keep output at full employment
  * Can't stabilize wages and prices and output

• Optimal rate of inflation is zero (ignores zero lower bound on nominal interest)

– Do not get Friedman rule because
  * reduced money holdings due to the positive nominal interest rate impose no costs because money does not yield utility
  * if put money in the utility function, then optimal is somewhere between Friedman rule and zero inflation
3 Optimal Policy

- Loss function

\[ L_t = E_t \sum_{i=0}^{\infty} \frac{1}{2} \beta^i \left\{ \pi^2_{t+i} + \lambda x_{t+i}^2 \right\} \]

- Commitment

- Central bank chooses \( i_{t+i}, \pi_{t+i}, \) and \( x_{t+i} \) to minimize Lagrangian

\[
E_t \sum_{i=0}^{\infty} \beta^i \left\{ \frac{1}{2} \left( \pi^2_{t+i} + \lambda x_{t+i}^2 \right) \\
+ \theta_{t+i} \left( x_{t+i} - E_{t+i} + x_{t+i+1} + \left( \frac{1}{\sigma} \right) (i_{t+i} - E_{t+i} \pi_{t+i+1}) - u_{t+i} \right) \\
+ \psi_{t+i} \left( \pi_{t+i} - \beta E_{t+i} \pi_{t+i+1} - \kappa x_{t+i} - e_{t+i} \right) \right\}
\]
\(- i_{t+i}\)

\[ \sigma^{-1} E_t \theta_{t+i} = 0 \]

choose interest rate to make multiplier zero, that is to satisfy the demand constraint exactly

\(- \pi_{t+i}\)

\[ E_t \left( \beta^i \left( \pi_{t+i} + \psi_{t+i} \right) - \beta^{i-1} \beta \psi_{t+i-1} \right) = 0 \]

\[ E_t \left( \pi_{t+i} + \psi_{t+i} - \psi_{t+i-1} \right) = 0 \quad i \geq 1 \]

\(- \pi_t\)

\[ E_t \left( \pi_t + \psi_t \right) = 0 \]
\[-x_{t+i}\]

\[\beta^i \left( \lambda x_{t+i} - \kappa \psi_{t+i} \right) = 0\]

\[\psi_{t+i} = \frac{\lambda}{\kappa} x_{t+i}\]

- Dynamic inconsistency of policy

* at time \(t\), set

\[\pi_t = -\psi_t = -\frac{\lambda}{\kappa} x_t\]

planning to set

\[\pi_{t+i} = - \left( \psi_{t+i} - \psi_{t+i-1} \right) = -\frac{\lambda}{\kappa} (x_{t+i} - x_{t+i-1})\]

\[x_{t+i} = x_{t+i-1} - \frac{\kappa}{\lambda} \pi_{t+i}\]
* when inflation is high, raise interest rate enough that output gap falls

* promise to keep large negative output gap next period by keeping interest rate high even though shock is gone, reducing expected inflation and improving tradeoff today

* when time $t + i$ comes, reneg on promise and set

$$x_{t+i} = -\frac{\kappa}{\lambda} \pi_{t+i}$$
– **Timeless perspective**: think of policy as having been chosen in the distant past and current values are values chosen from earlier perspective

\[ \pi_{t+i} = -\frac{\lambda}{\kappa} (x_{t+i} - x_{t+i-1}) \]

* Substitute into New Keynesian Phillips Curve

\[ -\frac{\lambda}{\kappa} (x_t - x_{t-1}) = -\beta E_t \frac{\lambda}{\kappa} (x_{t+1} - x_t) + \kappa x_t + e_t \]

\[ (x_t - x_{t-1}) = \beta E_t (x_{t+1} - x_t) - \frac{\kappa^2}{\lambda} x_t - \frac{\kappa}{\lambda} e_t \]

\[ \left(1 + \beta + \frac{\kappa^2}{\lambda}\right) x_t = x_{t-1} + \beta E_t x_{t+1} - \frac{\kappa}{\lambda} e_t \]
* Expectational difference equation: solve using undetermined coefficients

\[ x_t = a_xx_{t-1} + b_x e_t \]

\[ E_t x_{t+1} = a_xx_t + b_x E_t e_{t+1} \]

Let

\[ e_t = \rho e_{t-1} + \varepsilon_t \]

\[ E_t x_{t+1} = a_xx_t + b_x \rho e_t \]

* Substitute into equation in \( x_t \) and equate coefficients to solve for \( a_x \) and \( b_x \)

* Solution for \( x_t \)

\[ x_t = a_xx_{t-1} - \frac{\kappa}{\lambda [1 + \beta (1 - \rho - a_x)]} + \kappa^2 e_t \]
where $a_x$ is stable solution to quadratic equation

$$0 < a_x < 1$$

* solution for $\pi_t$

$$\pi_t = \frac{\lambda}{\kappa} (1 - a_x) x_{t-1} + \frac{\lambda}{\kappa [1 + \beta (1 - \rho - a_x)]} + \kappa^2 e_t$$

* Precommitment policy introduces inertia into inflation and output gap processes even if $\rho = 0$

  · Let $\rho = 0$, and consider optimal response to an increase in $e_t$ (higher inflation for any output gap)
  
  · Optimal policy requires that $x_t$ fall, so raise $i_t$ to get $x_t$ down
  
  · The increase in $e_t$ raises inflation, but less than if output gap had not fallen
Next period, reduce inflation due to last period’s negative output gap, maintaining negative output gap, even though shock has vanished – reduces $E_t \pi_{t+1}$ yielding less inflation for any output gap today, improving trade-off

Time inconsistent since no need to keep output gap negative after shock has vanished

* Optimal equilibrium outcome – nothing about how to attain it
• Discretion

- Minimize loss function period by period, choosing only $x_t$, not all $x_{t+i}$

$$\min_{x_t} \frac{1}{2} \left( \left( \beta E_t \pi_{t+1} + \kappa x_t + e_t \right)^2 + \lambda x_t^2 \right)$$

- First order condition is same as for time 0 problem

$$\kappa \pi_t + \lambda x_t = 0$$

$$\pi_t = -\frac{\lambda}{\kappa} x_t$$

- Substitute into New Keynesian Phillips Curve

$$-\frac{\lambda}{\kappa} x_t = -\beta E_t \frac{\lambda}{\kappa} x_{t+1} + \kappa x_t + e_t$$
\[ x_t = \beta E_t x_{t+1} - \frac{\kappa^2}{\lambda} x_t - \frac{\kappa}{\lambda} e_t \]

\[ \left(1 + \frac{\kappa^2}{\lambda}\right) x_t = \beta E_t x_{t+1} - \frac{\kappa}{\lambda} e_t \]

- Solve expectational difference equation with undetermined coefficients

\[ x_t = \delta e_t \quad E_t x_{t+1} = \rho \delta e_t \]

\[ x_t^d = \frac{-\kappa}{\lambda (1 - \rho \beta) + \kappa^2 e_t} \]

\[ \pi_t^d = \frac{\lambda}{\lambda (1 - \rho \beta) + \kappa^2 e_t} \]

- Unconditional expected value of inflation is zero implying no inflation bias (remember \( x^* = 0 \))
- Stabilization bias since response to $e_t$ differs and $x_{t-1}$ is missing
  * Denominator under discretion is larger, so respond too little to $e_t$ under discretion
  * Equivalently, don’t raise $i_t$ enough, making output gap fall too little and inflation rise too much

- Solve for interest rate implied by policy

\[ i_t^d = E_t \pi_{t+1} + \sigma \left( E_t x_{t+1} - x_t + u_t \right) \]

\[ E_t x_{t+1}^d = \frac{-\kappa}{\lambda(1 - \rho \beta) + \kappa^2 \rho e_t} \]

\[ E_t \pi_{t+1}^d = \frac{\lambda}{\lambda(1 - \rho \beta) + \kappa^2 \rho e_t} \]
\[ i_t^d = E_t \pi_{t+1} + \sigma (E_t x_{t+1} - x_t - u_t) \]

\[ i_t^d = \frac{\lambda \rho + \sigma \kappa (1 - \rho)}{\lambda (1 - \rho \beta) + \kappa^2} e_t + \sigma u_t \]

- Commit to a rule

\[ x_t = b_x e_t \]

- New Keynesian Phillips Curve

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t \]

- Substitute rule into Phillips Curve

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa b_x e_t + e_t \]
– Expectational difference equation in $\pi$ – solve using undetermined coefficients

$$\pi_t = b_\pi e_t \quad E_t \pi_{t+1} = \rho b_\pi e_t$$

$$\pi_t = \frac{1 + \kappa b_x}{1 - \rho \beta} e_t$$

– Substitute into loss function

$$E_t \sum_{i=0}^{\infty} \frac{1}{2} \beta^i (\pi_{t+i}^2 + \lambda x_{t+i}^2) = \sum_{i=0}^{\infty} \frac{1}{2} \beta \left[ i \left( \frac{1 + \kappa b_x}{1 - \rho \beta} \right)^2 + \lambda b_x^2 \right] \sigma_e^2$$

– Minimize with respect to $b_x$

$$b_x = \frac{-\kappa}{\lambda (1 - \rho \beta)^2 + \kappa^2}$$
Substituting into equation for $x_t$

$$x_t = \frac{-\kappa}{\lambda (1 - \rho \beta)^2 + \kappa^2} e_t$$

* Denominator is smaller than under discretion, so gap responds more than under discretion

* Substitute into inflation

$$\pi_t = \left[ \frac{1}{1 - \rho \beta} + \left( \frac{\kappa}{1 - \rho \beta} \right) \left( \frac{-\kappa}{\lambda (1 - \rho \beta)^2 + \kappa^2} \right) \right] e_t$$

$$= \frac{\lambda (1 - \rho \beta)^2}{(1 - \rho \beta) \left( \lambda (1 - \rho \beta)^2 + \kappa^2 \right)} e = \frac{\lambda (1 - \rho \beta)}{\left( \lambda (1 - \rho \beta)^2 + \kappa^2 \right)} e$$

- if $\rho = 0$, identical to discretion

- if $0 < \rho < 1$, inflation responds less to $e$ than under discretion
• to get rule to equal discretionary response, appoint a Rogoff conservative central banker with $\hat{\lambda} = \lambda (1 - \rho \beta)$

• Policy when New Keynesian Phillips Curve has past inflation

$$\pi_t = (1 - \phi) \beta E_t \pi_{t+1} + \phi \pi_{t-1} + \kappa x_t + e_t$$

  – Justify by saying that $\phi$ of firms do not follow Calvo pricing, but update their price based on average, last period’s actual, or last period’s optimal, inflation

  – Optimal commitment

    * Backward-looking term adds forward-looking term to optimal policy

    * Implies optimal inflation depends both on past output gap and on expected future output gap
– Discretion
  * Backward-looking term increases persistence of output gap and inflation gap even when $\rho = 0$

• Targeting Regimes
  – Modify loss function to include target value, inflation under inflation-targeting regime
  – Does not imply that central bank cares only about target variable
    * Flexible targeting – cares about other variables in social loss
    * Strict targeting – cares only about target variable
- Loss function for inflation targeting regime

\[
L^{IT} = \left(\frac{1}{2}\right) E_t \sum_{i=0}^{\infty} \beta^i \left\{ (\pi_{t+i} - \pi_T)^2 + \lambda_{IT} x_{t+i}^2 \right\}
\]

* where \( \lambda_{IT} \) can differ from \( \lambda \) in social loss function

* \( \lambda_{IT} > 0 \) is a flexible inflation target

* if central bank must set interest rate before know current values of variables then loss function becomes

\[
L^{IT} = \left(\frac{1}{2}\right) E_{t-1} \sum_{i=0}^{\infty} \beta^i \left\{ (\pi_{t+i} - \pi_T)^2 + \lambda_{IT} x_{t+i}^2 \right\}
\]
- Optimal policy under assumption that do not know current values:
  maximize subject to

\[ E_{t-1}x_t = E_{t-1}x_{t+1} - \left( \frac{1}{\sigma} \right) (i_t - E_{t-1}\pi_{t+1}) \]

\[ E_{t-1}\pi_t = \beta E_{t-1}\pi_{t+1} + \kappa E_{t-1}x_t + E_{t-1}e_t \]

* Under discretion, first order conditions imply

\[ E_{t-1}x_t = - \left( \frac{\kappa}{\lambda_{IT}} \right) E_{t-1}(\pi_t - \pi_T) \]

* If Fed’s expectation of inflation exceeds target, it should raise the interest rate to reduce expectations of the output gap

* Problems with multiple equilibrium depending on assumptions about how forecasts are determined
Instrument Rules

- Taylor Rule for setting the nominal interest rate
  \[ i_t = r^* + \pi^T + \alpha_x x_t + \alpha_\pi (\pi_t - \pi^T) \]

- Focus is on setting coefficients on output gap and inflation deviation large enough to assure model is globally unstable

- If need globally unstable model for equilibrium to be determinate, then all other policies could yield different equilibria from those intended

  * Walsh is careful to say that the solution for interest rate in response to other variables it the equilibrium response, not something like the instrument rule above

  * Unclear how the central bank establishes that equilibrium response
Optimal policy when economic model is uncertain

- Let effect of output gap on inflation be stochastic
  \[ \pi_t = \beta E_t \pi_{t+1} + \kappa_t x_t + e_t \quad \kappa_t = \bar{\kappa} + \nu_t \]
  - where \( \nu_t \) is distributed iid with zero mean

- Loss function
  \[ L = \frac{1}{2} E_t \left( \pi_t^2 + \lambda x_t^2 \right) \]

- First order conditions with respect to \( x_t \) with inflation determined by New Keynesian Phillips Curve with all shocks iid is
  \[ E_t (\kappa_t \pi_t + \lambda x_t) = 0 \]
- Use New Keynesian Phillips Curve with expected inflation of zero to substitute

\[ E_t (\kappa_t (\kappa_t x_t + e_t) + \lambda x_t) = \kappa^2 (\bar{\kappa} + \sigma_v^2) + \bar{\kappa}e_t + \lambda x_t = 0 \]

* Solving for \( x_t \)

\[ x_t = - \left( \frac{\bar{\kappa}}{\lambda + \bar{\kappa}^2 + \sigma_v^2} \right) e_t \]

* Presence of \( \sigma_v^2 \) in denominator implies output gap should respond less to supply shock (raise interest rate less), implying that inflation should respond more

* Intuition: reducing variability of \( x_t \) offsets additional variability of \( \kappa_t \) on inflation since \( x_t \) and \( \kappa_t \) enter multiplicatively in determining inflation
– Result that interest rate should respond less aggressively to supply shocks when there is model uncertainty is not robust and depends on source of model uncertainty