1 One-Period Wage Rigidity in a Linearized MIU Model

Three rules for linearization about steady state (ss) values (hat’s denote log deviations from ss)

\[ uw = u^{ss} (1 + \hat{u}) w^{ss} (1 + \hat{w}) \approx u^{ss} w^{ss} (1 + \hat{u} + \hat{w}) \]

\[ u^a = (u^{ss})^a (1 + \hat{u})^a \approx (u^{ss})^a (1 + a \hat{u}) \]

\[ \ln u = \ln [u^{ss} (1 + \hat{u})] = \ln u^{ss} + \ln (1 + \hat{u}) \approx \ln u^{ss} + \hat{u} \]
• CES utility with \( b = \phi \) yielding utility separable in consumption and money

\[
    u(c_t, m_t, l_t) = \frac{[ac_t^{1-\phi} + (1 - a) m_t^{1-\phi}]}{1 - \phi} + \psi \frac{l_t^{1-\eta}}{1 - \eta}
\]

• Equilibrium equations of the model where all variables are log deviations from steady state (dropped hat’s)

  – Production function with fixed capital and productivity shock \( e_t \)

    \[
    y_t = (1 - \alpha) n_t + e_t
    \]

  – Goods market equilibrium

    \[
    y_t = c_t
    \]
- Labor demand equates the marginal product of labor with the real wage

\[ y_t - n_t = w_t - p_t \]

- Labor supply equates marginal disutility of work relative to marginal utility of consumption with the relative price of work

\[ \eta \left( \frac{n^{ss}}{1 - n^{ss}} \right) n_t + \phi c_t = w_t - p_t \]

- Euler equation

\[ \phi E_t (c_{t+1} - c_t) - r_t = 0 \]

- Money demand

\[ m_t - p_t = c_t - \frac{1}{\phi i^{ss} i_t} \]
– Fisher relation

\[ i_t = r_t + E_t p_{t+1} - p_t \]

– Money supply process

\[ m_t = \gamma m_{t-1} + s_t \]
• Solution with flexible prices
  – Neoclassical dichotomy with real variables independent of nominal variables

• Solution with sticky wages
  – Assume nominal wage for period $t$ is set in period $t - 1$ at the wage which equates expected labor supply and demand
  – Solve for equilibrium wage under flexible prices and wages
    * labor demand with $y_t$ substituted out
      \[(1 - \alpha) n_t + e_t - n_t = w_t - p_t\]
* solving for $n_t$

$$n_t = \frac{e_t - (w_t - p_t)}{\alpha}$$

* labor supply with $c_t = y_t$ substituted out

$$\tilde{\eta} n_t + \phi [(1 - \alpha) n_t + e_t] = w_t - p_t$$

* solve both for $w_t - p_t$ and equate to solve for $n_t$

$$n_t^* = \left[ \frac{1 - \phi}{1 + \tilde{\eta} + (1 - \alpha)(\phi - 1)} \right] e_t = b_0 e_t$$

* solve for real wage $\omega_t = w_t - p_t$

$$\omega_t^* = \left[ \frac{\tilde{\eta} - \phi}{1 + \tilde{\eta} + (1 - \alpha)(\phi - 1)} \right] e_t = b_1 e_t$$
- Contract wage

\[ w_t^c = E_{t-1} (\omega_t^* + p_t) \]

* real wage

\[ w_t^c - p_t = E_{t-1} (\omega_t^*) + E_{t-1} p_t - p_t \]

- Employment is determined by labor demand

\[-\alpha n_t + e_t = w_t^c - p_t = E_{t-1} (\omega_t^*) + E_{t-1} p_t - p_t \]

\[ n_t = \frac{e_t - E_{t-1} (\omega_t^*) + (p_t - E_{t-1} p_t)}{\alpha} \]

\[ = \left( e_t - E_{t-1} e_t \right) + \left( p_t - E_{t-1} p_t \right) + E_{t-1} n_t^* \]

* implying that price surprises raise employment: an unexpected increase in price reduces the real wage and raises employment
* an unexpected increase in productivity raises labor demand further than expected

- Output is a Lucas aggregate supply equation

* substitute into the production function

\[ y_t = (1 - \alpha)n_t + e_t \]
\[ = \frac{1 - \alpha}{\alpha} [(e_t - E_{t-1}e_t) + (p_t - E_{t-1}p_t)] + (1 - \alpha) E_{t-1}n^*_t + e_t \]

* take \( t - 1 \) expectation

\[ E_{t-1}y_t = - (1 - \alpha) E_{t-1}n^*_t + E_{t-1}e_t \]
and subtract
\[ y_t - E_{t-1}y_t = \frac{1 - \alpha}{\alpha} [(e_t - E_{t-1}e_t) + (p_t - E_{t-1}p_t)] + (e_t - E_{t-1}e_t) \]
\[ = \frac{(e_t - E_{t-1}e_t) + (1 - \alpha)(p_t - E_{t-1}p_t)}{\alpha} \]

* implying that price surprises create output surprises

• Simplified model
  
  – Let productivity disturbances be serially uncorrelated so \( E_{t-1}y_t = 0 \) and let \((e_t - E_{t-1}e_t) = \varepsilon_t\)
  
  \[ y_t = a (p_t - E_{t-1}p_t) + (1 + a) \varepsilon_t \]
  
  – Assume money demand does not depend on the interest rate
  
  \[ m_t - p_t = y_t \]
- Solve for price surprises

\[ p_t - E_{t-1}p_t = m_t - E_{t-1}m_{t-1} - (y_t - E_{t-1}y_t) = s_t - y_t \]

- Substituting into output

\[ y_t = a(s_t - y_t) + (1 + a)\varepsilon_t \]

* solving for output

\[ y_t = \frac{a}{1 + a} s_t + \varepsilon_t \]

* money surprises affect output, but their effect vanishes after one period

  - no persistence

  - persistence in money shocks does not help since persistent part is expected
2 Taylor’s Staggered Nominal Adjustment

- Assumptions
  - Wages are set for two periods with half of wages set each period
  - $x_t$ is the log of the contract wage set at time $t$
  - average wage facing firm at $t$ is
    \[
    w_t = \frac{x_t + x_{t-1}}{2}
    \]
  - assume that price is a constant markup over the wage
    \[
    p_t = w_t + \mu
    \]
    and set $\mu = 0$
– for workers, average expected real wage over contract life is

\[ .5 \left[ x_t - p_t + (x_t - E_t p_{t+1}) \right] = x_t - .5 \left[ p_t + E_t p_{t+1} \right] \]

– assume the average contract wages increases with expected prices and with output

\[ x_t = .5[ p_t + E_t p_{t+1} ] + k y_t \]

• Equilibrium

  – Constant markup implies

\[ p_t = .5 \left( x_t + x_{t-1} \right) \]
- Substituting for $x_t$ into $p_t$

$$p_t = .5 \left[ .5 (p_t + E_{t}p_{t+1}) + ky_t + .5 (p_{t-1} + E_{t-1}p_t) + ky_{t-1} \right]$$

$$= .25 \left[ 2p_t + E_{t}p_{t+1} + p_{t-1} + (E_{t-1}p_t - p_t) \right] + .5k \left[ y_t + y_{t-1} \right]$$

- Solving for $p_t$

$$p_t = .5 (E_{t}p_{t+1} + p_{t-1}) + k [y_t + y_{t-1}] + .5 (E_{t-1}p_t - p_t)$$

- The value for price has forward and backward looking terms, giving price inertia
– Solve for inflation

\[ p_t - p_{t-1} \]

\[ = \frac{1}{2} (E_t p_{t+1} + p_{t-1}) - p_{t-1} + k [y_t + y_{t-1}] + \frac{1}{2} (E_{t-1} p_t - p_t) \]

\[ = \frac{1}{2} (E_t p_{t+1} - p_{t-1}) + k [y_t + y_{t-1}] + \frac{1}{2} (E_{t-1} p_t - p_t) \]

\[ = \frac{1}{2} (E_t p_{t+1} - p_t + p_t - p_{t-1}) + k [y_t + y_{t-1}] + \frac{1}{2} \eta_t \]

\[ = (E_t p_{t+1} - p_t) + 2k [y_t + y_{t-1}] + \eta_t \]

\[ \pi_t = E_t \pi_{t+1} + 2k [y_t + y_{t-1}] + \eta_t \]

no persistence to inflation because inflation does not depend on past inflation
3 Quadratic Adjustment Cost to Changing Prices

- Assumptions
  - All variables are in logs
  - All firms are identical
  - Desired price depends on aggregate price and on economic activity (output gap)

\[ p_t^* (j) = p_t + \alpha x_t \]
– Profits are a decreasing quadratic function of the deviation of actual price from desired price

\[ \Pi_t (j) = -\delta [p_t (j) - p_t^* (j)]^2 = -\delta [p_t (j) - p_t - \alpha x_t]^2 \]

– Costs of adjusting price are quadratic

\[ c_t (j) = \phi [p_t (j) - p_{t-1} (j)]^2 \]
• Firm problem

  – Choose $p_t(j)$ to maximize

  $$
  \sum_{i=0}^{\infty} \beta^i E_t \left[ \prod_{t+i} (j) - c_{t+i} (j) \right]
  = \sum_{i=0}^{\infty} \beta^i E_t \left[ -\delta \left[ p_{t+i} (j) - p_{t+i} - \alpha x_{t+i} \right]^2 - \phi \left[ p_{t+i} (j) - p_{t+i-1} (j) \right]^2 \right]
  $$

• First order condition

  $$
  -\delta \left[ p_t (j) - p_t - \alpha x_t \right] - \phi \left[ p_t (j) - p_{t-1} (j) \right]
  + \beta E_t \phi \left[ p_{t+1} (j) - p_t (j) \right]
  = 0
  $$
– use fact that all firms are identical

\[ \delta [\alpha x_t] - \phi [p_t - p_{t-1}] + \beta E_t \phi [p_{t+1} - p_t] = 0 \]

– solve for inflation

\[ \pi_t = \beta E_t \pi_{t+1} + \frac{\delta}{\phi} [\alpha x_t] \]

– no persistence because no backward looking terms
4 Monopolistic Competition

4.1 Price setting behavior

- Perfect competition – firms are price takers
  - cannot set a higher price because would sell nothing
  - no reason to set a lower price because can sell all want at market price

- Monopolistic competition – firms are price setters
  - Determine how monopolistically competitive firms set price
4.2 Assumptions

- $Y_t$ is a final good produced with a continuum of intermediate inputs

\[ Y_t = \left[ \int Y_t(i)^q \, di \right]^{q^{-1}} \quad 0 < q \leq 1 \]

- Final goods producers operate in competitive markets and maximize profits

\[ P_t Y_t - \int P_t(i) Y_t(i) \, di \]

- First order condition with respect to $Y_t(i)$

\[ P_t \frac{1}{q} \left[ \int Y_t(i)^q \, di \right]^{q^{-1}-1} q Y_t(i)^{q-1} - P_t(i) = 0 \]
Solve for demand for intermediate inputs

\[ P_t Y_t^{1-q} Y_t(i)^{q-1} - P_t(i) = 0 \]

\[ P_t \left( \frac{Y_t(i)}{Y_t} \right)^{q-1} = P_t(i) \]

\[ Y_t^d(i) = Y_t \left( \frac{P_t(i)}{P_t} \right)^{\frac{1}{q-1}} = Y_t \left( \frac{P_t}{P_t(i)} \right)^{\frac{1}{1-q}} \]

where \( q < 1 \) reflects elastic demand
• Zero profits determine the aggregate price level

\[ P_t = \frac{\int P_t(i) Y_t(i) \, di}{Y_t} = \int P_t(i) \frac{Y_t(i)}{Y_t} \, di \]

\[ = \int P_t(i) \left( \frac{P_t(i)}{P_t} \right)^{\frac{1}{q-1}} \, di = \int P_t(i)^{1+\frac{1}{q-1}} P_t^{\frac{1}{1-q}} \, di \]

\[ P_t^{1-\frac{1}{1-q}} = \int P_t(i)^{\frac{q}{q-1}} \, di \]

\[ = \int P_t(i)^{\frac{q}{q-1}} \, di \]

\[ = P_t^{\frac{q}{q-1}} = \int P_t(i)^{\frac{q}{q-1}} \, di \]

\[ P_t = \left[ \int P_t(i)^{\frac{q}{q-1}} \, di \right]^{\frac{q-1}{q}} \]
Intermediate goods production has monopolistic competition

- output

\[ Y_t(i) = K_t(i)^\alpha L_t(i)^{1-\alpha} \]

- factor markets are competitive with prices \( r \) and \( W \)

- profits in intermediate goods firms

\[ \Pi_t(i) = P_t(i) Y_t(i) - r_t K_t(i) - W_t L_t(i) \]

\[ = [P_t(i) - P_t V_t] \left( \frac{P_t}{P_t(i)} \right)^{\frac{1}{1-q}} Y_t \]

\( V_t \) is minimized unit costs of production
First order condition for profit-maximizing price

\[
0 = \frac{\partial \Pi_t(i)}{\partial P_t(i)} = \left\{ \left( \frac{P_t}{P_t(i)} \right)^{\frac{1}{1-q}} - [P_t(i) - P_tV_t] \left( \frac{1}{1-q} \right) \left( \frac{P_t}{P_t(i)} \right)^{\frac{1}{1-q}-1} \frac{P_t}{P_t(i)^2} \right\} Y_t
\]

\[
1 - [P_t(i) - P_tV_t] \left( \frac{1}{1-q} \right) \left( \frac{P_t(i)}{P_t} \right) \frac{P_t}{P_t(i)^2} = 0
\]

\[
1 - [P_t(i) - P_tV_t] \left( \frac{1}{1-q} \right) \frac{1}{P_t(i)} = 0
\]

\[
P_t(i) [(1 - q) - 1] = -P_tV_t
\]
\[ P_t(i) = \frac{P_t V_t}{q} \]

* price of intermediate goods is a markup \( \frac{1}{q} > 1 \) over nominal unit costs

- Labor demand is FO condition on profit maximization with respect to labor
  - for a monopolistic competitor, price will change when output changes
  - use demand curve to eliminate \( P_t(i) \)

\[ P_t \left( \frac{Y_t(i)}{Y_t} \right)^{q-1} = P_t(i) \]

\[ P_t Y_t^{1-q} Y_t(i)^q = P_t(i) Y_t(i) \]
\[ \Pi_t(i) = P_t(i) Y_t(i) - r_t K_t(i) - W_t L_t(i) \]

\[ \frac{\partial \Pi_t(i)}{\partial L_t(i)} = \left( \frac{\partial P_t(i) Y_t(i)}{\partial Y_t(i)} \right) \left( \frac{\partial Y_t(i)}{\partial L_t(i)} \right) - W_t \]

\[ = q P_t Y_t^{1-q} Y_t(i)^q - (1 - \alpha) \frac{Y_t(i)}{L_t(i)} - W_t \]

\[ = q (1 - \alpha) P_t(i) \frac{Y_t(i)}{L_t(i)} - W_t = 0 \]

- solve for real wage

\[ \frac{W_t}{P_t(i)} = q (1 - \alpha) \frac{Y_t(i)}{L_t(i)} < MPL \]
Aggregate symmetric equilibrium

- all prices and outputs are identical

\[ L_t = \frac{q (1 - \alpha) Y_t}{W_t/P_t} \]
5 Time-Dependent Pricing

- The probability that a firm adjusts its price does not depend on the state of the economy, that is on how far its price is from equilibrium.

- Models are more tractable than state-dependent pricing models.

5.1 Taylor Model

- Assume that prices are set for two periods by intermediate goods producers to maximize profits.
• Firm chooses $\tilde{P}_t$ to maximize

$$E_t \left\{ \left[ \tilde{P}_t - P_tV_t \right] \left( \frac{P_t}{\tilde{P}_t} \right)^{\frac{1}{1-q}} Y_t + R_{t+1}^{-1} \left[ \tilde{P}_t - P_{t+1}V_{t+1} \right] \left( \frac{P_{t+1}}{\tilde{P}_t} \right)^{\frac{1}{1-q}} Y_{t+1} \right\}$$

– First order condition

$$E_t \left( \frac{P_t}{\bar{P}_t} \right)^{\frac{1}{1-q}} Y_t \left\{ 1 - \left[ \tilde{P}_t - P_tV_t \right] \left( \frac{1}{1-q} \right) \frac{1}{\tilde{P}_t} \right\}$$

$$+ E_t R_{t+1}^{-1} \left( \frac{P_{t+1}}{\tilde{P}_t} \right)^{\frac{1}{1-q}} Y_{t+1} \left\{ 1 - \left[ \tilde{P}_t - P_{t+1}V_{t+1} \right] \left( \frac{1}{1-q} \right) \frac{1}{\tilde{P}_t} \right\}$$

$$= 0$$
- Solve for $\tilde{P}_t$

\[
E_t P_t^{1-q} Y_t \left\{ \tilde{P}_t (1 - q) - [\tilde{P}_t - P_t V_t] \right\} \\
+ E_t R_{t+1}^{-1} P_{t+1}^{1-q} Y_{t+1} \left\{ \tilde{P}_t (1 - q) - [\tilde{P}_t - P_{t+1} V_{t+1}] \right\} \\
= 0
\]

\[
q \tilde{P}_t E_t \left[ P_t^{1-q} Y_t + R_{t+1}^{-1} P_{t+1}^{1-q} Y_{t+1} \right] = E_t \left[ \frac{2-q}{P_t^{1-q}} V_t Y_t + R_{t+1}^{-1} \frac{2-q}{P_{t+1}^{1-q}} V_{t+1} Y_{t+1} \right]
\]

\[
\tilde{P}_t = \frac{E_t \left[ \frac{2-q}{P_t^{1-q}} V_t Y_t + R_{t+1}^{-1} \frac{2-q}{P_{t+1}^{1-q}} V_{t+1} Y_{t+1} \right]}{q E_t \left[ P_t^{1-q} Y_t + R_{t+1}^{-1} P_{t+1}^{1-q} Y_{t+1} \right]}
\]
– Log linearize equation (1) about steady state in which

\[ \bar{P}^{ss}_t = \frac{P^{ss}V^{ss}}{q} \]

– Note when all firms adjust, as in steady state

\[ V^{ss} = q \]

– Let \( \theta = \frac{2-q}{1-q} \)

– Let \( \bar{p}, p, \) and \( v \) denote percent deviations about zero inflation steady state

\[ \bar{p} = \frac{\bar{P} - \bar{P}^{ss}}{\bar{P}^{ss}} \quad p = \frac{P - P^{ss}}{P^{ss}} \quad v = \frac{V - V^{ss}}{V^{ss}} \]
\[ q [1 + \bar{p}_t] \bar{P}^{ss} P^{ss} \frac{1}{1-q} \left( 1 + \frac{1}{1-q} p_t \right) Y^{ss}(1 + y_t) \]
\[ + q [1 + \bar{p}_t] \bar{P}^{ss} E_t P^{ss} \frac{1}{1-q} \left( 1 + \frac{1}{1-q} p_{t+1} \right) Y^{ss}(1 + y_{t+1}) \]
\[ = V^{ss} Y^{ss} P^{ss} \theta \left( 1 + \theta p_t \right) \left( 1 + v_t \right) \left( 1 + y_t \right) \]
\[ + V^{ss} Y^{ss} P^{ss} \theta E_t \left( 1 + \theta p_{t+1} \right) \left( 1 + v_{t+1} \right) \left( 1 + y_{t+1} \right) \]

- Use \( q = V^{ss}, \bar{P}^{ss} = P^{ss} \), and let all cross products be zero since small changes

\[ 2\bar{p}_t + 1 + \frac{1}{1-q} p_t + y_t + 1 + E_t \left( \frac{1}{1-q} p_{t+1} + y_{t+1} \right) \]
\[ = 1 + \frac{2 - q}{1-q} p_t + v_t + y_t + 1 + E_t \left( \frac{2 - q}{1-q} p_{t+1} + v_{t+1} + y_{t+1} \right) \]
– solve for $\bar{p}_t$ to reveal that price is set at the average of expected future nominal cost

$$\bar{p}_t = \frac{p_t + v_t + E_t (p_{t+1} + v_{t+1})}{2}$$

• Aggregate price level

$$p_t = \frac{1}{2} (\bar{p}_{t-1} + \bar{p}_t)$$

$$E_t p_{t+1} = \frac{1}{2} E_t (\bar{p}_t + \bar{p}_{t+1})$$

– Substitute for aggregate price and expected price into the firm’s choice for price

$$\bar{p}_t = \frac{1}{2} \left[ \frac{1}{2} (\bar{p}_{t-1} + \bar{p}_t) + \frac{1}{2} E_t (\bar{p}_t + \bar{p}_{t+1}) + v_t + v_{t+1} \right]$$
Solving for firm price yields firm price with a backward and a forward-looking component

\[ \bar{p}_t = \frac{1}{2} (\bar{p}_{t-1} + E_t \bar{p}_{t+1}) + (v_t + E_t v_{t+1}) \]

- Place pricing in an aggregate model
  - Assume the deviation of minimized unit costs is proportional to output such that
    \[ v_t = \gamma y_t \]

\[ \bar{p}_t = \frac{1}{2} (\bar{p}_{t-1} + E_t \bar{p}_{t+1}) + \gamma (y_t + E_t y_{t+1}) \]

- Assume simple quantity equation for money demand
  \[ m_t - p_t = y_t \]
\[ \bar{p}_t = \frac{1}{2} \left( \bar{p}_{t-1} + E_t \bar{p}_{t+1} \right) + \gamma (m_t - p_t + E_t (m_{t+1} - p_{t+1})) \]

- Substituting from above for \( p_t \) and \( E_t p_{t+1} \) and solving yields

\[ \bar{p}_t = \frac{1}{2} \left( \frac{1 - \gamma}{1 + \gamma} \right) (\bar{p}_{t-1} + E_t \bar{p}_{t+1}) + \left( \frac{\gamma}{1 + \gamma} \right) (m_t + E_t m_{t+1}) \]

- Assume money follows a random walk and solve

\[ \bar{p}_t = \frac{1}{2} \left( \frac{1 - \gamma}{1 + \gamma} \right) (\bar{p}_{t-1} + E_t \bar{p}_{t+1}) + \left( \frac{\gamma}{1 + \gamma} \right) (2m_t) \]

* method of undetermined coefficients

* guess a solution

\[ \bar{p}_t = a \bar{p}_{t-1} + bm_t \]
* use guess to express

\[ E_t \bar{p}_{t+1} = a \bar{p}_t + bm_t = a^2 \bar{p}_{t-1} + b (1 + a) m_t \]

* substitute into equation for \( \bar{p}_t \)

\[
\begin{align*}
  \frac{a \bar{p}_{t-1} + bm_t}{2} \\
  = & \frac{1}{2} \left( \frac{1 - \gamma}{1 + \gamma} \right) (\bar{p}_{t-1} + a^2 \bar{p}_{t-1} + b (1 + a) m_t) + \left( \frac{\gamma}{1 + \gamma} \right) (2m_t)
\end{align*}
\]

• coefficient on \( \bar{p}_{t-1} \)

\[
a - \frac{1}{2} \left( \frac{1 - \gamma}{1 + \gamma} \right) (1 + a^2) = 0 \quad a = \frac{1 - \sqrt{\gamma}}{1 + \sqrt{\gamma}}
\]

• coefficient on \( m_t \)

\[
b \left( 1 - \frac{1}{2} \left( \frac{1 - \gamma}{1 + \gamma} \right) (1 + a) \right) = 2 \left( \frac{\gamma}{1 + \gamma} \right) \quad b = \frac{2 \sqrt{\gamma}}{1 + \sqrt{\gamma}}
\]
solution

\[ \bar{p}_t = a\bar{p}_{t-1} + (1 - a)(m_t) \]

strong persistence with large \( a \) requiring small \( \gamma \), equivalently a flat aggregate supply curve

solving for price

\[ p_t = \frac{1}{2} (\bar{p}_{t-1} + \bar{p}_t) = a p_{t-1} + \frac{1}{2} (1 - a)(m_t + m_{t-1}) \]
6 Calvo model

• Assumptions
  
  – In any period, a firm can adjust prices with probability $1 - \omega$
  
  – The interval between adjustments is random with mean $\frac{1}{1-\omega}$
    
    * probability that a firm can adjust in one period is $1 - \omega$
    
    * in two periods is probability that did not adjust in period 1 times probability can adjust in period 2 $\omega (1 - \omega)$
    
    * in three periods is $\omega^2 (1 - \omega)$
* expected duration between price adjustments is

\[
1 - \omega + 2\omega (1 - \omega) + 3\omega^2 (1 - \omega) + ... = \frac{1}{1 - \omega}
\]

- When a firm is allowed to adjust (because the Calvo fairy came by) he adjusts price \(p_t\) to minimize discounted expected squared deviation between his price and optimal price going forward

\[
E_t \left\{ (p_t - p_t^*)^2 + \omega \beta (p_t - p_{t+1}^*)^2 + (\omega \beta)^2 (p_t - p_{t+2}^*)^2 + \ldots \right\}
\]

note that the optimal price can be changing, but the agent's price choice is constant

- Optimal choice for price

\[
E_t \sum_{i=0}^{\infty} 2 (\omega \beta)^i (p_t - p_{t+i}^*) = 0
\]
\[ pt \left( \frac{1}{1 - \omega \beta} \right) = E_t \sum_{i=0}^{\infty} (\omega \beta)^i p_{t+i}^* \]

- Let \( x_t \) be optimal price for all firms adjusting at time \( t \)

\[ p_t = x_t = (1 - \omega \beta) E_t \sum_{i=0}^{\infty} (\omega \beta)^i p_{t+i}^* \]

price set today is a weighted average of current and expected future desired prices

\[ x_t = (1 - \omega \beta) p_t^* + \omega \beta E_t x_{t+1} \]

price set today is a weighed average of current optimal price and expected future prices
• Add to macro model

  – Assume

    \[ p_t^* = p_t + \gamma y_t + \epsilon_t \]

  – Aggregate price level \((p_t)\)

    * \(\omega\) of firms retain previous price

    * \(1 - \omega\) of firms are drawn at random and allowed to change price

    * random drawing leaves distribution of non-adjusting firms unchanged
      and therefore aggregate price of non-adjusting firms unchanged

    \[ p_t = (1 - \omega) x_t + \omega p_{t-1} \]
* using equations for \( x_t \) and \( p_t^* \)

\[
x_t = (1 - \omega \beta)(p_t + \gamma y_t + \epsilon_t) + \omega \beta E_t x_{t+1}
\]

* take equation for \( p_t \) forward one period and solve for \( E_t x_{t+1} \)

\[
E_t x_{t+1} = \frac{E_t p_{t+1} - \omega p_t}{1 - \omega}
\]

* substitute into \( x_t \)

\[
x_t = (1 - \omega \beta)(p_t + \gamma y_t + \epsilon_t) + \omega \beta \frac{E_t p_{t+1} - \omega p_t}{1 - \omega}
\]

* substitute into \( p_t \) and rearrange to get a Phillips curve

\[
p_t = (1 - \omega) \left[ (1 - \omega \beta)(p_t + \gamma y_t + \epsilon_t) + \omega \beta \frac{E_t p_{t+1} - \omega p_t}{1 - \omega} \right] + \omega p_{t-1}
\]
\[ p_t = (1 - \omega) [(1 - \omega \beta) (p_t + \gamma y_t + \epsilon_t)] + \omega \beta (E_t p_{t+1} - p_t + p_t - \omega p_t) + \omega p_{t-1} \]

\[ p_t (1 - (1 - \omega) (1 - \omega \beta) - \omega \beta (1 - \omega)) = (1 - \omega) (1 - \omega \beta) (\gamma y_t + \epsilon_t) + \omega \beta (E_t p_{t+1} - p_t) + \omega p_{t-1} \]

\[ (p_t - p_{t-1}) \omega = (1 - \omega) (1 - \omega \beta) (\gamma y_t + \epsilon_t) + \omega \beta (E_t \pi_{t+1}) \]

\[ \pi_t = \frac{(1 - \omega) (1 - \omega \beta)}{\omega} (\gamma y_t + \epsilon_t) + \beta (E_t \pi_{t+1}) \]

- no backward-looking terms in the aggregate – individual prices can be stuck for substantial periods

- depends on output and expected future inflation, but as \( \omega \) increase, number of adjusting firms falls, inflation is less responsive to output
7 State-Dependent Pricing

- Firms with prices furthest from their optimal values will choose to adjust compared with a random sample in Calvo or those who adjusted longest ago in Taylor
  - Firms which adjust will be further away from the optimum and will make larger adjustments
  - For a large shock, more firms will adjust than for a small shock
  - On both counts, aggregate price tends to be less sticky
  - Caplin and Spulber (1987) have a model in which individual prices are sticky, but aggregate prices are not
• Models are less tractable but very interesting
  
  – If effectiveness of monetary policy depends on price not reacting to monetary shocks, state dependent pricing could have very different implications
8 Empirical Evidence on Price Stickiness

- Five facts (Nakamura and Steinsson (2008))

1. Many price changes are associated with sales. Excluding sales doubles the median duration between price changes from 4.5 months, including sales, to 10 months, excluding them.

2. One-third of non-sale price changes are price decreases

3. The frequency of price increases is positively correlated with the inflation rate, while the frequency of price decreases and the size of price changes are not correlated with inflation

4. The frequency of price changes is seasonal, with higher frequency in the first quarter
5. The probability that the price of an item changes declines during the first few months after a price change

Facts 1, 2, 3 are consistent with fixed menu costs and Calvo fairy, while 4 and 5 are not.

- More facts (Klenow and Kryvtsov 2008)

1. Higher frequency of price changes, 7 months excluding sales

2. While many price changes are large, many are also small

3. Some prices remain fixed for long periods
4. Variation in the size of price changes rather than variation in their frequency is responsible for variance in aggregate inflation.
9 Empirical Evidence on New Keynesian Phillips Curve

\[
\pi_t = \frac{(1 - \omega)(1 - \omega\beta)}{\omega} (\gamma y_t + \epsilon_t) + \beta (E_t \pi_{t+1})
\]

\[
= \beta (E_t \pi_{t+1}) + \kappa (\gamma y_t + \epsilon_t)
\]

\[
= \kappa \sum_{i=0}^{\infty} \beta^i E_t (\gamma y_{t+i} + \epsilon_{t+i})
\]

- Persistence in inflation
  - none due to price stickiness itself
  - persistence if $\gamma y_{t+i} + \epsilon_{t+i}$ is persistent
– when we embed in fully-specified model, we will show no persistence to $\gamma Y_{t+i}$

– data has persistence in inflation
• Does the model fit the data without the standard backward-looking term

  – general answer is no

  – inflation in the data is persistent, implying equation needs a backward-looking term

  – fixes add a backward-looking term by assuming that non-adjusting firms index their price to past inflation
• Output term really measures marginal cost (see this in general equilibrium model later)
  
  – use Cobb-Douglas production function to construct a term for marginal cost and replace output with marginal cost

  – early results positive, but weaker over time