Financial Market Imperfections
Uribe, Ch 8
1 Imperfect Credibility of Policy: Trade Reform

1.1 Model

- Assumptions

  - Output is exogenous constant endowment \((y)\), not useful for consumption, but can be exported
  
  - Consumption goods must be imported
  
  - Relative price of exports to imports (TOT) is unity
  
  - \(\beta(1 + r) = 1\)
• Consumer’s problem

  – Utility

  \[ U_1 = \sum_{t=0}^{\infty} \beta^t U(c_t) \]

  – Household flow budget constraint

  \[ d^h_t = (1 + r) d^h_{t-1} - y - x_t + c_t (1 + \tau_t) \]

  * debt this period equals interest and principle on debt from previous period \((1 + r) d^h_{t-1}\) less the endowment \(y\) less the household’s lump-sum transfer from the government \(x_t\) plus expenditures on consumption together with their tax

  * since consumption is all imported, tax on consumption has the interpretation of a tariff
– NPG constraint

\[
\lim_{t \to \infty} \frac{d_{t+j}^h}{(1 + r)^j} \leq 0
\]

* optimality implies that agents will not want negative present-value
debt (positive assets) in the limit such that the above holds with equality

– Consumer’s intertemporal budget constraint with the zero limit term

\[
(1 + r)^h d_{t-1}^h = \sum_{t=0}^{\infty} \left( \frac{1}{1 + r} \right)^t [y + x_t - c_t (1 + \tau_t)]
\]
\( L = \sum_{t=0}^{\infty} \beta^t U(c_t) + \lambda_0 \left\{ \sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t [y + x_t - c_t (1 + \tau_t)] \right\} \\
- \lambda_0 (1 + r) d_{h-1}^h \\
= \sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t \{U(c_t) + \lambda_0 [y + x_t - c_t (1 + \tau_t)]\} - \lambda_0 (1 + r) d_{h-1}^h \\

* FO condition with respect to \( c_t \) \\
\( U'(c_t) = \lambda_0 (1 + \tau_t) \)
- Government

  - flow budget constraint

  \[ d_t^g = (1 + r) d_{t-1}^g - \tau c_t + x_t \]

  * government accumulates debt equal to interest and principle on last period’s debt, less import tariffs, plus transfers

  * primary fiscal surplus is tax revenue less transfers

  \[ \tau c_t - x_t \]

  * actual fiscal surplus subtracts interest on debt

  \[ \tau c_t - x_t - r d_{t-1}^g \]
assume fiscal policy assures that the present value of government debt is zero in the limit

\[ \lim_{t \to \infty} \frac{d_{t+j}^g}{(1 + r)^j} = 0 \]

- government intertemporal budget constraint

\[ (1 + r) d_{-1}^g = \sum_{t=0}^{\infty} \left( \frac{1}{1 + r} \right)^t [c_t \tau_t - x_t] \]

* Present discounted value of primary surpluses must equal outstanding debt with interest
• Country intertemporal government budget constraint

  – add household and government intertemporal budget constraints letting
    \[ d_t = d^h_t + d^g_t \]
    
    \[ (1 + r) d_{-1} = \sum_{t=0}^{\infty} \left( \frac{1}{1 + r} \right)^t [y - c_t] \]

* Present discounted value of income less consumption must equal outstanding debt with interest
1.2 Policy 1: Permanent tariff reduction

- At time $t = 0$, tariff unexpectedly falls from $\tau^H$ to $\tau^L$
  
  - FO condition on consumption implies that consumption is constant over time ($\bar{c}$) after reduction in tariff
  
  - Intertemporal country budget constraint implies
    
    $$(1 + r) d_{-1} = \left(\frac{1 + r}{r}\right) (y - \bar{c})$$

    $$\bar{c} = y - rd_{-1}$$

- Change in the tariff has no effect on consumption
1.2.1 Policy 2: Temporary tariff reduction

- Tariff reduction lasts for a period of time $T$
  \[
  \tau_t = \tau^L \quad \text{for} \quad 0 \leq t < T
  \]
  \[
  \tau_t = \tau^H \quad \text{for} \quad t \geq T
  \]

- FO condition on consumption implies that consumption is constant at different values over two periods
  \[
  U'(c_t) = \lambda_0 (1 + \tau_t)
  \]
  
  - Let consumption be given by
    \[
    c_t = c^1 \quad \text{for} \quad 0 \leq t < T
    \]
    \[
    c_t = c^2 \quad \text{for} \quad t \geq T
    \]
- Since marginal utility of consumption must be higher when the tariff is high, consumption must be lower when the tariff is high

\[ c^1 = (1 + \kappa) c^2 \]

- If period utility is CRRA

\[ U(c) = \frac{c^{1-\sigma}}{1 - \sigma} \]

then relative marginal utility is given by

\[ \left( \frac{c_1}{c_2} \right)^{-\sigma} = \frac{1 + \tau^L}{1 + \tau^H} \]
implying

\[(1 + \kappa) = \left(\frac{1 + \tau^H}{1 + \tau^L}\right)^{\frac{1}{\sigma}}\]
Consumption must respect the intertemporal resource constraint

$$(1 + r) d_{-1} = \sum_{t=0}^{\infty} \left( \frac{1}{1 + r} \right)^t [y - c_t]$$

$$(1 + r) d_{-1} = \frac{1 + r}{r} y + \sum_{t=0}^{T-1} \beta^t c^{1} + \sum_{t=T}^{\infty} \beta^t c^{2}$$

$$\frac{1}{1 - \beta} \bar{c} = \sum_{t=0}^{T-1} \beta^t c^{1} + \sum_{t=T}^{\infty} \beta^t c^{2}$$
Sums

$$\sum_{t=0}^{T-1} \beta^t = A$$

$$\frac{1}{1 - \beta} = \sum_{t=0}^{\infty} \beta^t = A + \beta^T \left( \frac{1}{1 - \beta} \right)$$

$$A = \frac{1}{1 - \beta} \left( 1 - \beta^T \right)$$

$$\sum_{t=T}^{\infty} \beta^t = \beta^T \left( \frac{1}{1 - \beta} \right)$$

Substituting the sums

$$\bar{c} = \left( 1 - \beta^T \right) c^1 + \beta^T c^2$$
– Consumption magnitudes
\[ c^1 > \bar{c} > c^2 \]

• Interpretation

– Temporary tariff reduction increases consumption until period \( T \) when the tariff disappears

– Afterwards, consumption falls below its initial level and remains there forever

– Policy reduces welfare because it causes consumers to deviate from perfectly smooth consumption, which is feasible
Effect on trade balance, current account, and debt

- Assume that \( d_{-1} = 0 \)

- Policy of tariff reduction increases consumption, creating a trade and current account deficit

- Since consumption remains above output by the same amount debt increases by the difference plus the interest on debt

- Therefore, debt and the current account deficit increase at increasing rates

- Once the tariff increases, consumption falls sufficiently below income to pay interest on the accumulated debt and the current account is zero while the trade balance is in surplus
1.3 Overborrowing

- Define overborrowing as borrowing which exceeds the social optimum

- Reform causes household to engage in period in which borrowing exceeds the social optimum

- Current account deficit ends with a sudden stop of borrowing such that the current account is balanced
• Relation of temporary policy to one of imperfect credibility

  – Government announces a permanent tariff reduction

  – Agents believe the reduction will last until period $T$

  – Raise consumption as in above example

  – When period $T$ arrives, even if the government is able to convince them that the policy is permanent, consumption must fall due to the accumulated debt
2 Financial Externalities

- Could lead agents to borrow more than the socially optimal quantity, yielding overborrowing

2.1 Borrowing Constraints

2.1.1 Aggregate Borrowing Constraint

- Agents do not take account of their decision to increase debt on aggregate debt
• Model

  – Utility

  \[ E_0 \sum_{t=0}^{\infty} \theta_t U(c_t, h_t) \]

  – Discount factor

  \[ \theta_{t+1} = \beta(C_t, H_t) \theta_t \quad \beta_C < 0; \beta_H > 0 \]

  * agent’s discount factor depends on aggregate values for consumption and labor hours, not individual ones (simplifies)

  * necessary to make model stationary
– Production

\[ e^{z_t F(k^*, h_t)} \]

* \( z_t \) is exogenous productivity shock

* \( k^* \) is a fixed factor like land

– flow budget constraint

\[ d_t = (1 + r_{t-1}) d_{t-1} + c_t - e^{z_t F(k^*, h_t)} \]

– NPG constraint together with optimality imply

\[ \lim_{j \to \infty} \frac{d_{t+j}}{(1 + r)^j} = 0 \]
• Consumer problem

  – Choose \( \{c_t, d_t, h_t\} \) to maximize utility subject to constraints

    * Euler equation
      \[
      U_c (c_t, h_t) = \beta (C_t, H_t) (1 + r_t) U_c (c_{t+1}, h_{t+1})
      \]

    * Labor leisure choice
      \[
      \frac{-U_h(c_t, h_t)}{U_c(c_t, h_t)} = e^{zt} F_h (k^*, h_t)
      \]

  – Constraint on aggregate debt
    \[
    D_t \leq \bar{d}
    \]
• Equilibrium

  – All agents are identical, implying

      \[ D_t = d_t \]

  – When borrowing constraint does not bind

      * agents choose less debt than the maximum

      * domestic interest rate equals world rate
– When the borrowing constraint does bind

* Excess demand for loans at world interest rate, requiring that domestic interest rate rises above world interest rate

* Quantity

\[(r_t - r) \, dt\]

is a pure financial rent

* Consider case in which rent is appropriated by domestic banks and distributed to agents in lump-sum fashion

  · Equilibrium requires FO conditions, including Euler equation with interest rate equal to the domestic rate, resource constraint with interest equal to the world rate, and

\[d_t \leq \bar{d} \quad r_t \geq r \quad (d_t - \bar{d})(r_t - r) = 0\]
Last condition requires that if the borrowing constraint is not binding, then the interest rate equals the world rate, and if the interest rate is strictly above the world rate, the borrowing constraint must be binding.
2.1.2 Individual Borrowing Constraint

- Competitive equilibrium will be the social optimum

- Agents do take account of the effect of their decision to increase debt on the borrowing constraint
• Consumer problem

  – Maximize same utility subject to same budget constraint plus additional constraint on individual debt

  \[ d_t \leq \bar{d} \]

  – relevant interest rate will be world rate, \( r \)

  * unnecessary for domestic interest rate to rise to keep agents from exceeding the aggregate debt limit since no agent can exceed his own debt limit
– Lagrangian

\[ E_0 \sum_{t=0}^{\infty} \theta_t \{ U(c_t, h_t) + \lambda_t [d_t - (1 + r) d_{t-1} - c_t + e^{\gamma_t} F(k^*, h_t)] \} \]

\[ + E_0 \sum_{t=0}^{\infty} \theta_t \lambda_t \mu_t [\bar{d} - d_t] \]

– FO conditions

* \( c_t \)

\[ U_c(c_t, h_t) = \lambda_t \]

* \( d_t \)

\[ \lambda_t - \lambda_t \mu_t = \beta(C_t, H_t) (1 + r) \lambda_{t+1} \]
\[ \lambda_t = \beta (C_t, H_t) \frac{(1 + r)}{1 - \mu_t} \lambda_{t+1} \]

- Note 0 ≤ \( \mu_t < 1 \) to assure marginal utility of consumption is neither infinite or negative

* inequality constraint

\[ \mu_t \geq 0 \quad \mu_t [\bar{d} - d_t] = 0 \]

- When borrowing constraint binds (\( \mu_t > 0 \)), marginal utility of extra unit of debt is not the marginal utility of consumption, but something smaller (\( \lambda_t (1 - \mu_t) \))

* extra unit of debt tightens the borrowing constraint and carries a shadow punishment (\( \lambda_t \mu_t \))
• Equilibrium debt, consumption, and labor hours are identical whether the borrowing constraint is individual or aggregate

− Define the shadow interest rate

\[ 1 + \tilde{r}_t = \frac{1 + r}{1 - \mu_t} \]

− Use the substitution

\[
\mu_t = \frac{\tilde{r}_t - r}{1 + \tilde{r}_t}
\]

to write the slackness condition as

\[
(\tilde{r}_t - r) \left[ \bar{d} - d_t \right] = 0
\]

− two systems are identical, implying that the financial externality does not have equilibrium effects
market interest rate with aggregate borrowing constraint conveys identical information as Lagrange multiplier $\mu_t$ with individual borrowing constraint
2.1.3 **Rents are expropriated by foreign banks**

- Aggregate borrowing constraint
  - Interest rate in resource constraint becomes domestic interest rate
  - Implies that at the debt limit, transfer difference in interest rates multiplied by debt to foreigners

- Simulate models with aggregate borrowing constraint
  - No debt limit
  - Rents accrue to domestic agents
  - Rents accrue to foreign agents
– Later two are virtually identical

* Agents engage in precautionary savings so seldom hit the debt limit

* When the debt limit does bind, it produces a very small interest rate premium

  · Agents never hold so much debt that the premium becomes large
2.2 Collateral Constraints

- Borrowing constraint depends on an endogenous variable
  - as debt approaches the collateral constraint, the price of collateral can collapse endogenously, tightening the borrowing constraint
  - possible downward spiral
2.2.1 Aggregate Collateral Constraint

- Model

  - Production takes labor \((h_t)\) and land \((k)\) as inputs

\[ y_t = e^{zt} F(k^*, h_t) \]

  - Aggregate supply of land is fixed by \(k^* > 0\)

  - \(q\) is the market price for land in terms of consumption goods

  * additional financial externality because agents do not take account of their actions on the equilibrium price of land which constraints aggregate debt
– collateral constraint at the aggregate level

\[ D_t \leq \kappa q_t k^* \]

– household’s flow budget constraint

\[ d_t = (1 + r_{t-1}) d_{t-1} + q_t (k_{t+1} - k_t) + c_t - e^{zt} F(k_t, h_t) \]
• Optimization problem with collateral constraint at aggregate level

  – Choose processes $\{c_t, h_t, d_t, k_{t+1}\}$ to maximize utility subject to flow budget constraint, the NPG condition

  – FO conditions for debt, leisure, and consumption are identical to no-land counterparts

  – FO condition on land

    * Lagrangian

    \[
    E_t \sum \theta_t U (c_t, h_t) + \\
    E_t \sum \theta_t \lambda_t [d_t - (1 + r_{t-1}) d_{t-1} - q_t (k_{t+1} - k_t) - c_t + e^{zt} F (k_t, h_t)]
    \]

    * FO condition on land

    \[
    \theta_t \lambda_t q_t = E_t \theta_{t+1} \lambda_{t+1} [q_{t+1} + e^{zt+1} F_k (k_{t+1}, h_{t+1})]
    \]
* Endogenous discount factor

\[ \theta_{t+1} = \beta (C_t, H_t) \theta_t \]

* Substitute endogenous discount factor

\[ U_c (c_t, h_t) q_t = \beta (C_t, H_t) E_t U_c (c_{t+1}, h_{t+1}) [q_{t+1} + e^{zt+1} F_k (k_{t+1}, h_{t+1})] \]

* Solve for the price of land

\[ q_t = E_t \frac{U_c (c_{t+1}, h_{t+1})}{U_c (c_t, h_t)} \beta (C_t, H_t) [q_{t+1} + e^{zt+1} F_k (k_{t+1}, h_{t+1})] \]

* Define a stochastic discount factor

\[ \Lambda_{t,t+j} \equiv \frac{U_c (c_{t+j}, h_{t+j})}{U_c (c_t, h_t)} \prod_{s=0}^{j-1} \beta (c_{t+s}, h_{t+s}) \]
– Solve equation for $q_t$ forward

* price of land is the expected present discounted value of its future marginal products

$$q_t = E_t \sum_{j=1}^{\infty} \Lambda_{t,t+j} e^{zt+j} F_k \left( k_{t+j}, h_{t+j} \right)$$
– Equilibrium

* FO conditions including Euler equation

\[ \lambda_t = \beta (C_t, H_t) (1 + r_t) \lambda_{t+1} \]

* Budget constraints

* Inequality constraint on collateral where \( d_t = D_t \)

\[ d_t \leq \kappa q_t k^* \]

* Slackness condition

\[ (r - r_t) (d_t - \kappa q_t k^*) = 0 \]
2.2.2 Collateral Constraint at Individual Level

- Individual collateral constraint

  - agent’s decisions on debt and land
  - market price of land

\[ d_t \leq \kappa q_t k_{t+1} \]

- in equilibrium all external loans are extended at the world interest rate
• Lagrangian with additional multiplier and inequality constraint

\[ E_t \sum \theta_t \{U(c_t, h_t) - \lambda_t \mu_t (d_t - \kappa q_t k_{t+1})\} \]

\[ + E_t \sum \theta_t \lambda_t [d_t - (1 + r) d_{t-1} - q_t (k_{t+1} - k_t) - c_t + e^{z^t} F(k_t, h_t)] \]

– FO condition with respect to \( k_{t+1} \) where additional \( k_{t+1} \) relaxes the collateral constraint

\[ \theta_t q_t \lambda_t (1 - \mu_t \kappa) = E_t \theta_{t+1} \lambda_{t+1} [q_{t+1} + e^{z_{t+1}} F_k(k_{t+1}, h_{t+1})] \]

* substitute for \( \lambda_t \)

\[ U_c(c_t, h_t) q_t (1 - \mu_t \kappa) \]

\[ = \beta(C_t, H_t) E_t U_c(c_{t+1}, h_{t+1}) [q_{t+1} + e^{z_{t+1}} F_k(k_{t+1}, h_{t+1})] \]

– in solving for \( q_t \), the discount factor is smaller so that \( q_t \) is larger
because additional land has value in relaxing the collateral constraint

\[ q_t = E_t \beta (C_t, H_t) U_c (c_{t+1}, h_{t+1}) \frac{U_c (c_t, h_t)}{U_c (c_t, h_t) (1 - \mu_t \kappa)} [q_{t+1} + e^{zt+1} F_k (k_{t+1}, h_{t+1})] \]

– FO condition with respect to \( d_t \)

\[ \theta_t \lambda_t (1 - \mu_t) = \theta_{t+1} \lambda_{t+1} (1 + r) \]

\[ \lambda_t = \beta (C_t, H_t) \frac{(1 + r)}{1 - \mu_t} \lambda_{t+1} \]

– Equilibrium looks same except for equation on land, giving land more value
2.2.3 Simulate aggregate and individual collateral constraint models

- Models are very similar except that the value of land is higher when the borrowing limit is internalized by the agent
  
  - Since the value of land constrains debt, then the individual borrowing constraint allows more debt
  
  - Slightly higher value of land represents the effect of the financial externality

- When debt is high, the interest rate must rise to convince agents to postpone consumption, and the increase in the interest rate (represented by an increase in the stochastic discount factor) causes a sharp drop in the value of land
• Market price of debt is same as social price of debt whether borrowing constraint is internal (shadow interest rate adjusts) or external (actual interest rate adjusts)
  – This source of financial externality does not affect the equilibrium as before

• No overborrowing in the sense equilibrium debt with individual constraint is not higher than debt with aggregate constraint
  – Market price of debt is identical to social price of debt whether the constraint is internal or external
  – Interest rate (and pseudo interest rate) equals the world interest rate except very near the constraints
– Price of land is very similar whether the constraint is aggregate or individual

– When the constraint binds, it binds for all agents simultaneously
2.3 Heterogeneous Agents

- Assumptions
  - 2-period endowment economy with certainty
  - constant debt ceiling of $\kappa$ per capita
  - continuum of agents of measure one
  - agents are heterogeneous in their period-2 endowments
    * in period 1 all agents receive endowments of $y$
    * in period 2, half of agents receive $y^a > y$ and half receive $y^b < y$
• Model without a debt ceiling and

\[ c^a > y + \kappa \quad c^b < y + \kappa \]

– Aggregate per capita external debt (unconstrained) is

\[ d^u = \frac{c^a + c^b}{2} - y \]
• Impose debt ceiling of $\kappa$ on each agent

  – $a$ agents are constrained and consume

    $$c^a = y + \kappa$$

  – $b$ agents are unconstrained and consume

    $$c^b$$

  – per capita external debt with individual constraint is below the ceiling $\kappa$

    $$d^i = \frac{y + \kappa + c^b}{2} - y < d^u$$
• Impose debt ceiling of $\kappa$ at the aggregate level

  – Case 1

    • Unconstrained debt is less than the maximum

      \[ d^u \leq \kappa \]

    • The equilibrium interest rate equals the world rate and each agent’s consumption is determined as if there were no borrowing constraint

      \[ d^a = d^u > d^i \]
- Case 2

* Unconstrained debt exceeds the maximum

\[ d^u > \kappa \]

* The equilibrium domestic interest rate exceeds the world interest rate

* Per capita external debt with aggregate constraint equals the ceiling and exceeds debt with individual constraint

\[ d^a = \kappa > d^i \]
• With aggregate borrowing constraint, per capita debt is higher
  
  – Financial externality when the more frugal consumer borrows less than the per capita limit
  
  – This externality is not priced in the market
  
  – Overborrowing due to absence of a market equalizing intertemporal marginal rates of substitution across households

  * Allow a domestic financial market in which frugal agents (those with relatively low expected future endowments) could borrow and lend to more lavish agents (those with relatively high future endowments)

  * Requires that foreign lenders maintain the upper bound on debt of more lavish agents of $\kappa$ even though their total debt has increased
With heterogeneous agents welfare function of social planner is not specified, so the overborrowing result is not necessarily suboptimal from the perspective of the social planner.

- Social planner could care only about agents who want to borrow heavily, implying that aggregate borrowing constraint moves economy closer to social optimum.
2.4 Debt Elastic Interest Rate

- Financial externality because agents do not take account of their actions on the interest rate
  - Externality whereby the country borrows more than socially optimal quantity

2.4.1 Debt elastic interest rate at aggregate level

\[ r_t = r + \rho (D_t) \quad \rho' > 0 \]
• Equilibrium

  – Aggregate debt equals individual debt

    \[ D_t = d_t \]

  – Steady state equilibrium value of debt is \( d^* > 0 \)

  – From Euler equation, steady state interest rate must satisfy

    \[ 1 = (1 + r + \rho(d^*)) \beta \]
2.4.2 Social Optimum: debt-elastic interest rate imposed at individual level

\[ r_t = r + \rho(d_t) \quad \rho' > 0 \]

- Euler equation

\[ U'(c_t) = \beta u'(c_{t+1}) \left[ 1 + r + \rho(d_{t+1}) + d_{t+1}\rho'(d_{t+1}) \right] \]

- Steady state equilibrium, \( d^{**} \)

\[ 1 = \beta \left[ 1 + r + \rho(d^{**}) + d^{**}\rho'(d^{**}) \right] \]

- Since \( \rho'(d^{**}) > 0, \rho(d^{**}) < \rho(d^*) \), implying

\[ d^{**} < d^* \]
- Agents would borrow less if they were forced to take into account the effect of their borrowing on the aggregate interest rate

- Implies that model with debt-elastic interest rate yields over-borrowing
2.5 Nontraded Exogenous Output as Collateral

2.5.1 Model

- small, open economy with preferences

\[ E_0 \sum_{t=0}^{\infty} \beta^t U(c_t) \]

- utility is CRRA

\[ U(c) = \frac{c^{1-\sigma}}{1-\sigma} \]

- consumption is a CES composite of tradeables and nontradeables

\[ c_t = A(c_t^N, c_t^T) \equiv \left[ \omega (c_t^T)^{1-1/\eta} + (1 - \omega) (c_t^N)^{1-1/\eta} \right]^{1/(1-1/\eta)} \]
- budget constraint with one period risk-free asset with constant interest rate and stochastic endowments

\[ d_t = (1 + r) d_{t-1} + c_t^T - y_t^T + p_t^N (c_t^N - y_t^N) \]

- Individual borrowing constraint

\[ d_t \leq \kappa^T y_t^T + \kappa^N p_t^N y_t^N \]

- Borrowing constraint yields a financial externality because agents do not take account of their actions on \( p_t^N \)
• Lagrangian

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left[ \omega \left( c_t^T \right)^{1-1/\eta} + (1 - \omega) \left( c_t^N \right)^{1-1/\eta} \right]^{1/(1-1/\eta)} \right\}^{1-\sigma} \\
+ E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \left[ d_t - (1 + r) d_{t-1} - c_t^T + y_t^T - p_t^N \left( c_t^N - y_t^N \right) \right] \\
- E_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left( d_t - \kappa^T y_t^T - \kappa^N p_t^N y_t^N \right)
\]
• FO conditions

\[- c_t^T\]

\[G \left( c_t^T, c_t^N \right) \]

\[\equiv \omega \left[ A \left( c_t^N, c_t^T \right) \right]^{-\sigma} \left[ \omega \left( c_t^T \right)^{1-\frac{1}{\eta}} + (1 - \omega) \left( c_t^N \right)^{1-\frac{1}{\eta}} \right]^{1 \over \eta - 1} \left( c_t^T \right)^{-1/\eta} \]

\[G \left( c_t^T, c_t^N \right) = \lambda_t\]

* marginal utility of tradeable consumption equals \( \lambda_t \)
\[- c_t^N \]

\[
(1 - \omega) \left[ A \left( c_t^N, c_t^T \right) \right]^{-\sigma} \\
\times \left[ \omega \left( c_t^T \right)^{1 - \frac{1}{\eta}} + (1 - \omega) \left( c_t^N \right)^{1 - \frac{1}{\eta}} \right]^{\frac{1}{\eta - 1}} \left( c_t^N \right)^{-1/\eta} \\
= \lambda_t p_t^N
\]

* combining two FO conditions

\[
p_t^N = \frac{1 - \omega}{\omega} \left( \frac{c_t^T}{c_t^N} \right)^{1/\eta}
\]

* relative price equals relative marginal utilities

\[- d_t \]

\[
\lambda_t = \beta (1 + r) \lambda_{t+1} + \mu_t
\]
\[ G \left( c_t^T, c_t^N \right) = \beta (1 + r) G \left( c_{t+1}^T, c_{t+1}^N \right) + \mu_t \]

- * marginal benefit of additional debt is marginal utility of tradeables
  
  * during tranquil times, when the collateral constraint does not bind, the marginal cost of debt is the present discounted value of the reduction in future utility it generates

  * when the collateral constraint binds, the marginal cost is higher reflecting a shadow punishment to additional debt
Competitive Equilibrium

- requires FO conditions and flow budget constraint on debt

- goods market clearing for non-traded goods
  \[ c^N_t = y^N_t \]

- collateral constraint
  \[ d_t \leq \kappa^T y^T_t + \kappa^N \left( \frac{1 - \omega}{\omega} \right) \left( c^T_t \right)^{1/\eta} \left( y^N_t \right)^{1-1/\eta} \]

- slackness condition
  \[ \mu_t \left[ d_t - \kappa^T y^T_t - \kappa^N \left( \frac{1 - \omega}{\omega} \right) \left( c^T_t \right)^{1/\eta} \left( y^N_t \right)^{1-1/\eta} \right] = 0 \]

- non-negative multiplier
  \[ \mu_t \geq 0 \]
Financial externality

- Additional consumption of tradeables relaxes the collateral constraint on debt
- Agents do not take this into account when they choose their consumption of tradeables
- Effect on economy only during contractions when the constraint binds
  * reduction in consumption of tradeables tightens the constraint
  * reducing ability of agents to borrow
2.5.2 Social Planner’s Problem

- Design to internalize the externality by solving following problem

  - maximize utility with respect to $c^T_t$ and $d_t$ subject to two equilibrium conditions

  \[
  E_0 \sum_{t=0}^{\infty} \beta^t U \left( A \left( c^N_t, c^T_t \right) \right)
  \]

  \[
  d_t = (1 + r) d_{t-1} + c^T_t - y^T_t
  \]

  \[
  d_t \leq \kappa^T y^T_t + \kappa^N \left( \frac{1 - \omega}{\omega} \right) \left( c^T_t \right)^{1/\eta} \left( y^N_t \right)^{1-1/\eta}
  \]

  - right hand side of collateral constraint contains the evolution of consumption of tradeables, taken as exogenous by the household but as endogenous by the social planner
social planner replaces consumption of nontradeables everywhere by output of nontradeables since these will be equal

- Lagrangian

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U \left( A \left( c_t^N, c_t^T \right) \right) + \lambda_t \left[ d_t - (1 + r) d_{t-1} - c_t^T + y_t^T \right] \right\}$$

$$- E_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left( d_t - \kappa^T y_t^T - \kappa^N \left( \frac{1 - \omega}{\omega} \right) (c_t^T)^{1/\eta} (y_t^N)^{1-1/\eta} \right)$$

- First order conditions

- $c_t^T$

$$G \left( c_t^T, c_t^N \right) + \left( \frac{1}{\eta} \right) \mu_t \kappa^N \left( \frac{1 - \omega}{\omega} \right) (c_t^T)^{1/\eta-1} (y_t^N)^{1-1/\eta} = \lambda_t$$
\[
G \left( c_t^T, c_t^N \right) + \mu_t \Gamma \left( c_t^T, c_t^N \right) = \lambda_t
\]

* in periods in which the borrowing constraint is binding, the marginal utility of consumption of tradeables includes the direct marginal utility \( G \left( c_t^T, c_t^N \right) \) plus the factor \( \mu_t \Gamma \left( c_t^T, c_t^N \right) \), reflecting that an extra unit of tradeables raises the relative price of nontradeables relaxing the collateral constraint

\[
- d_t
\]

\[
\lambda_t - \mu_t = \beta \left( 1 + r \right) \lambda_{t+1}
\]

- combining

\[
G \left( c_t^T, c_t^N \right) + \mu_t \Gamma \left( c_t^T, c_t^N \right) = \beta \left( 1 + r \right) E_T \left[ G \left( c_{t+1}^T, c_{t+1}^N \right) + \mu_{t+1} \Gamma \left( c_{t+1}^T, c_{t+1}^N \right) \right] + \mu_t
\]
- slackness condition

\[
\mu_t \left[ d_t - \kappa^T y_t^T - \kappa^N \left( \frac{1 - \omega}{\omega} \right) \left( c_t^T \right)^{1/\eta} \left( y_t^N \right)^{1-1/\eta} \right] = 0
\]
• implementation of social planning solution
  
  – equilibria differ only in Euler equations, where the social planning one includes the effect of extra tradeables consumption on the relative price of nontradeables and therefore on the tightness of the collateral constraint
  
  – achieve the social optimum with a proportional tax on external debt and a lump-sum transfer of the proceeds

* Household budget constraint

\[
d_t = (1 + r)(1 + \tau_{t-1})d_{t-1} + c^T_t - y^T_t + p^N_t(c^N_t - y^N_t) - s_t
\]

* Individual Euler equation becomes

\[
G(c^T_t, c^N_t) = \beta (1 + r)(1 + \tau_t) E_t G(c^T_{t+1}, c^N_{t+1}) + \mu_t
\]

  • the tax on debt raises the cost of increasing debt
optimal fiscal policy makes the individual and the social planning Euler equations identical

\[
\beta (1 + r) E_t \left[ G \left( c^T_{t+1}, c^N_{t+1} \right) + \mu_{t+1} \Gamma \left( c^T_{t+1}, c^N_{t+1} \right) \right] - \mu_t \Gamma \left( c^T_t, c^N_t \right) = \beta (1 + r) \left( 1 + \tau_t \right) E_t G \left( c^T_{t+1}, c^N_{t+1} \right)
\]

* solve for the tax rate which equates

\[
\frac{(1 + \tau_t)}{\beta (1 + r) E_t G \left( c^T_{t+1}, c^N_{t+1} \right)} = \frac{\beta (1 + r) \left[ G \left( c^T_{t+1}, c^N_{t+1} \right) + \mu_{t+1} \Gamma \left( c^T_{t+1}, c^N_{t+1} \right) \right] - \mu_t \Gamma \left( c^T_t, c^N_t \right)}{\beta (1 + r) E_t G \left( c^T_{t+1}, c^N_{t+1} \right)}
\]

* in tranquil periods, when the borrowing constraint does not bind and is not expected to bind in the future, the government does not tax external debt
in periods in which the borrowing constraint does not bind currently ($\mu_t = 0$), but binds with some probability in the future ($E_t\mu_{t+1} > 0$), the government taxes debt to discourage external borrowing.
2.5.3 Simulate to Compare Competitive Equilibrium (CE) and Social Planning Equilibrium (SP)

- Behavior of debt away from and near the collateral constraint
  - debt is increasing in the level of debt away from the collateral constraint, implying current account deficits
  - near the constraint, debt is falling, implying current account surpluses
- Debt in CE tends to be larger
  - 15% probability that debt in the CE is higher than the upper bound of the support for debt in SP economy
  - overborrowing

- Borrowing constraint binds at a higher level of debt in the SP equilibrium

- Define crisis as period when the collateral constraint binds
  - Probability of crisis in CE is 5.5%
  - in SP equilibrium is only 0.5%

- Welfare is not much lower in CE than in SP
2.6 Non-traded Endogenous Output as Collateral

2.6.1 Model

- Utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U (c_t, h_t)$$

- utility is CRRA

$$U (c, h) = \frac{\left( c - \frac{h_t^\delta}{\delta} \right)^{1-\sigma} - 1}{1 - \sigma}$$

- consumption is a composite of tradeables and nontradeables
• Flow budget constraint

\[ d_t = (1 + r) d_{t-1} + c_t^T - y_t^T + p_t^N c_t^N - w_t h_t \]

- tradeables take form of stochastic endowment
- non-tradeables are produced with labor hours

• Collateral constraint is expressed as a fraction of income

\[ d_t \leq \kappa \left( y_t^T + w_t h_t \right) \]

- right hand side of borrowing constraint contains a variable endogenous to households
- households internalize the benefit that working more hours relaxes the borrowing constraint
2.6.2 Equilibrium

- Same equations as with exogenous income except for a FO condition on hours

\[-U_h(c_t, h_t) = \lambda_t (w_t + \kappa \mu_t / \lambda_t)\]

- When the borrowing constraint is binding, \((\mu > 0)\), the shadow real wage \([(w_t + \kappa \mu_t / \lambda_t)]\) exceeds market real wage

- Working additional hours confers benefits in the form of the real wage and a relaxation of the borrowing constraint
2.6.3 Firms

- production equals labor hours

\[ y_t^N = h_t \]

- firm profits

\[ p_t^N y_t^N - w_th_t = (p_t^N - w_t) h_t \]

- free entry guarantees zero profits implying

\[ p_t^N = w_t \]
2.6.4 Borrowing Limit in Equilibrium

- identical to that in model with exogenous output

\[ d_t \leq \kappa \left( y_t^T + p_t^N y_t^N \right) \]

- increasing labor hours can actually tighten the borrowing constraint instead of relaxing it if the larger supply of non-traded goods causes their relative price to fall substantially

  - could occur if the elasticity of substitution between traded and non-traded goods is low

2.6.5 Simulations Show this Model can generate Underborrowing
3 Asymmetric Information (O&R Ch6)

3.1 Model

- agents know own output, but not output of other countries

- 2 period model
  - date 1 exogenous output endowment has mean $\bar{Y}$
  - half receive low output of $\bar{Y}$ and half receive high output of $\bar{Y}$
  - date 2 output is identical for all
• continuum of very small countries, indexed by \([0, 1]\)

• no interest and no discounting

• utility

\[ E \left[ \log (C_1) + \log (C_2) \right] \]
3.2 Financial instruments to achieve risk-sharing

- Arrow-Debreu securities would have agents contract before period 1 to all receive net payouts of $Y$ in period 1
  - Countries receiving more than $Y$ would pay
  - Countries receiving less than $Y$ would receive payments
  - Not possible if actual period 1 output is not observable
• Bonds

- no need to reveal income

- once countries learn income in period 1, trade bonds
  * high-output countries trade away $\frac{1}{2}(\bar{Y} - Y)$ and get it back next period

$$C^H_1 = C^H_2 = \frac{1}{2}(\bar{Y} + Y)$$

* low-output countries get $\frac{1}{2}(Y - \bar{Y})$ and repay next period

$$C^L_1 = C^L_2 = \frac{1}{2}(\bar{Y} + Y)$$
where

\[
\frac{1}{2} (\bar{Y} - Y) = \frac{1}{2} (Y - \bar{Y})
\]

* allocations are efficient for both countries because intertemporal marginal rates of substitution are unity

* ex ante expected utility is lower with bonds since consumption differs across states

– contracts involving payment must make agents choose to reveal their true types
4 Moral Hazard

4.1 Assumptions

- small country with a fixed world interest rate $r$

- large number of 2-period-lived entrepreneurs who invest on date 1 and consume only on date 2

- Utility is period-2 consumption

\[ U_1 = C_2 \]
• Endowment can be invested \((I)\) abroad at the risk-free rate \(r\) or at home in a risky venture yielding

- \(z\) with probability \(\pi(I)\)

- \(0\) with probability \(1 - \pi(I)\)

- higher investment raises expected future output at a declining rate

\[
\pi'(I) > 0 \quad \pi''(I) < 0 \quad \pi(0) = 0 \quad \pi'(0) z > 1 + r
\]
4.2 Optimization Problem

- Choose $I$ to maximize present-value of expected profits

$$-I + \frac{z \pi (I)}{1 + r}$$

- FO condition

$$z \pi' (\bar{I}) = 1 + r$$

- Assume that optimal investment $(\bar{I})$ exceeds $Y_1$

- Finance constraint
- $L$ is gross foreign lending by the small country

- $D$ is gross foreign borrowing by the small country

$$I + L = Y_1 + D$$

- domestic agents can borrow from abroad to lend abroad instead of investing at home yielding capital flight
4.2.1 Optimal Contract

- foreign lenders are risk-neutral and competitive with requirement for expected return of $r$

- if domestic lenders could commit to $\bar{I}$, they would borrow $\bar{I} - Y_1$ and do no foreign lending since $L$ has a lower rate of return

- Promised payments are state-contingent $P(Y_2)$ with

\[ P(0) = 0 \]

$P(z)$, satisfying

\[ \pi(\bar{I}) P(z) = (1 + r)(\bar{I} - Y_1) \]
– expected returns equal the opportunity cost of the project

- Borrower might not be able to credibly commit to $\bar{I}$
4.2.2 Contract with Asymmetric Information

- Asymmetric information
  - lenders observe $Y_1, D, Y_2$, but not $I$ or $L$
  - borrowers choose $I$ and $L$ after the lender sets $D$ and $P(Y_2)$
• Optimization problem for the borrower

- maximizes expected period-2 consumption with respect to \( I \) and \( L \) taking \( P(0) \) and \( P(z) \) as given

\[
EC_2 = \pi(I) [z - P(z)] - [1 - \pi(I)] P(0) + (1 + r)L
\]

- expected consumption equals the probability of good state less payout in good state, less the probability of the bad state times the payout in the bad state plus interest and principle on loans

- substitute budget constraint for loans

\[
EC_2 = \pi(I) [z - P(z)] - [1 - \pi(I)] P(0) + (1 + r)(Y_1 + D - I)
\]

- FO condition with respect to \( I \)

\[
\pi'(I) [z - P(z)] + \pi'(I) P(0) - (1 + r) = 0
\]
\[ \pi'(I)[z + P(0) - P(z)] = (1 + r) \]

- At \( \tilde{I} \),

\[ z\pi'(\tilde{I}) = 1 + r \]

Substituting, optimality at \( \tilde{I} \) requires

\[ 1 + r + \pi'(\tilde{I})[P(0) - P(z)] = (1 + r) \]

implying that the marginal benefits of investing \( \tilde{I} \) are less than those from lending abroad since \( P(0) - P(z) < 0 \)

- Agent will not invest enough, \( \pi'(I) \) must be larger, requiring that \( I \) be smaller

- Borrower is not willing to invest to get the probability of a good outcome up since he has to pay less in the event of a bad outcome

* Lenders know this and will not write the contract
• Incentive-compatible contract

  – Contract maximizes expected period-2 consumption

  \[ EC_2 = \pi(I) [z - P(z)] - [1 - \pi(I)] P(0) + (1 + r)(Y_1 + D - I) \]

  * subject to Lender’s zero profit condition ZP

  \[ \pi(I) P(z) + [1 - \pi(I)] P(0) = (1 + r)D \]

  * Incentive compatibility constraint from agent’s optimal behavior (FO condition) determining investment IC

  \[ \pi'(I) [z + P(0) - P(z)] = (1 + r) \]

  * When \( Y_2 = 0 \), could pay only if used assets held abroad, but these are unobservable. Therefore,

  \[ P(0) = 0 \]
- Solve IC for $P(z)$

\[ P(z) = z - \frac{1 + r}{\pi'(I)} \]

* As required payouts in good times ($P(z)$) increase, willing to invest less to assure good times

* Get optimal investment only if $P(z) = P(0) = 0$

* Expected marginal product of capital exceeds the world interest rate

\[ z\pi'(I) = P(z)\pi'(I) + 1 + r \]
Solve $ZP$ for $P(z)$ letting $D = I - Y_1$, where $L = 0$ at the optimum

$$P(z) = \frac{(1 + r)(I - Y_1)}{\pi(I)}$$

* take derivative

$$\frac{\partial P(z)}{\partial I} = \frac{\pi(I)(1 + r) - (1 + r)(I - Y_1)\pi'(I)}{[\pi(I)]^2}$$

$$= \frac{1 + r}{[\pi(I)]^2} \left[ \pi(I) - \pi'(I)(I - Y_1) \right] > 0$$

* concavity of $\pi(I)$ implies average product exceeds marginal product

$$\pi(I)/I > \pi'(I)$$

* investing more in $\pi(I)$ cannot raise the probability sufficiently that agents could pay less in the event of a good outcome
* as investment and loans rise, payouts have to rise in good times, yielding upward slope for ZP

* horizontal intercept is at $I = Y_1$ from ZP equation for $P(z)$
– Graph shows that equilibrium investment is strictly below $\bar{I}$

– Comparative statics

* higher period-1 output reduces borrowing and shifts ZP right reducing $P(\varepsilon)$ and raising $I$
  
  · therefore the richer the country the smaller is $P(\varepsilon)\pi'(I)$ and the closer is expected marginal product to the interest rate

* increase in profitability in good times with an increase in $\varepsilon$ increases investment demand shifts IC right, raising $P(\varepsilon)$ and raising $I$

* higher $r$ pivots both curves left and reduces $\bar{I}$, reducing investment and having an ambiguous effect on $P(\varepsilon)$
4.2.3 Two-country Model with Asymmetric Information

- Assumptions

  - home and foreign have equal populations

  - $s$ of each population is a saver and $1 - s$ are entrepreneurs

  - savers acquire securities issued by entrepreneurs and are able to diversify and assure a gross riskless rate of return of $1 + r$

  - home savers and entrepreneurs have date 1 endowments of $y_1$, while foreign have $y_1^*$

  $$y_1 > y_1^*$$
– utility is linear in $c_2$ implying that all period 1 endowment is invested in period 2 output

– entrepreneurs have access to identical technologies
• Solution with no informational asymmetries

\[ \pi'(I)z = 1 + r = \pi'(I^*)z \]

– All resources are invested

\[ I = I^* = \frac{y_1 + y_1^*}{2(1 - s)} \]

Since only \( 1 - s \) per cent of agents are entrepreneurs, must divide world per capita output by \( 1 - s \) to convert to investment per entrepreneur

– And the world interest rate is

\[ 1 + r = \pi' \left( \frac{y_1 + y_1^*}{2(1 - s)} \right)z \]
Solution with informational asymmetries

- assume that $y_1 < \bar{I}$ and $y_1^* < \bar{I}^*$ so that entrepreneurs need resources of savers to invest optimal quantity

- incentive compatibility constraints under asymmetric information

$$P(z) = z - \frac{1 + r}{\pi'(I)} \quad P(z)^* = z - \frac{1 + r}{\pi'(I^*)}$$

- zero-profit constraints for the lender

$$P(z) = \frac{(1 + r)(I - y_1)}{\pi(I)} \quad P(z)^* = \frac{(1 + r)(I^* - y_1^*)}{\pi(I^*)}$$
substitute \( P(z) \) from incentive compatibility constraint into zero-profit constraint for each country and solve for the interest rate

\[
1 + r = \frac{\pi'(I) z}{1 + \frac{\pi'(I)(I-y_1)}{\pi(I)}} \equiv \rho(I, y) \quad \rho_I < 0; \rho_y > 0
\]

* as investment increases, its marginal product, given by increase in probability of a good outcome, falls

* as output increases, amount borrowed falls and payout falls increasing interest rate

* analogous interest rate solution for foreign country

* equilibrium requires domestic and foreign interest rates equal

\[
\rho(I, y_1) = 1 + r = \rho^*(I^*, y_1^*)
\]
resource constraint

\[ \frac{y_1 + y_1^*}{(1 - s)} = I + I^* \]

in equilibrium saving flows to richer country

* Assume

\[ y_1 > y_1^* \]

* If \( I = I^* \), then returns to investing in richer country are larger

\[ \rho (I, y_1) > \rho^* (I^*, y_1^*) \]

* Need \( I > I^* \), to equilibrate the returns to investing in both countries