Monetary Policy Operating Procedures
Walsh Chapter 11
1 Monetary Policy Terminology

- Instruments – policy variables directly controlled by central bank
  - interest rate charged on bank reserves (discount rate)
  - required reserve ratio (zero now)
  - central bank assets (Treasury securities, private market assets)
• Operating targets – endogenous variables closely affected by instrument
  – bank reserves
  – short-term interest rate (Fed funds rate)

• Intermediate targets – endogenous variables which provide information useful in forecasting variables of ultimate interest (goal variables)
  – money growth rate
  – exchange rate
• Policy goals – endogenous variables central bank ultimately seeks to affect
  – in US, central bank has dual mandate: inflation and unemployment
  – in Europe, central bank responsible only for inflation
2 What Instrument and/or Operating Target?

2.1 Poole (1970)

- Simple IS-LM model with fixed prices

\[ y_t = -\alpha i_t + u_t \]

\[ m_t = y_t - c i_t + v_t, \]

where disturbances are mean zero, iid, and uncorrelated

- Policy-maker chooses either \( i_t \) or \( m_t \) to minimize loss function given by

\[ E(y_t)^2, \]

where expectation does not include knowledge of disturbances
– When $m_t$ is choice variable, solution of IS-LM for output yields

$$y_t = \frac{\alpha m_t + cu_t - \alpha v_t}{\alpha + c}$$

* Choosing $m_t$ to minimize expected squared deviations of output yields

$$E_m(y_t)^2 = \frac{c^2 \sigma^2_u + \alpha^2 \sigma^2_v}{(\alpha + c)^2}$$

* Variance of both disturbances matter with coefficients less than unity

– When $i_t$ is the choice variable, solution of IS-LM for output yields

$$E_i(y_t)^2 = \sigma^2_u$$

* only variance of aggregate demand shock matters

* completely eliminate effects from variance of money demand
- Prefer interest rate iff

\[ E_i (y_t)^2 < E_m (y_t)^2 \]

\[ \sigma_u^2 < \frac{c^2 \sigma_u^2 + \alpha^2 \sigma_v^2}{(\alpha + c)^2} \]

\[ \sigma_v^2 > \left( 1 + \frac{2c}{\alpha} \right) \sigma_u^2. \]

* When variance of money demand is high, prefer to fix the interest rate because this eliminates effects due to variance of money demand.

* Slopes matter too because they affect responsiveness of output to interest rate.
2.2 Extension: Endogenous Money

- Central bank has control over base money \((b_t = \text{currency} + \text{bank reserves})\)
  
  - Actual money determined endogenously by bank decisions to lend
    
    \[ m_t = b_t + h i_t + \omega_t \]

  - As the interest rate increases, banks lend more, increasing demand deposits, thereby increasing M1 (currency + checkable deposits)

  - With instrument being the monetary base, choose \(b\) which minimizes expected squared output to yield output as
    
    \[ y_t = \frac{(c + h) u_t - \alpha (v_t - \omega_t)}{\alpha + c + h} \]
\[ E_b(y_t)^2 = \frac{(c + h)^2 \sigma_u^2 + \alpha^2 (\sigma_v^2 + \sigma_\omega^2)}{(\alpha + c + h)^2} \]

– With interest rate policy, expected squared output deviations unchanged

– Prefer interest rate policy iff

\[ \sigma_v^2 + \sigma_\omega^2 > \left( 1 + \frac{2(c + h)}{\alpha} \right) \sigma_u^2 \]

– Since money supply disturbances do not affect output under interest rate rule, but do under money base rule, interest rate rule more likely to dominate
2.3 General Comments

• Realistic assumption that monetary policy cannot continuously view disturbances or endogenous variables and react to them

• Prefer interest rate policy if disturbances in financial markets are dominate

• Model ignores inflation and inflationary expectations – modern central banks include inflation in objective function
3 Policy Rules

- Adjust monetary base (policy instrument) in response to interest rate movements (intermediate target).

\[ b_t = \mu i_t \]

- Implying that money is given by

\[ m_t = (\mu + h) i_t + \omega_t \]

Substituting for \( m_t \) using

\[ m_t = - (\alpha + c) i_t + u_t + v_t \]

and solving for \( i_t \)

\[ i_t = \frac{v_t + \omega_t + u_t}{\alpha + c + \mu + h} \]

* Note, a large value for \( \mu \) reduces the variance of the interest rate
– Special cases

* monetary base operating procedure: monetary base is fixed and does not respond to interest rates $\mu = 0$

* money supply operating procedure $\mu = -h$ implies $m_t = \omega_t$ such that monetary is on average fixed at zero, but subject to money supply disturbances

* interest rate operating procedure: fix interest rate by setting $\mu \to \infty$
– Solution for output with this policy rule

\[ y_t = \frac{(c + \mu + h) u_t - \alpha (v_t - \omega_t)}{\alpha + c + \mu + h} \]

* with output variance given by

\[ \sigma_y^2 = \frac{(c + \mu + h)^2 \sigma_u^2 + \alpha^2 \left( \sigma_v^2 + \sigma_\omega^2 \right)}{(\alpha + \mu + c + h)^2} \]

* minimizing output variance with respect to \( \mu \) yields the optimal value for \( \mu \) as

\[ \mu^* = - \left[ c + h - \frac{\alpha \left( \sigma_v^2 + \sigma_\omega^2 \right)}{\sigma_u^2} \right] \]

* None of the special cases is optimal
– Special cases

* No variance to money supply or money demand \( \sigma_v^2 = \sigma_\omega^2 = 0 \)
  
  - Monetary base should fall as interest rate rises
  
  - Interest rate will rise as aggregate demand rises
  
  - Reduce money to offset rise in aggregate demand further increasing interest rate
  
  - Poole’s analysis would have chosen a monetary base procedure failing to reduce the money supply
* With positive variances in financial markets, a rise in the interest rate can be due to an increase in aggregate demand, a decrease in money supply or an increase in money demand

  · If the disturbance is in the goods market, want decrease in the money base to increase rise in the interest rate

  · If in the money market, want increase in money base to offset increase in interest rate (leaning against the wind)

  · Appropriate response depends on relative magnitudes
• Signal extraction problem

  – If the monetary authority could observe disturbances and respond to them

    \[ b_t = \mu_u u_t + \mu_v v_t + \mu_\omega \omega_t \]

  * Solve for output with this policy rule

    \[ y_t = \frac{(c + \alpha \mu_u + h) u_t - \alpha [(1 - \mu_v) v_t - (1 + \mu_\omega) \omega_t]}{\alpha + c + \mu + h} \]

  * With perfect information about disturbances can set output to zero by choosing

    \[ \mu_u = -\frac{c + h}{\alpha} \quad \mu_v = 1 \quad \mu_\omega = -1 \]

    \[ b = -\frac{c + h}{\alpha} u_t + v_t - \omega_t \]
* In reality, monetary authority observes interest rate and must use the interest rate to infer the disturbances such that forecasts are given by

\[ \hat{u}_t = \delta u i_t \quad \hat{v}_t = \delta v i_t \quad \hat{\omega}_t = \delta \omega i_t \]

· Substituting into instrument rule yields

\[ b = -\frac{c + h}{\alpha} \hat{u}_t + \hat{v}_t - \hat{\omega}_t = \left( -\frac{c + h}{\alpha} \delta u + \delta v + \delta \omega \right) i_t \]
4 Intermediate Targets

- Monetary policy sets an instrument to achieve a value for a goal variable
  - Goal variable could be observed infrequently
  - Other variables, which are closely related to the goal variable, could be observed more frequently
  - Use these other variables as intermediate targets
  - Set the instrument to achieve intermediate targets with the expectation that achieving intermediate targets will achieve goal targets
• Example

– Equations of the model

\[ y_t = a (\pi_t - E_{t-1} \pi_t) + z_t \]

\[ y_t = -\alpha (i_t - E_t \pi_{t+1}) + u_t \]

\[ m_t - p_t = m_t - \pi_t - p_{t-1} = y_t - c_i + v_t \]

– Disturbances are AR(1) with \( \rho' \)'s inside the unit circle

\[ z_t = \rho_z z_{t-1} + e_t \]

\[ u_t = \rho_u u_{t-1} + \varphi_t \]

\[ v_t = \rho_v v_{t-1} + \psi_t \]
– Monetary authority chooses interest rate (instrument) to minimize a loss function specified in terms of inflation (goal)

\[ V = \frac{1}{2} E (\pi_t - \pi^*)^2 \]

\[ \frac{\partial V}{\partial i_t} = E (\pi_t - \pi^*) \frac{\partial \pi}{\partial i_t} = 0 \]

\[ E\pi_t = \pi^* \]

– Information structure to determine expectation

* \( i_t \) must be set before knowledge about \( e_t, \varphi_t, \) or \( \psi_t \) is available

* allows policy to respond to anything past such that

\[ E_{t-1} \pi_t = E_t \pi_{t+1} = \pi^* \]
substituting into supply and demand equations and solving for inflation

$$\pi_t = \frac{(a + \alpha) \pi^* - \alpha i_t + u_t - z_t}{a}$$

- inflation is independent of money market disturbances

- with full information, would set it such that \(\pi_t = \pi^*\) yielding

$$i_t = \pi^* + \frac{u_t - z_t}{\alpha}$$

- with imperfect information, set \(i_t\) equal to the expectation of its optimal value

$$\hat{i}_t = \pi^* + \frac{\rho_u u_{t-1} - \rho_z z_{t-1}}{\alpha}$$
· yielding a value for inflation of

\[ \pi_t(\hat{\pi}_t) = \pi^* + \frac{\varphi_t - e_t}{a} \]

· and a value of

\[ V(\hat{\pi}_t) = \frac{1}{2} E (\pi_t(\hat{\pi}_t) - \pi^*)^2 = \frac{1}{2} \left( \frac{\sigma^2_\varphi + \sigma^2_e}{\alpha^2} \right) \]
• Alternative solution – derive the money supply consistent with achieving target inflation $\pi^*$ and set the interest rate to achieve this money supply

– eliminate $i_t$ from aggregate demand and money market equilibrium and solve for output

$$y_t = \frac{\alpha (m_t - \pi_t - p_{t-1} - v_t) + c (u_t + \alpha \pi^*)}{\alpha + c}$$

– substituting into aggregate supply and using $E_{t-1} \pi_t = \pi^*$ yields

$$\pi_t = \pi^* + \frac{1}{a} \left[ \frac{\alpha (m_t - \pi_t - p_{t-1} - v_t) + c (u_t + \alpha \pi^*)}{\alpha + c} - z_t \right]$$

– solving for $m_t^*$ to set $\pi_t = \pi^*$ yields

$$m_t^* = \pi^* (1 - c) + p_{t-1} + v_t - \frac{c}{\alpha} u_t + \frac{\alpha + c}{\alpha} z_t$$
– if the monetary authority lacks information on current shocks, must set the interest rate to achieve the expectation of this optimal value for money

\[ \hat{m}_t = \pi^* (1 - c) + p_{t-1} + \rho_v v_{t-1} - \frac{c}{\alpha} \rho_u u_{t-1} + \frac{\alpha + c}{\alpha} \rho_z z_{t-1} \]

– interest rate required to achieve this value for the money supply is identical to that previously derived (takes a few steps)
Set up problem to use money as an intermediate target – monetary authority can observe $m_t$ and respond to it

- Under policy with $i_t$ set to equal $\hat{i}_t$, money is

  $$m_t(\hat{i}) = \hat{m}_t - \frac{1}{a} e_t + \left(1 + \frac{1}{a}\right) \varphi_t + \psi_t,$$

  implying that observation of actual money supply provides information on current disturbances

- Implies that can use money as an intermediate target

- Special case: $e \equiv \psi \equiv 0$, implying that aggregate demand shocks are the only disturbance

  * Observation of the actual money supply yields perfect information on $\varphi_t$
* In the event of a positive demand shock, the money supply should be decreased to allow the nominal interest rate to rise to offset the shock.

* A policy of setting the interest rate to keep money at its intermediate target value will achieve this.
– General case: all three disturbances are present, and observation of money does not yield information on which disturbances have occurred

* Solve money market equilibrium for $m_t$ as a function of $i_t$, and disturbances

* Set resulting expression equal to $\hat{m}_t$ and solve for the target interest rate

$$i^T = \hat{i}_t + \frac{(1 + a) \varphi_t - e_t + a \psi_t}{ac + \alpha (1 + a)}$$

* Implies inflation equal to

$$\pi_t = \pi^* + \frac{c \varphi_t - (\alpha + c) e_t - \alpha \psi_t}{ac + \alpha (1 + a)}$$

· Impact of aggregate demand shock on inflation is reduced compared to its effect when information on money supply is not used
Now, money demand shocks ($\psi_t$) affect inflation since positive $\psi_t$ raises money above target, triggering increase in interest rate to bring it back and reducing inflation.

* Loss function under money targeting

\[
V(\hat{i}_t) = \frac{1}{2} \left( \frac{c^2 \sigma^2_\varphi + (\alpha + c)^2 \sigma^2_e + \alpha^2 \sigma^2_\psi}{[ac + \alpha (1 + a)]^2} \right)
\]

* Money targeting does better than alternative of setting interest rate without information on money as long as variance of money-market disturbances is not too large

* Write the intermediate targeting procedure as

\[
i_t^T - \hat{i}_t = \left[ \frac{\alpha}{ac + \alpha (1 + a)} \right] [\hat{m}_t(i) - \hat{m}] = \mu^T [\hat{m}_t(i) - \hat{m}]
\]
It the money supply realized under the initial policy setting \((\hat{m}_t (\hat{i}))\) differs from its expected level \((\hat{m})\), then the interest rate is adjusted.

- Ignoring information because this rule does not take into account the probability that the deviation is caused by particular shocks.
Optimal Policy response to fluctuations in money (intermediate target)

Let

\[ i_t - \hat{i}_t = \mu (m_t - \hat{m}) = \mu x_t \]

\[ x_t = \frac{-1}{a} e_t + \left(1 + \frac{1}{a}\right) \varphi_t + \psi_t \]

with \( x_t \) representing the new information available from observing \( m_t \)

Choose \( \mu \) to minimize \( E(\pi_t - \pi^*)^2 \), not \( E(m_t - \hat{m})^2 \) to yield

\[ \mu^* = \frac{1}{\alpha} \left[ \frac{a (1 + a) \sigma_{\varphi}^2 + a \sigma_e^2}{(1 + a)^2 \sigma_{\varphi}^2 + \sigma_e^2 + \alpha^2 \sigma_{\psi}^2} \right] \]

* The larger the variance of money demand disturbances, the smaller the response to the deviation of money from its expectation should be
• Practical implications for using money as an intermediate target
  
  – Works best if money demand (and supply) relatively stable, but this does not seem to reflect actual markets
  
  – More complicated with lagged effects
5 Operating Procedures and Policy Measures

5.1 Monetary aggregates – What is $M_t$ in a macro model?

- Quantity of means of payment requires broad measure including currency and checkable deposits, $M_1$

- Policy instrument of central bank require measure as monetary base or high-powered money, currency plus bank reserves

- Two are related by money multiplier
5.2 Money multiplier

- Monetary base is total reserves plus currency
  \[ MB = TR + C \]

- Break total reserves into required reserves and excess reserves
  \[ TR = RR + ER \]

- Denote aggregates as a fraction of deposits \((D)\) by small letters
  \[ MB = (rr + ex + c)D \]

- \(M1\) is currency plus deposits
  \[ M1 = C + D = (1 + c)D = \left( \frac{1 + c}{rr + ex + c} \right) MB, \]
where money multiplier is

\[
\frac{1 + c}{rr + ex + c}
\]

cautions: ratios are endogenous choices and can change with MB

Interest rate and money multiplier

Traditionally reserves did not pay interest

Therefore, banks would choose lower ratio of excess reserves to deposits the higher the interest rate

As \( ex \) falls with a higher interest rate, \( M1 \) rises for a given \( MB \)

Money multiplier is increasing in interest rate
– Would not occur now if interest on reserves follows interest rate
5.3 Traditional Model of Market for Bank Reserves

- Reserve demand – depends negatively in fed funds rate \( (i^f) \) with a disturbance

\[ TR^d = -ai^f + v^d \]

- disturbance reflects variations in deposit demand, perhaps due to income

- Reserve supply – reflects reserves borrowed from central bank \( (BR) \) plus non-borrowed reserves \( (NBR) \)

\[ TR^s = BR + NBR \]

- Bank demand for borrowed reserves \( (BR) \)
* Depends positively in $i^f$ and negatively on discount rate ($i^d$) because can borrow at $i^d$ and lend at $i^f$

* Non-price rationing prevents continuous borrowing and implies banks borrow less today if expect high $i^f - i^d$ in future such that would prefer to borrow in future

\[
BR = b_1 (i^f_t - i^d_t) - b_2 E_t (i^f_{t+1} - i^d_{t+1}) + v^b_t
\]

* In 2003, Fed raised $i^d_t$ above $i^f_t$, charging a penalty rate on borrowing (model with penalty rate later)

* Simpler model

\[
BR = b (i^f_t - i^d_t) + v^b_t
\]
– Fed controls $NBR$ through open market operations

$$NBR = \phi^d v_t^d + \phi^b v_t^b + v_t^s$$  \hspace{1cm} (1)

where $v_t^s$ is a policy shock,

• Market equilibrium requires that reserve demand equal reserve supply

$$TR^d = TR^s = BR + NBR$$

$$-ai_t^f + v_t^d = b (i_t^f - i_t^d) + v_t^b + \phi^d v_t^d + \phi^b v_t^b + v_t^s$$

• Solving for Fed funds rate

$$i_t^f = \left( \frac{b}{a+b} \right) i_t^d + \left( \frac{1}{a+b} \right) \left[ (\phi^d - 1) v_t^d + (1 + \phi^b) v_t^b + v_t^s \right]$$
- For BR
\[ BR_t = -\left( \frac{ab}{a+b} \right) \dot{i}_t^d - \left( \frac{1}{a+b} \right) \left[ b v_t^s - (a - b\phi^b) v_t^b - b \left( 1 - \phi^d \right) v_t^d \right] \]

- For TR
\[ TR_t = -\left( \frac{ab}{a+b} \right) \dot{i}_t^d + \left( \frac{1}{a+b} \right) \left[ a v_t^s + a \left( 1 + \phi^b \right) v_t^b + (b + a\phi^d) v_t^d \right] \]
• Federal funds operating procedure

– Offset effects of shocks to demand deposits \( v_t^d \) and to borrowed reserves \( v_t^b \) on Fed funds rate by setting

\[
\phi^d = 1 \quad \phi^b = -1
\]

– Shock to borrowed reserves requires offsetting shock to nonborrowed reserves to keep total reserves constant

– Shock to total reserve demand through deposits \( v_t^d \) leads to equal increase in total reserve supply through adjustment of NBR

– Under this policy

\[
NBR = v_t^d - v_t^b + v_t^s,
\]

and therefore does not reflect solely exogenous policy shocks \( v^s \)
• Non-borrowed reserve operating procedure

\[ \phi^d = 0 \quad \phi^b = 0 \]

– Implying that innovations to NBR are policy shocks \((v_t^s)\)

– Fed funds rate becomes

\[ i^f = \left( \frac{b}{a + b} \right) \dot{i}_t^d + \left( \frac{1}{a + b} \right) \left[ v_t^d - v_t^b - v_t^s \right] \]

and therefore does not reflect only exogenous policy shocks

– If \(v_t^d\) increases demand deposits raising M1, then money and interest rates will be positively correlated
Borrowed reserve policy

\[ \phi^d = 1 \quad \phi^b = \frac{a}{b} \]

- non-borrowed reserves adjust to accommodate fluctuations in total reserve demand

- Fed funds rate becomes

\[ i_t^f = \left( \frac{b}{a+b} \right) i_t^d - \left( \frac{1}{a+b} \right) \left[ (1 + \frac{a}{b}) v_t^b + v_t^s \right] \]
Identify monetary policy shock \( (v_t^s) \)

- Data on \( i_t^f, BR_t, \) and \( NBR_t \)

- Equations for the three as functions of three shocks \( v_t^d, v_t^b, \) and \( v_t^s \)

- Solve for the three shocks as a function of \( i_t^f, BR_t, \) and \( NBR_t, \) given values for \( \phi^d \) and \( \phi^b \) implied by operating procedure

\[
v_t^s = (b\phi^b - a\phi^d) i_t^f - (\phi^d + \phi^b) BR_t + (1 - \phi^d) NBR_t
\]

- Fed funds operating procedure

\[
v_t^s = -(b + a) i_t^f
\]

- NBR procedure

\[
v_t^s = NBR_t
\]
− BR procedure

\[ v_t^s = - \left( 1 + \frac{a}{b} \right) BR_t \]

− TR procedure

\[ v_t^s = \left( 1 + \frac{a}{b} \right) TR_t \]
5.4 Response to shocks under different operating procedures

- Supply and demand for reserves
  - Supply is vertical at NBR until funds rate equals the discount rate and then upward sloping
  - Demand is downward sloping in funds rate

- Positive shock to non-borrowed reserves \((v^s > 0)\)

- Reserve supply shifts right due to increase in NBR (open market purchase)
– NBR operating procedure – interpret shock as policy shock and fed funds rate falls while TR rises

– Fed funds operating procedure – fall in fed funds rate would be represented by reserve supply shock

• Reserve demand increases \( (v^d > 0) \)

– With no policy response, equivalently a NBR procedure, fed funds rate rises and total reserves increase

– Fed funds operating procedure, Fed increases NBR to keep fed funds rate constant, accommodating increase in demand

– Total reserves operating procedure, NBR falls requiring larger increase in Fed funds rate
6 Channel System

• Rules

  – Central bank targets an interest rate of $i^*$
  
  – Sets penalty borrowing rate at $i^* + s$
  
  – Sets interest rate on reserves at $i^* - s$
  
  – Private market interest rate will stay within the channel
    
    $i^* - s \leq i \leq i^* + s$
  
  – Central bank can affect market rate by moving target rate – open mouth operations
• Decision for a private bank

  – Define $T$ as target value for end of the day reserves

  – Actual end-of-the day reserves are stochastic $T + \varepsilon$

  – Bank balances two costs

    * Cost of positive reserve balance is that it earns $i^* - s$ instead of $i$

    * Cost of negative reserve balance is that borrows from central bank at rate $i^* + s$ instead of from banks at rate $i$

  * Bank chooses $T$ to minimize

\[
\int_{-T}^{\infty} (i - (i^* - s))(T + \varepsilon) \, dF(\varepsilon) - \int_{-\infty}^{-T} (i^* + s - i)(T + \varepsilon) \, dF(\varepsilon)
\]
• First term occurs when $T + \varepsilon > 0$, such that the bank has positive reserves, thereby forfeiting interest in the private market ($i$) and earning only interest on reserves ($i^* - s$)

• The second occurs when $T + \varepsilon < 0$, such that the bank has negative reserves and is forced to borrow at the penalty rate of $i^* + s$ instead of at the market rate of $i$

• FO condition with respect to $T$

\[
(i - (i^* - s)) \int_{-T}^{\infty} dF(\varepsilon) - (i^* + s - i) \int_{-\infty}^{-T} dF(\varepsilon) = 0
\]

\[
(i - (i^* - s)) [1 - F(-T^*)] - (i^* + s - i) F(-T^*) = 0
\]

\[
F(-T^*) = \frac{i - i^* + s}{2s}
\]
- When market rate equals target rate

\[ i = i^* \]

\[ F(-T^*) = \frac{1}{2} \]

such that if \( F \) is symmetric, \( T^* = 0 \)

- Target rate controls market interest rate instead of quantity of reserves
7 Federal Reserve Operating Procedures

7.1 Post- Bretton Woods Fed Funds operating procedure (1972-1979)

- Innovations in funds rate were an appropriate measure of monetary policy shocks

- Problem – prices rise and reserve demand rises – fully accommodated by reserve supply to keep interest rate constant – validates increase in price level
7.2 Non-Borrowed Reserves operating procedure (1979-1982)

- Part of policy shift to bring inflation down
  - Increase in inflation would no longer lead to increase in reserves (and money) to keep interest fixed
  - Fed funds rate was both high and volatile

- Targets for growth of monetary aggregates

- Relationship between NBR and M1 broke down in early 1980’s
7.3 Borrowed Reserves operating procedure (1982-1988)

- Similar to Fed Funds operating procedure in face of shocks to reserve demand