Imperfect Information and Market Segmentation
Walsh Chapter 5
1 Why Does Money Have Real Effects?

- Add market imperfections to eliminate short-run neutrality of money

- Imperfect information keeps price from fully reacting to money

- Limited participation in markets implies that only some agents are affected by a change in money

- Monetary policy can have real effects in the presence of market imperfections
2 Imperfect Information

2.1 Lucas – Monetary Surprises

- Assumptions
  - Each individual is a consumer and a producer.
  - Each individual consumes many different goods and produces only a single good.
  - Each individual is a price-taker in all markets
  - Each individual has imperfect information
All variables are logs

Information

* Agents know their own price $P_t(z)$

* Everything dated $t - 1$ or earlier including $E_{t-1}P_t = \bar{P}_t$

* And the model for price

Price in market $z$ has an aggregate component ($P_t$) and an idiosyncratic component ($z_t$)

$$P_t(z) = P_t + z_t$$

Where variance of $P_t$ is $\sigma^2$, variance of $z_t$ is $\tau^2$, and covariance is zero
Each individual wants to work harder and produce and sell more of his
good in periods when the price of his good is high relative to the price
of all other goods he consumes. Since agents do not know the prices
of other goods, must form an expectation

\[ y_t(z) = y + \gamma [P_t(z) - E(P_t|I_t(z))] \]

- Expectations

\[ E(P_t|P(z), \tilde{P}_t) = (1 - \theta) P_t(z) + \theta \tilde{P}_t \]

where

\[ \theta = \frac{\tau^2}{\tau^2 + \sigma^2} \]
- Prove that this is a rational expectation: Choose $a, b$ to

$$
\min E \left[ P_t - a (P_t + z_t) - b \bar{P}_t \right]^2
$$

$$
= \min E \left[ (1 - a) (P_t - \bar{P}_t) - a z - (b - 1 + a) \bar{P}_t \right]^2
$$

$$
= \min \left[ (1 - a)^2 \sigma^2 + a^2 \tau^2 + (b - 1 + a)^2 \bar{P}_t^2 \right]
$$

- Substitute for expectations

$$
y_t(z) = y + \gamma \left[ P_t(z) - (1 - \theta) P_t(z) - \theta \bar{P}_t \right] = y + \gamma \theta \left[ P_t(z) - \bar{P}_t \right]
$$

- Aggregate over markets by integrating over $z$

$$
y_t = y + \gamma \theta \left[ P_t - \bar{P}_t \right]
$$
output is increasing in price relative to its expectation conditioned on information one period ago

effect of an increase in price on output is greater the larger the fraction of idiosyncratic variance relative to total variance

* large fraction: an increase in price fools agents into thinking that an increase in aggregate price is an increase in relative price

* small fraction: agents know that if their price has risen, it is probably due to aggregate price and they are not fooled

* countries with volatile monetary policy will have steep supply curves relative to countries with stable monetary policy

* equivalently, if use monetary policy often, will not work to raise output
Aggregate demand is the cash-in-advance constraint, determined by money

\[ y_t + P_t = M_t \]

\[ M_t = M_{t-1} + \delta + \epsilon_t \]

where \( \epsilon_t \) is iid with mean zero

Equilibrium sets aggregate supply equal to aggregate demand

\[ y_t = y + \gamma \theta \left[ P_t - \bar{P}_t \right] = M_t - P_t \]

Solution

- Remember that

\[ \bar{P}_t = E_{t-1} P_t \]
Undetermined coefficients

* conjecture a solution for price

\[ P_t = \pi_0 + \pi_1 M_t + \pi_2 M_{t-1} \]

* take the expectation conditional on \( t-1 \) information

\[ E_{t-1} P_t = \pi_0 + \pi_1 (M_{t-1} + \delta) + \pi_2 M_{t-1} \]

* substitute both into equilibrium equation and equate coefficients on variables to zero

\[ y + \gamma \theta \pi_1 (M_t - M_{t-1} - \delta) - M_t + \pi_0 + \pi_1 M_t + \pi_2 M_{t-1} = 0 \]

\[ \pi_1 = \frac{1}{1 + \gamma \theta}; \quad \pi_2 = \frac{-\gamma \theta}{1 + \gamma \theta}; \quad \pi_0 = \frac{\gamma \theta \delta}{1 + \gamma \theta} - y \]
\[ P_t = \frac{\gamma \theta (\delta + M_{t-1})}{1 + \gamma \theta} - y + \frac{M_t}{1 + \gamma \theta} \]

* Price surprises become

\[ P_t - E_{t-1}P_t = \frac{\epsilon_t}{1 + \gamma \theta} \]

* Money affects output only if it is unexpected

\[ y_t = y + \frac{\gamma \theta \epsilon_t}{1 + \gamma \theta} \]

* Magnitude of effect depends on \( \theta \), which reflects extent to which shocks to \( P(z) \) are usually idiosyncratic
2.2 Sticky Information (Mankiw and Reis)

- Assumptions

  - Firms can adjust price every period

  - Optimal price (all variables in logs)

    \[ p_t^* (j) = p_t + \alpha x_t \]

    * \( p_t \) is aggregate price level, implying that firms care about relative price

    * \( x_t \) is output gap, where higher output gap implies increasing marginal cost
all firms are identical implying that the optimal aggregate price

\[ p_t^* = p_t + \alpha x_t \]

- price is equal to optimal if output gap is zero

- Each period a fraction \( \lambda \) of firms are randomly chosen and allowed to update information

* a firm which updated \( i \) periods ago sets price at

\[ p_t^i = E_{t-i}p_t^* \]
Sticky Information Aggregate Price

- $\lambda$ of firms update information at time $t$, setting their price at

$$p^0_t = p^*_t$$

- do not update continuously due to cost of information processing

- of the $1 - \lambda$ firms which do not update at time $t$, $\lambda$ of them updated at time $t-1$, setting their price in period $t$ at

$$p^1_t = E_{t-1}p^*_t$$

- therefore the aggregate price in period $t$ is given by an infinite backward recursion
\[ p_t = \lambda p_t^* + (1 - \lambda) \lambda E_{t-1} p_t^* + (1 - \lambda)^2 \lambda E_{t-2} p_t^* + \ldots \]  
\[ = \lambda \sum_{i=0}^{\infty} (1 - \lambda)^i E_{t-i} p_t^* \]  
\[ = \lambda \sum_{i=0}^{\infty} (1 - \lambda)^i E_{t-i} (p_t + \alpha x_t) \]

- if $\lambda$ is close to one, then firms update often and prices are based largely on recent information

- multi-period surprises, compared with one-period surprises in Lucas
• Sticky Information Phillips Curve

- define optimal price

\[ z_t = p_t + \alpha x_t \]

\[ p_t = \lambda z_t + \lambda \sum_{i=1}^{\infty} (1 - \lambda)^i E_{t-i}z_t \]

\[ = \lambda z_t + \lambda (1 - \lambda) E_{t-1}z_t + \lambda (1 - \lambda)^2 E_{t-2}z_t + .... \]

\[ p_{t-1} = \lambda E_{t-1}z_{t-1} + \lambda \sum_{i=1}^{\infty} (1 - \lambda)^i E_{t-1-i}z_{t-1} \]

\[ = \lambda E_{t-1}z_{t-1} + \lambda (1 - \lambda) E_{t-2}z_{t-1} + \lambda (1 - \lambda)^2 E_{t-3}z_{t-1} + .... \]
– subtracting yields an expression for inflation

\[
\pi_t = p_t - p_{t-1} \\
= \lambda z_t + \left[ \lambda E_{t-1} (z_t - z_{t-1}) - \lambda^2 E_{t-1} z_t \right] \\
+ \left[ \lambda (1 - \lambda) E_{t-2} (z_t - z_{t-1}) - \lambda^2 (1 - \lambda) E_{t-2} z_t \right] + \ldots \\
= \lambda z_t + \lambda \sum_{i=0}^{\infty} (1 - \lambda)^i E_{t-1-i} \Delta z_t - \lambda^2 \sum_{i=0}^{\infty} (1 - \lambda)^i E_{t-1-i} z_t
\]

– from equation (1) above

\[
p_t = \lambda \sum_{i=0}^{\infty} (1 - \lambda)^i E_{t-i} (p_t + \alpha x_t) \\
= \lambda (p_t + \alpha x_t) + \lambda \sum_{i=1}^{\infty} (1 - \lambda)^i E_{t-i} (p_t + \alpha x_t)
\]
- solve for $p_t$ by subtracting $p_t$ from both sides and dividing by $1 - \lambda$

$$p_t = \frac{\lambda}{1 - \lambda} \alpha x_t + \frac{\lambda}{1 - \lambda} \sum_{i=1}^{\infty} (1 - \lambda)^i E_{t-i} (p_t + \alpha x_t)$$

$$= \frac{\lambda}{1 - \lambda} \alpha x_t + \lambda \sum_{i=0}^{\infty} (1 - \lambda)^i E_{t-1-i} z_t$$

- this implies

$$\lambda^2 \sum_{i=0}^{\infty} (1 - \lambda)^i E_{t-1-i} z_t = \lambda p_t - \frac{\lambda^2}{1 - \lambda} \alpha x_t$$

- substituting for the last term in equation (2) yields

$$\pi_t = \lambda z_t + \lambda \sum_{i=1}^{\infty} (1 - \lambda)^i E_{t-i} \Delta z_t - \lambda p_t + \frac{\lambda^2}{1 - \lambda} \alpha x_t$$
\[ \pi_t = \frac{\lambda}{1 - \lambda} \alpha x_t + \lambda \sum_{i=1}^{\infty} (1 - \lambda)^i E_{t-i} (\pi_t + \alpha \Delta x_t) \]

* inflation is increasing in the output gap, by a larger amount, the more frequently firms update (the larger is \( \lambda \))

* since firms update infrequently, a shock which occurs at a point in time in the past, slowly raises inflation

* expectations are not forward-looking because agents can adjust future prices to expectations of future inflation

• Add an equation for output (gap)

\[ m_t + v_t = p_t + x_t \]
– First differences

\[ m_t - m_{t-1} + \Delta v_t = \pi_t + \Delta x_t \]

– Money shock does not affect second term in Phillips curve in current period

* Therefore, money up must raise both inflation and output

* Simulations show that if money growth is positively correlated, inflation and output continue to rise before eventually falling

* Hump-shaped responses we see in data
3 Limited Participation

- Model in which increase in money growth lowers nominal interest rate initially

- Assumptions

  - Each household has a shopper, a firm manager, a worker, a bank
  
  - Household allocates money ($M_t$) to bank deposit ($D_t$) and to balances to use for purchases

\[ P_t C_t \leq M_t - D_t \]
– Firms must pay nominal wages ($P_t \omega_t$) to workers before they sell output, implying that they must use bank loans ($L_t$)

$$P_t \omega_t N_t^d \leq L_t$$

– Firm’s nominal profits

$$\Pi^f_t = P_t Y (N_t^d) - P_t \omega_t N_t^d - R^L_t L_t$$

– Banks accept deposits from households and pay $R^D$. Banks make loans to firms. Central bank makes transfers ($H$) to banks.

$$L_t = D_t + H_t$$

– Bank profits are

$$\Pi^b_t = R^L_t L_t + H_t - R^D_t D_t = (R^L_t - R^D_t) D_t + (1 + R^L_t) H_t$$
- Competition and profit max assure interest rates are equal
  \[ R^L_t = R^D_t = R_t \]

- Divide by \( P_t \), writing everything in real terms
  \[ m_t - d_t \geq C_t \]

- \( \lambda_1 \) is the marginal value of money in consumption
  \[ l_t \geq \omega_t N^d_t \]

- \( \lambda_3 \) is the marginal value of money in loans
• Household Problem

  – Budget constraint

\[
P_t \omega_t N_t^s + M_t - D_t + (1 + R_t) D_t + \Pi_t^b + \Pi_t^f - P_t C_t = M_{t+1}
\]

\[
P_t \omega_t N_t^s + M_t - D_t + (1 + R_t) D_t + \left(1 + R_t^L\right) H_t + P_t Y \left(N_t^d\right)
- P_t \omega_t N_t^d - R_t^L L_t - P_t C_t
= M_{t+1}
\]

  – Simplifying and dividing by price

\[
\omega_t N_t^s + m_t + R_t d_t + \left(1 + R_t^L\right) h_t + Y \left(N_t^d\right) - \omega_t N_t^d - R_t^L l_t - C_t
= m_{t+1} \left(\frac{P_{t+1}}{P_t}\right)
\]
- Household maximizes

\[ V(m_t) = \max_{d_t} E \left\{ \max_{C_t, N_t^s, N_t^D, l, m_{t+1}} \left[ u(C_t) - v(N_t^s) + \beta V(m_{t+1}) \right] \right\} \]

where \( d_t \) is chosen before observing \( h_t \)

- subject to budget constraint (\( \lambda_2 \)), cash-in-advance constraint (\( \lambda_1 \)), and loans-in-advance constraint (\( \lambda_3 \))

- First order conditions

  - \( d_t \) is determined before observing \( h_t \), so add \( E_h \)

\[ d_t \quad E_h \left[ -\lambda_{1t} + R^D \lambda_{2t} \right] = 0 \quad (3) \]

\[ C_t \quad u'(C_t) = \lambda_{1t} + \lambda_{2t} \quad (4) \]
– marginal utility of income differs from marginal utility of consumption when cia constraint binds

\[ N_t^s - v' (N_t^s) + \omega_t \lambda_{2t} = 0 \]  \hfill (5)

\[ N_t^d \quad \lambda_{2t} [Y' (N_t^d) - \omega_t] - \lambda_{3t} \omega_t = 0 \]  \hfill (6)

\[ l_t \quad - R_t^L \lambda_{2t} + \lambda_{3t} = 0 \]  \hfill (7)

– where \( \lambda_{3t} \) is the value of liquidity in the loan market

\[ m_{t+1} \quad - \lambda_{2t} \left( \frac{P_{t+1}}{P_t} \right) + \beta V_m (m_{t+1}) = 0 \]

Envelope condition

\[ V_m (m_t) = E_h [\lambda_{1t} + \lambda_{2t}] \]
• Interpretations

– Marginal value of liquidity in goods and loan markets

* Subtracting (4) from (7) yields marginal value of money for loans less its marginal value for consumption

\[ \lambda_{3t} - \lambda_{1t} = \left(1 + R_t^L\right) \lambda_{2t} - u'(C_t) \]

* Solving first order condition on money for

\[ \lambda_{2t} = \beta \left( \frac{P_t}{P_{t+1}} \right) V_m(m_{t+1}) \]

\[ = \beta \left( \frac{P_t}{P_{t+1}} \right) E_h \left[ \lambda_{1t+1} + \lambda_{2t+1} \right] \]

\[ = \beta \left( \frac{P_t}{P_{t+1}} \right) E_h u'(C_{t+1}) \]
Solving these two equations for marginal utility of consumption

\[ u'(C_t) = \left(1 + R^L_t\right) \lambda_{2t} - \lambda_{3t} + \lambda_{1t} \]

\[ = (1 + R_t) \beta \left(\frac{P_t}{P_{t+1}}\right) E_h u'(C_{t+1}) - \lambda_{3t} + \lambda_{1t} \]

when the marginal value of money in consumption and loans differs, there is a wedge in the normal Euler equation

* Combining equations (3) and (7)

\[ E_h [-\lambda_{1t} + \lambda_{3t}] = 0 \]

- when a household makes its portfolio choice, the value of sending cash to goods market must equal value of sending it to loan market by depositing it in a bank

- ex post, the values can differ because households cannot reallocate after central bank makes decision on \( h \)
– Labor market

* Together, equations (6) and (7) imply that the interest rate drives a wedge between the marginal product of labor and the real wage

\[ Y' \left( N^d_t \right) = (1 + R) \omega_t \]

* Equation (5) implies

\[ v' \left( N^s_t \right) = \omega_t \lambda_{2t} \]

* Combining the two implies that the interest rate drives a wedge between the marginal rate of substitution of labor and its marginal product

\[ \frac{v' \left( N^s_t \right)}{\lambda_{2t}} = \frac{Y' \left( N^d_t \right)}{1 + R_t} \]
• Unexpected increase in supply of money

  – Received by banks and so increases loans because $D_t$ is pre-determined by households

    \[ L_t = D_t + H_t \]

  – To get increase in loan demand, $R_t$ must fall

  – Fall in $R_t$ increases labor demand and the real wage

  – Increase in real wage causes households to increase labor supply

  – works because households cannot respond to the reduction in the interest rate on deposits by withdrawing them and having more money for consumption, driving up price