Walsh Chapter 4

Part 1: Fiscal Policy, Prices, and Inflation
1 Government Budget Constraint

1.1 Flow Budget Constraint

- Fiscal Authority

\[ G_t + i_{t-1}B^T_{t-1} = T_t + B^T_t - B^T_{t-1} + RCB_t \]

where superscript \( T \) denotes total bonds and \( RCB \) denotes receipts from the central bank

- Sources of revenue are taxes, increases in total bonds, and receipts from the central bank

- Uses of revenue are for spending and interest payments
• Monetary Authority

\[ B^m_t - B^m_{t-1} + RCB_t = i_{t-1}B^m_{t-1} + H_t - H_{t-1} \]

where \( H \) is high-powered money

– Sources of revenue are interest on bonds and increases in high-powered money

– Uses of revenue are for purchases of bonds and payments to treasury
- Consolidated budget constraint: add, letting $B$ denote bonds held by the public

\[ B = B^T - B^m \]

\[ G_t + i_{t-1}B_{t-1} = T_t + B_t - B_{t-1} + H_t - H_{t-1} \]

- Sources of revenue are taxes, increases in bonds held by the public, and increases in high-powered money

- Uses of revenue are for spending and interest payments
• Consolidated budget constraint in real terms

\[ b_t + h_t = g_t - t_t + \frac{(1 + i_{t-1}) b_{t-1} + h_{t-1}}{1 + \pi_t} \]

where \( t \) is real taxes and

\[ \frac{1}{1 + \pi_t} = \frac{P_{t-1}}{P_t} \]
• How does inflation affect the budget constraint, allowing uncertainty on inflation: must solve for $1 + i_{t-1}$

- Let consumer’s value function by

$$V_t(\omega) = \max \{ u(c_t, m_t) + \beta E_t V(\omega_{t+1}) \}$$

- Subject to

$$\omega_t = c_t + b_t + h_t + k_t$$
$$= f(k_{t-1}) + (1 - \delta) k_{t-1} - t_t + \left[ \frac{(1 + i_{t-1}) b_{t-1} + m_{t-1}}{1 + \pi_t} \right]$$
– FO conditions

\[ u_c(c_t, m_t) = \beta E_t V_\omega(\omega_{t+1}) \left( \frac{1 + i_t}{1 + \pi_{t+1}} \right) \]

\[ = \beta E_t V_\omega(\omega_{t+1}) (f_k(k_t) + (1 - \delta)) \]

\[ \equiv \beta E_t V_\omega(\omega_{t+1}) (1 + r_t) \]

– Assume the utility function yields superneutrality (making \( E_t V_\omega(\omega_{t+1}) \) independent of money) and solve for the nominal interest rate

\[ 1 + i_t = \frac{1 + r_t}{E_t \left( \frac{1}{1 + \pi_{t+1}} \right)} \]
Substitute into the government budget constraint

\[ b_t + h_t = g_t - t_t + \frac{(1 + r_{t-1}) b_{t-1}}{E_{t-1} \left( \frac{1}{1 + \pi_t} \right) (1 + \pi_t)} + \frac{h_{t-1}}{1 + \pi_t} \]

* inflation reduces the outstanding value of money, generating real revenue – seigniorage

* only unexpected inflation affects the real value of bonds – expected inflation causes a corresponding increase in the nominal interest rate
1.2 Intertemporal Budget Constraint

- flow budget constraint

\[ b_t + h_t = g_t - t_t + \frac{1 + i_{t-1}}{1 + \pi_t} b_{t-1} + \frac{h_{t-1}}{1 + \pi_t} \]

- define ex post real interest rate as

\[ 1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}} \]

- solve for present-value budget constraint
add present value debt terms

\[ b_0 - (1 + r_{-1})b_{-1} \]
\[ + \left( \frac{1}{1 + r_0} \right) (b_1 - (1 + r_0)b_0) \]
\[ + \left( \frac{1}{1 + r_0} \right) \left( \frac{1}{1 + r_1} \right) (b_2 - (1 + r_1)b_1) \]
\[ + \ldots \]
\[ = -(1 + r_{-1})b_{-1} + \lim_{T \to \infty} b_T \prod_{j=0}^{T-1} \left( \frac{1}{1 + r_j} \right) \]
• add present value high-powered money terms, defining

\[ R_{0,t} = \prod_{j=0}^{t-1} \left( \frac{1}{1 + r_j} \right) \]

– method 1

\[ \sum_{t=0}^{\infty} \left( h_t - \frac{h_{t-1}}{1 + \pi_t} \right) R_{0,t} \]

where first discount factor \( (R_{0,0}) \) is understood to be unity.

\[ h_t - \frac{h_{t-1}}{1 + \pi_t} = \frac{H_t}{P_t} - \frac{H_{t-1} P_{t-1}}{P_t} = \frac{H_t - H_{t-1}}{P_t} \]

\[ = \left( \frac{H_t - H_{t-1}}{H_{t-1}} \right) \frac{H_{t-1} P_{t-1}}{P_t} = \frac{\theta h_{t-1}}{1 + \pi_t} \]
* by definition

\[ h_t = \frac{(1 + \theta) H_{t-1}}{(1 + \pi_t) P_{t-1}} = \frac{1 + \theta}{1 + \pi_t} h_{t-1} \]

* solving for \( h_{t-1} \)

\[ h_{t-1} = \frac{1 + \pi_t}{1 + \theta} h_t \]

* therefore

\[ \frac{\theta h_{t-1}}{1 + \pi_t} = \frac{\theta}{1 + \theta} h_t \]

PV seigniorage = \( \sum_{t=0}^{\infty} \left( \frac{\theta}{1 + \theta} \right) h_t R_{0,t} \)
IBC becomes

\[(1+r_{-1})b_{-1} = \sum_{t=0}^{\infty} \left( t_t - g_t + \left( \frac{\theta}{1 + \theta} \right) h_t \right) R_{0,t} + \lim_{T \to \infty} b_T R_{0,T-1} \]
method 2 – collect terms on money for each particular period

\[ h_0 - \left( \frac{P_{-1}}{P_0} \right) h_{-1} \]
\[ + \left( \frac{1}{1 + r_0} \right) \left( h_1 - \left( \frac{P_0}{P_1} \right) h_0 \right) \]
\[ + \left( \frac{1}{1 + r_0} \right) \left( \frac{1}{1 + r_1} \right) \left( h_2 - \left( \frac{P_1}{P_0} \right) h_1 \right) \]
\[ + \ldots \ldots \]
\[ = -\left( \frac{P_{-1}}{P_0} \right) h_{-1} + h_0 \left( 1 - \left( \frac{1}{1 + r_0} \right) \frac{P_0}{P_1} \right) \]
\[ + \left( \frac{1}{1 + r_0} \right) h_1 \left( 1 - \left( \frac{1}{1 + r_1} \right) \frac{P_1}{P_0} \right) \]
\[ + \ldots + \lim_{T \to \infty} h_T \prod_{j=0}^{T-1} \left( \frac{1}{1 + r_j} \right) \]
\[ = -\left( \frac{P_{-1}}{P_0} \right) h_{-1} + \sum_{t=0}^{\infty} \left( \frac{i_t}{1 + i_i} \right) h_t R_{0,t} + \lim_{T \to \infty} h_T \prod_{j=0}^{T-1} \left( \frac{1}{1 + r_j} \right) \]
setting limit to zero implies

$$PV \text{ seigniorage} = \sum_{t=0}^{\infty} \left( \frac{\theta}{1 + \theta} \right) h_t R_{0,t} = \sum_{t=0}^{\infty} \left( \frac{i_t}{1 + i_t} \right) h_t R_{0,t} - \frac{H_{-1}}{P_0}$$

- Present value seigniorage is the present value of interest expenditures on money less initial real money valued at $P_0$

- treating seigniorage as interest expenditures on money overstates its value, since it does not subtract $(\frac{P_{-1}}{P_0}) h_{-1}$

- cannot get seigniorage without money growth.
* setting limit terms to zero, IBC becomes

$$\frac{(1 + i_{-1})B_{-1} + H_{-1}}{P_0} = \sum_{t=0}^{\infty} \left( t_t - g_t + \left( \frac{i_t}{1 + i_t} \right) h_t \right) R_{0,t}$$

* Fiscal Theory of the Price Level

  - Let $g_0$ increase with no offsetting increase in present value taxes
  - Requires $P_0$ to increase to reduce the government’s outstanding debt
2 Price and Inflation under perfect foresight

2.1 Monetary Policy

- Monetary policy determines expected inflation

\[
\frac{P_t}{P_{t+1}} = \frac{1 + r_t}{1 + i_t}
\]

- \(1 + r_t\) determined by real part of economy, independent of money or monetary growth

- monetary authority can choose \(i_t\), thereby controlling \(\pi_{t+1}\)
beginning with $t = 0$

\[
\frac{P_0}{P_1} = \frac{1 + r_0}{1 + i_0}
\]

choice of $i_0$ determines $P_1$ relative to $P_0$, but what determines $P_0$?
2.2 Determination of $P_0$

- Fiscal Policy and the Fiscal Theory of the Price Level

- Agent intertemporal budget constraint with endowment economy

$$\sum_{t=0}^{\infty} \left( y_t - c_t - t_t - \left( \frac{i_t}{1 + i_i} \right) h_t \right) R_{0,t} + \frac{(1 + i_{t-1}) B_{-1} + H_{-1}}{P_0} = 0$$

- Solving for consumption

$$\sum_{t=0}^{\infty} c_t R_{0,t} = \frac{(1 + i_{t-1}) B_{-1} + H_{-1}}{P_0} + \sum_{t=0}^{\infty} \left( y_t - t_t - \left( \frac{i_t}{1 + i_i} \right) h_t \right) R_{0,t}$$
- Goods market equilibrium

\[ y_t = c_t + g_t \]

- Substituting into agent's IBC yields government IBC

\[
\sum_{t=0}^{\infty} \left( g_t - t_t - \left( \frac{i_t}{1 + i_t} \right) h_t \right) R_{0,t} + \frac{(1 + i_{t-1}) B_{-1} + H_{-1}}{P_0} = 0
\]

- FTPL

\[
\sum_{t=0}^{\infty} s_t R_{0,t} = \frac{(1 + i_{t-1}) B_{-1} + H_{-1}}{P_0}
\]

* Consider a tax cut which raises disposable income

  - The tax cut is not offset by future tax increases or by government spending reduction
PV agent wealth is up, implying agent wants more consumption

As agent tries to raise consumption \( P_0 \) increases

The increase in price reduces wealth until they want the same consumption

Using FTPL, tax cut reduces PV surpluses, requiring reduction in liabilities, ie real debt
• Ricardian world

  – * Tax cut must be offset by future tax increases or by government spending reduction to that government IBC holds at any price level

  * \( P_0 \) is indeterminate
2.3 Controlling Inflation

- Sargent and Wallace "Unpleasant Monetarist Arithmetic"

  - Put time subscripts on monetary growth rates

\[
(1+r_{-1})b_{-1} = \sum_{t=0}^{\infty} \left( t_t - g_t + \left( \frac{\theta_t}{1 + \theta_t} \right) h_t \right) R_{0,t} + \lim_{T \to \infty} b_T R_{0,T-1}
\]

  - limit term must be zero from household’s transversality condition

  - bonds are real, and real interest rate is offered on real bonds, so inflation does not affect debt

  - increase in $g_t$ with no corresponding change in $t_t$ must raise $\theta$ eventually, and the longer the increase is postponed, the larger the eventual increase must be
– calls for independent monetary authority which cannot be forced to comply with fiscal policy
implications for current price level

\[
\sum_{t=0}^{\infty} \left( \frac{\theta_t}{1 + \theta_t} \right) h_t R_{0,t} = \sum_{t=0}^{\infty} \left( \frac{i_t}{1 + i_i} \right) h_t R_{0,t} - \frac{H_{-1}}{P_0}
\]

* increase in \( \theta_t \) will increase \( i_t \) and agents will want less real money

* cannot have future anticipated jump in \( P \)

* therefore, expectation that \( P \) will rise in the future (so real money right when \( i \) increases) implies that it rises today

* fiscal expansion causes inflation today even if the monetary authority does not accommodate with money growth until the future
• Compare effects of fiscal policy under UMA and FTPL

  – UMA

  \[(1 + r_{-1})b_{-1} = \sum_{t=0}^{\infty} \left( t_t - g_t + \left( \frac{\theta}{1 + \theta} \right) h_t \right) R_{0,t} \]

  – FTPL

  \[\frac{(1 + i_{t-1}) B_{-1} + H_{-1}}{P_0} = \sum_{t=0}^{\infty} \left( t_t - g_t + \left( \frac{i_t}{1 + i_i} \right) h_t \right) R_{0,t} \]

  With nominal bonds, price level increase has substantial affect on government debt, even if money growth does not rise.

  * Non-monetary economy – \( \lim_{H \to 0} \), can still determine price level
3 Price and Inflation under Uncertainty

3.1 Present value surplus shocks create shocks to $P_0$

- Define the present value of future real surpluses at time 0

$$\hat{s}_0 = \frac{1}{P_0} \sum_{t=0}^{\infty} P_t s_t \frac{1}{\prod_{j=1}^{t} (1 + i_{t-j})}$$

- Let nominal initial government debt be

$$D_{-1} = (1 + i_{t-1}) B_{-1} + M_{-1}$$
• Government intertemporal budget constraint

\[ P_0 \hat{s}_0 = D_{-1} \]

– Let \( D_{-1} = 0 \)

* \( \hat{s}_0 \) cannot be stochastic and must equal 0 to satisfy IBC

* FTPL cannot apply with zero outstanding debt

* initially a government must plan on no PV surpluses or deficits

* if a deficit in first period, must be planning PV surpluses after that
• PV surplus shocks create price shocks
  
  - $\hat{s}_1$ can be stochastic
    
    * if $D_0 > 0$

$$P_0 \hat{s}_0 = P_0 s_0 + \frac{P_1 \hat{s}_1}{1 + i_0}$$

$$P_1 \hat{s}_1 = D_0$$

  * positive jumps in $\hat{s}_1$ must be offset by negative jumps in $P_1$

$$\hat{s}_1 = \frac{D_0}{P_1}$$

  * going forward, government cannot plan future surplus adjustments that violate this budget constraint
future surplus shocks, which cause actual PV surpluses to deviate from $\hat{s}_1$, must be unexpected
• Surplus shocks cannot generate systematic revenue (no free lunch)
  
  – \( \hat{s}_1 \) is stochastic, but it is subject to restrictions
    
    * monetary authority sets \( E_0 \pi_1 \) with its choice of \( i_0 \)
      
      \[
      E_0 (\hat{s}_1) = E_0 \frac{D_0}{P_1} = \frac{d_0}{1 + \pi^*}
      \]
    
    * cannot generate revenue with "surprise" surplus reductions each period because will not be a surprise

• A government with no shocks to \( \hat{s}_1 \) has fiscal policy which is Ricardian and passive – no effect on prices

• A government with shocks to \( \hat{s}_1 \) has fiscal policy which is dominant, non-Ricardian, and active – determines price level
3.2 Observational Equivalence

- Government budget constraint has to hold whether debt responds to the surplus or the surplus responds to debt

- How does real debt respond to a fall in the surplus?
  - Canzonerie et al showed that real debt rises when the surplus falls
    * due to flow budget constraint, not accounting for change in price
  - If surpluses are autocorrelated, then \( s \) down implies \( \hat{s} \) down, and FTPL says real debt should fall
  - Cochrane shows that the only way to raise real debt with a fall in the surplus is for agents to believe \( \hat{s} \) has gone up so the government can pay the new debt
• Use policy regressions, with the surplus regressed on lagged debt