

The Fiscal Theory of the Price Level and Initial Government Debt*

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Abstract

It is widely believed that the Fiscal Theory of the Price Level (FTPL) does not work in an environment in which initial government debt is zero. This paper demonstrates that this view is incorrect when the government issues a set of financial assets restricted to standard nominal debt contracts and money. In particular, it is possible to define a non-Ricardian fiscal policy for which the set of equilibrium price sequences under non-Ricardian fiscal policy is a proper subset of the set of equilibrium price sequences under Ricardian fiscal policy.

Key Words: *Fiscal Theory of the Price Level, Active and Passive Fiscal Policy, Ricardian and non-Ricardian Fiscal Policy, Monetary Policy*

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1 Introduction

The Fiscal Theory of the Price Level (FTPL) states that the expected price level is determined to equate the expected present value of government budget surpluses with the real value of outstanding nominal government debt. If there is no outstanding nominal debt, then the present value of surpluses must be zero, and this relationship cannot determine the equilibrium price level. However, could the FTPL work in periods after the government has issued debt, and therefore in actual economies? Niepelt (2004) claims that if government debt begins at zero, then the FTPL does not apply. Explicitly he claims that "the non-Ricardian policy assumption and, by implication, fiscal price level determination are inconsistent with an equilibrium in which all asset holdings reflect optimal household choices" (Niepelt 2004, 277).

This paper demonstrates that the FTPL does work in a standard dynamic macroeconomic model as long as the government issues a restricted set of securities. We assume that government debt takes the form of a standard nominal debt contract and money. The government issues no state-contingent claims. We show that it is possible to define a non-Ricardian (active) fiscal policy, consistent with the equilibrium conditions derived from household optimizing behavior, under which the government's intertemporal budget constraint restricts the set of equilibrium price sequences to be a proper subset of the set of equilibrium price sequences admissible under Ricardian policy.

The Fiscal Theory of the Price Level and Initial Government Debt

The paper is organized as follows. The next section characterizes equilibrium price sequences under Ricardian and non-Ricardian fiscal policy in a simple two-period economy and provides the main results. Section 3 extends the results to an infinite horizon economy, and Section 4 contains conclusions.

2 A Two-Period Economy

Let the economy begin at $t = 0$ with the creation of a government, consisting of a monetary and a fiscal authority, and the birth of a representative agent. The representative agent receives an endowment of y in periods 0 and 1, and the government spends g in both periods. Immediately following period 1, the economy ends.

2.1 Government

The newly-created government has no outstanding debt, but issues nominal debt in the initial period. The fiscal authority purchases a fixed quantity of goods (g) each period. The financing in the initial period is determined by the choice of a fiscal policy, with some combination of a lump-sum tax (τ_0) and one-period nominal bonds (B_0). The monetary authority buys government bonds in exchange for money (M_0), fixing the nominal interest rate at i^* .¹ In the next period, the government pays interest and principle on its debt and finances its spending of g by levying taxes (τ_1) and issuing any new debt that agents are willing to hold. The consolidated monetary and fiscal authority faces flow budget constraints

¹ The assumption that the interest rate is fixed follows Niepelt (2004) and much of the FTPL literature. It simplifies the analysis, but is not essential for what follows.

The Fiscal Theory of the Price Level and Initial Government Debt

in the two periods given by

$$M_0 + B_0 = P_0(g - \tau_0), \quad (1)$$

$$M_1 + B_1 = (1 + i^*)(M_0 + B_0) + P_1(g - \tau_1) - i^*M_0,$$

where P_t is the price of the single good in the economy at time t , and $M_0 + B_0 > 0$ under the assumption that the government issues debt in the initial period.

The government's real primary surplus inclusive of seigniorage revenue (s_t) is given by

$$s_t = \tau_t - g + \left(\frac{i^*}{1 + i^*} \right) \frac{M_t}{P_t}. \quad (2)$$

Letting D_t denote government debt inclusive of interest at the end of period t ,

$$D_t = (1 + i^*)B_t + M_t, \quad (3)$$

the government's period 0 and 1 flow budget constraints become

$$D_0 = -(1 + i^*)(s_0P_0), \quad (4)$$

$$D_1 = (1 + i^*)(D_0 - s_1P_1). \quad (5)$$

Combining equations (4) and (5) and imposing $D_1 = 0$ yields the government's intertemporal budget constraint as

$$0 = s_0 + \frac{P_1}{P_0} \left(\frac{1}{1 + i^*} \right) s_1. \quad (6)$$

The intertemporal budget constraint requires that the present value of surpluses over the two periods equal the initial value of outstanding debt, which is set at zero.

With government spending fixed and the monetary authority pegging the nominal interest rate, the behavior of taxes over time characterizes fiscal policy. Following Kocherlakota

The Fiscal Theory of the Price Level and Initial Government Debt

and Phelan (1999), we define fiscal policy as a function. Let $P = \{P_t\}_{t=0}^1$ denote a stochastic process of positive prices and let \mathbb{P} denote the space of these processes. Similarly, let $(\tau, D, M) = \{\tau_t, D_t, M_t\}_{t=0}^1 \in \mathbb{T} \times \mathbb{D} \times \mathbb{M}$ denote fiscal policy processes.

Definition 1 *A fiscal policy is a function $\Omega_j : \mathbb{P} \rightarrow \mathbb{T} \times \mathbb{D} \times \mathbb{M}$ such that for all price sequences $\{P_t\}_{t=0}^1$, the corresponding policy sequence $\{\tau_t, D_t, M_t\}_{t=0}^1$ satisfies the government's flow budget constraints, given by equations (4) and (5).*

Fiscal policy can be either Ricardian or non-Ricardian.

Definition 2 *A Ricardian (passive) fiscal policy is a particular fiscal policy for which the government's intertemporal budget constraint (equation 6) is satisfied for any price sequence.*

Definition 3 *A non-Ricardian (active) fiscal policy is a particular fiscal policy for which there exists at least one price sequence for which the government's intertemporal budget constraint (equation 6) is not satisfied.*

2.2 Agent

The representative agent receives a fixed perishable endowment of goods each period (y), where $y - g > 0$, and maximizes a utility function, which is separable in consumption and real money balances, subject to a sequence of budget constraints and borrowing constraints.

Expected utility is given by

$$E_t \sum_{t=0}^1 \beta^t \left[u(c_t) + v\left(\frac{M_t}{P_t}\right) \right], \quad (7)$$

where c_t denotes real consumption at time t and the functions u and v are assumed to be twice differentiable, strictly increasing and strictly concave with $u'(0) = \infty$. E_t denotes the expectation conditioned on all variables dated t or earlier. The flow budget constraints in

The Fiscal Theory of the Price Level and Initial Government Debt

nominal terms for the two periods are given by²

$$M_0 + B_0 \leq P_0(y - \tau_0 - c_0), \quad (8)$$

$$M_1 + B_1 \leq (1 + i^*)(M_0 + B_0) + P_1(y - \tau_1 - c_1) - i^*M_0. \quad (9)$$

Using the definition of D_t from equation (3), the agent's flow budget constraints can be expressed as

$$D_t \leq (1 + i^*) \left[D_{t-1} + P_t(y - \tau_t - c_t - \frac{i^*}{1 + i^*} \frac{M_t}{P_t}) \right], \quad t \in (0, 1), \quad (10)$$

where $D_{-1} = 0$.

The borrowing constraints require that the agent borrow no more than the expected present value of future income

$$\frac{D_0}{P_0} \geq -E_0 \left(\frac{P_1(y - \tau_1)}{P_0} \right),$$

and

$$\frac{D_1}{P_1} \geq 0.$$

The flow budget constraints and the borrowing constraints can be combined to yield the agent's intertemporal budget constraint as

$$\sum_{t=0}^1 \left(c_t + \frac{i^*}{1 + i^*} \frac{M_t}{P_t} \right) \frac{P_t}{P_0} \left(\frac{1}{1 + i^*} \right)^t \leq \sum_{t=0}^1 (y - \tau_t) \frac{P_t}{P_0} \left(\frac{1}{1 + i^*} \right)^t. \quad (11)$$

First order conditions for the agent yield the standard Euler equation,

$$E_0 \left[\frac{\beta (1 + i^*) P_0 u'(c_1)}{P_1 u'(c_0)} \right] = 1, \quad (12)$$

² Agents can trade in state-contingent claims. However, since the government does not trade in these claims they will be in zero net supply and will not appear in the representative agent's budget constraint. The results do not require that risk-sharing among agents be restricted.

The Fiscal Theory of the Price Level and Initial Government Debt

and an implicit money demand equation,

$$\frac{i^*}{1+i^*} = \frac{v' \left(\frac{M_t}{P_t} \right)}{u'(c_t)}, \quad t \in (0, 1). \quad (13)$$

Equation (12) implies that the household's flow and intertemporal budget constraints, equations (10) and (11) respectively, hold with equality along the optimal path.

2.3 Equilibrium

Definition 4 *Given outstanding values for nominal debt and the real surplus ($D_{-1} = 0, s_{-1} = 0$), a fiscal policy, Ω_j , and a monetary policy fixing $i_t = i^*$, an equilibrium is a collection of sequences for real consumption $\{c_t\}_{t=0}^1$, real taxes $\{\tau_t\}_{t=0}^1$, nominal asset quantities $\{M_t, D_t\}_{t=0}^1$ and prices $\{P_t\}_{t=0}^1$ such that goods market equilibrium holds ($c = y - g$); the agent's first order conditions, given by equations (12) and (13), hold; flow and intertemporal budget constraints for the agent, given by equations (10) and (11), hold; expectations are rational; all prices are positive; and all realizations of nominal money are non-negative.*

Under the simplifying assumptions of constant values for both the endowment and government spending, equilibrium consumption is constant at $y - g$. Substituting the equilibrium value for consumption, the Euler equation (12) simplifies to

$$E_0 \left[\frac{\beta (1+i^*) P_0}{P_1} \right] = 1. \quad (14)$$

Equilibrium real money balances are constant at

$$\frac{M_t}{P_t} = v'^{-1} \left(\frac{i^*}{1+i^*} u'(y-g) \right), \quad t \in (0, 1). \quad (15)$$

In equilibrium real money demand must equal real money supply, implying that this equation determines the equilibrium quantity of nominal money, given an equilibrium value for price and the fixed equilibrium value for real money.

Using goods market equilibrium ($c = y - g$) to substitute for consumption in the agent's flow and intertemporal budget constraints with equality yields the government's flow and

The Fiscal Theory of the Price Level and Initial Government Debt

intertemporal budget constraints. Therefore, the government's flow and intertemporal budget constraints (equations 4, 5, and 6) must hold in equilibrium. In what follows, we use the government's budget constraints as equilibrium restrictions, in place of the agent's budget constraints.

To summarize, equations necessary in equilibrium include: good market equilibrium ($c + g = y$), determining c ; money market equilibrium from equation (15), determining nominal money $\{M_t\}_{t=0}^1$ needed to keep the interest rate fixed at i^* for an equilibrium realization of price; the Euler equation, given by (14); the government's flow budget constraints, equations (4) and (5); and its intertemporal budget constraint, equation (6). Only equations (14), (4), and (6) have implications for the set of equilibrium price sequences.

Consider the determination of the initial price (P_0) in equilibrium.

2.3.1 Initial Price in Equilibrium

Lemma 1 *When outstanding government debt is zero ($D_{-1} = 0$), the government's intertemporal budget constraint is satisfied in equilibrium for any positive value for the initial price level (P_0).*

Proof. If equation (6) holds, and it must hold in equilibrium, then it holds for any positive value of P_0 . Equation (6) restricts only the ratio $\frac{P_1}{P_0}$. ■

Lemma 2 *Fiscal policy places no restrictions on the equilibrium value of the initial price level (P_0) when D_0 is endogenous.*

Proof. From equation (4), the assumption that $D_0 > 0$ implies that $s_0 < 0$. Equation (4) does not restrict the initial price level since the value for D_0 can adjust to any positive value of P_0 . Additionally, Lemma 1 shows that P_0 is not restricted by equation (6). Therefore,

The Fiscal Theory of the Price Level and Initial Government Debt

fiscal policy places no restrictions on P_0 .³ ■

Corollary 1 *Any positive initial price level (P_0) is an equilibrium value when D_0 is endogenous.*

Proof. No other requirements for equilibrium in the economy restrict P_0 . ■

The results in Lemmas 1 and 2 and in Corollary 1 are identical to those in Niepelt's (2004) two-period perfect-foresight economy. When outstanding debt is zero ($D_{-1} = 0$), the present value of future surpluses must be identically zero irrespective of the value for P_0 . The value for P_0 is indeterminate.

2.3.2 Future Price in Equilibrium

Lemma 3 *When fiscal policy is Ricardian, any positive price sequence, satisfying the Euler equation (14), is an equilibrium sequence.*

Proof. The Euler equation (14), which must hold in equilibrium, restricts the expectation for future price. By the definition of Ricardian policy, equation (6) provides no additional restrictions. No other equilibrium requirements provide restrictions. ■

Therefore, in equilibrium, the future price (P_1) can be stochastic, but its expectation is determined by the previous period's value for price together with the monetary authority's choice of the interest rate peg. Equivalently, the expectation of inflation is stochastic and the expectation of its inverse is determined by the inverse of the nominal interest rate peg.

To consider non-Ricardian fiscal policy, it is useful to define two particular non-Ricardian policies, a non-stochastic, non-Ricardian policy, Ω_n , and a stochastic, non-Ricardian policy,

³ If the government were able to devise a strategy for which D_0 and s_0 were both exogenous, then equilibrium P_0 would be uniquely determined by equation (4).

The Fiscal Theory of the Price Level and Initial Government Debt

Ω_s . Not all non-Ricardian fiscal policies are consistent with equilibrium. The two non-Ricardian policies we consider satisfy equation (14), and are consistent with equilibrium.

Definition 5 *The non-stochastic, non-Ricardian fiscal policy, Ω_n , is a fixed value for $s_0 = \bar{s}_0 < 0$ and a fixed value for $s_1 = \bar{s}_1 = -\frac{\bar{s}_0}{\beta}$.*

Definition 6 *The stochastic, non-Ricardian policy, Ω_s , is a fixed value for $s_0 = \bar{s}_0 < 0$ and a stochastic realization for $s_1 = \bar{s}_1 + \epsilon$, where $\bar{s}_1 = -\frac{\bar{s}_0}{\beta}$, and ϵ is stochastic with mean zero and is restricted such that $s_1 > 0$.*

Theorem 1 *Let P^* be the set of equilibrium price level sequences for a given Ricardian policy. Under either non-Ricardian fiscal policy, Ω_n or Ω_s , the set of equilibrium price level sequences is a proper subset of P^* .*

Proof. By Lemma 3, P^* contains all positive price sequences that satisfy equation (14).

Now, consider the set of equilibrium price sequences under the two non-Ricardian policies, Ω_n and Ω_s . Combining equations (6) and (4) yields

$$P_1 s_1 = D_0 = -P_0 s_0 (1 + i^*). \quad (16)$$

Once period 1 has arrived, both D_0 and P_0 are pre-determined, consistent with equation (4). In period 1, a value for s_1 is realized. Given pre-determined values for D_0 and P_0 and the realization of a value for s_1 , $P_1 = \hat{P}_1$ uniquely solves equation (16).

To support an equilibrium, \hat{P}_1 must also satisfy equation (14). Solving the first part of equation (16) for s_1 and taking its time 0 expectation using equation (14), yields

$$E_0 s_1 = D_0 E_0 \left(\frac{1}{P_1} \right) = \frac{D_0}{\beta (1 + i^*) P_0}.$$

By construction, both non-Ricardian fiscal policies, Ω_n and Ω_s , satisfy

$$E_0 s_1 = -\frac{\bar{s}_0}{\beta}.$$

The Fiscal Theory of the Price Level and Initial Government Debt

The government's initial flow budget constraint, equation (4), implies that two expressions for $E_0 s_1$ are equivalent. Therefore, under either Ω_n or Ω_s , \hat{P}_1 satisfies equation (14). This implies that a non-Ricardian fiscal policy, characterized by either Ω_n and Ω_s , yields a unique equilibrium value of P_1 , given initial values and a realized value for s_1 .

Therefore, the set of equilibrium price sequences under either type of non-Ricardian fiscal policy is restricted by equations (14) and (16), while P^* is restricted only by equation (14). It follows that the set of equilibrium price sequences under either Ω_n or Ω_s is a proper subset of P^* ■

When fiscal policy is Ricardian, the government's intertemporal budget constraint must hold for any positive realization for P_1 . Therefore, under Ricardian fiscal policy, agents could coordinate on a sunspot equilibrium value for P_1 . Given this value for P_1 , Ricardian fiscal policy requires that s_1 take on a value assuring intertemporal government budget balance, thereby validating the sunspot equilibrium value for P_1 . This implication of Ricardian policy, combined with the indeterminate initial price level, is familiar from Sargent and Wallace (1975). An interest rate peg under Ricardian fiscal policy involves both price level indeterminacy in the initial period and self-fulfilling sunspot equilibria in future periods (Bassetto 2005).

Kocherlakota and Phelan (1999) emphasize that non-Ricardian fiscal policy serves to choose among the multiple equilibria possible under Ricardian fiscal policy. This proposition remains valid even when outstanding initial debt is zero. Under either non-Ricardian fiscal policy (Ω_n or Ω_s), a realized value for s_1 , together with pre-determined period 0 values,

The Fiscal Theory of the Price Level and Initial Government Debt

implies a value for P_1 as the unique solution to (16). Although nothing pins down initial price when outstanding debt is zero, non-Ricardian policy does restrict the set of equilibrium price sequences by making the equilibrium value for P_1 a unique function of initial values and the realized value for s_1 .

This same result can be explained alternatively by rewriting equation (6) as

$$s_0 + \frac{s_1}{(1+i^*)(P_0/P_1)} = 0. \quad (17)$$

When outstanding debt is zero in period 0 ($D_{-1} = 0$), the actual present value of future surpluses, discounted at the actual real interest rate, must be zero. Under a Ricardian fiscal policy, a sunspot realization for the real interest rate requires s_1 to adjust to assure that equation (17) holds for all realizations of P_1 . Therefore, all positive realizations of P_1 which satisfy equation (14) are equilibrium realizations. Alternatively, when s_1 is fixed by the non-Ricardian policy Ω_n , there is a unique real interest rate that solves equation (17). With P_0 predetermined in period 1, the equilibrium value for P_1 is unique, ruling out sunspot equilibria. When s_1 is determined by the stochastic non-Ricardian policy Ω_s , equilibrium requires that a surplus innovation in period 1 be offset by a corresponding innovation in the real rate of return on government debt. This change in the real rate of return generates the revenue necessary for the government's intertemporal budget to balance from period 0, and is an alternative, but equivalent, way of viewing the capital gains and losses on debt created by price surprises.⁴ Under either non-Ricardian policy, the equilibrium real interest rate is unique, given initial values and a realization for s_1 .

⁴ As before, an innovation in the real return on debt can change real surpluses only if $s_1 \neq 0$, which requires that outstanding debt is not zero. In this example, $s_0 \neq 0$. When $s_0 = 0$, s_1 must be zero.

The Fiscal Theory of the Price Level and Initial Government Debt

Under any fiscal policy consistent with equilibrium, expected inflation is determined by the monetary authority's choice of the interest rate peg from equation (14). A fiscal policy which does not respect this expectation is not consistent with equilibrium. Therefore, the mean for the present value of future surpluses must be determined such that the government plans to balance its budget at the expected future price level, implied by the current price level and by the monetary authority's choice of an interest rate peg. Under Ricardian policy, this expectation is the only restriction on the equilibrium value for actual price. Under the non-Ricardian policies Ω_n and Ω_s , the equilibrium value for P_1 is further restricted by the realized value for s_1 , either as a fixed value consistent with equation (14), or as a realization from a distribution whose mean is consistent with equation (14).

A central characteristic of non-Ricardian fiscal policy is that the unexpected capital gains and losses on government debt and, equivalently, deviations of real returns from expectations, cannot provide systematic revenue. This feature of stochastic non-Ricardian fiscal policy is not original to this paper and is standard in papers with stochastic non-Ricardian fiscal policy (Christiano and Fitzgerald 2000 and Woodford 1998).

2.3.3 Niepelt's Two-Period Example

Niepelt (2004, p.283) derives an identical result in his two-period economy when he allows taxes to be stochastic in the second period, subject to a restriction on their mean to assure that the intertemporal budget is expected to balance, analogous to the mean on ϵ above. He notes that this restriction is equivalent to a restriction on the expected rate of return, analogous to equation (14). However, he interprets this stochastic tax policy as a Ricardian

The Fiscal Theory of the Price Level and Initial Government Debt

policy on the grounds that "Fiscal policy in this stochastic environment must still, on average, deliver the required rate of return..."

Contrary to Niepelt's claim, stochastic taxes, subject to a restriction on their mean such that the government is expected to satisfy its intertemporal budget constraint, or, equivalently, a restriction on the expected return, is an example of non-Ricardian fiscal policy. This is according to the definition of non-Ricardian policy above, which is virtually equivalent to Niepelt's formal definition (2004, p.288). The two definitions differ in that Niepelt defines fiscal policy as a sequence of numbers instead of as a function of a stochastic process. However, if taxes were a sequence of numbers, than almost no policy could be Ricardian, since policy could not respond to stochastic price realizations.⁵ Therefore, to allow some fiscal policies to be Ricardian, fiscal policy must be defined as a function. All fiscal policies admissible in equilibrium must satisfy the expected rate of return restriction (equation 14), but all fiscal policies satisfying this restriction are not Ricardian by the formal definition. This is because all price sequences satisfying equation (14) do not assure that the government's intertemporal budget constraint (equation 6) is satisfied.

This claim, that Niepelt's fiscal policy with stochastic taxes is non-Ricardian, is also consistent with other authors who consider the FTPL in a stochastic context. Christiano and Fitzgerald (2000) and Woodford (1998) are both careful to require that stochastic future surpluses satisfy a mean implying that the government plans to balance its intertemporal budget at the expected future price level. Finally, labeling exogenous stochastic taxes as a Ricardian policy appears at odds with Niepelt's own assumption that: "(b) Non-Ricardian

⁵ The author thanks Associate Editor, Marco Bassetto, for this point.

The Fiscal Theory of the Price Level and Initial Government Debt

fiscal policy: the FTPL assumes that the fiscal policy in period 0 can "move before" the price level,..." (2004, p.281). In the framework considered here, the exogenous stochastic tax, τ_1 , clearly moves in advance of the price level P_1 , even though τ_1 is constrained in mean. Therefore, Niepelt (2004) provides a counterexample in his paper to his claim that non-Ricardian policy and equilibrium are inconsistent when initial outstanding debt is zero. He presents a fiscal policy, that satisfies the definition of a non-Ricardian policy, and solves for the equilibrium restrictions on price under this policy.

Additionally, the fiscal policy in Niepelt's non-stochastic economy is Ω_n , a non-Ricardian policy. A fixed value for taxes in period 0, subject to the constraint that future taxes balance the government's budget when future price takes on its expected value, does not satisfy the government's intertemporal budget constraint for all possible realizations for P_1 . Intertemporal budget balance fails when agents coordinate on a sunspot equilibrium value for price which deviates from its expected value.

Niepelt (2004) also argues that the stochastic realization of an equilibrium value for P_1 , conditioned on P_0 , is a realization of inflation, not a price level, since the equilibrium value for P_0 is not restricted by the requirements for equilibrium. We can view the equation determining P_1 as equality of the outer terms in equation (16), as Niepelt does, leading to the interpretation of a "fiscal theory of inflation," in which the equilibrium value for P_1 is conditional on the pre-determined equilibrium value for P_0 , or as the first equality in equation (16), leading to the interpretation as a "fiscal theory of the price level" with the pre-determined value for debt (D_0) as a nominal anchor as in the FTPL. The two represen-

The Fiscal Theory of the Price Level and Initial Government Debt

tations are equivalent, given the government's budget constraint in the initial period. Papers presenting the FTPL are often written with the price level as the endogenous variable, but Woodford (1998) discusses the FTPL with the endogenous price variable represented as inflation. Either representation is consistent with the model and therefore correct. Additionally, Niepelt's claim (2004, p. 284) that "It rather confirms what we already knew from Sargent and Wallace (1981) and Sargent (1987), namely that the interaction of monetary and fiscal policy determines state-contingent *inflation* rates, but not price levels," is not relevant to the central issue of the validity of the FTPL in this economy. The method of representing the restrictions imposed by non-Ricardian fiscal policy on the equilibrium value for P_1 does not change the substantive result that non-Ricardian policy yields a set of equilibrium price sequences that is a proper subset of that under Ricardian fiscal policy.

3 Infinite Horizon Economy

In this section, we demonstrate that these results extend to an infinite horizon economy which begins with no outstanding government debt. Using equation (5), the government's flow budget constraint, governing the evolution of real government debt $\left(d_t = \frac{D_t}{P_t}\right)$, can be expressed as

$$d_t = (1 + i^*) \left(d_{t-1} \frac{P_{t-1}}{P_t} - s_t \right). \quad (18)$$

Under the assumption that $d_{-1} = 0$, period 0 debt is given by

$$d_0 = \frac{D_0}{P_0} = - (1 + i^*) s_0. \quad (19)$$

The Fiscal Theory of the Price Level and Initial Government Debt

Taking the expected present value of equation (18), summing over time, and taking the limit yields

$$\lim_{T \rightarrow \infty} E_t d_{t+T} \frac{P_{t+T}}{P_t} \left(\frac{1}{1+i^*} \right)^{T+1} = \frac{P_{t-1}}{P_t} d_{t-1} - E_t \sum_{j=0}^{\infty} s_{t+j} \frac{P_{t+j}}{P_t} \left(\frac{1}{1+i^*} \right)^j.$$

The sequence of intertemporal budget constraints for the government, one for each date $t \geq 0$, is given by

$$\lim_{T \rightarrow \infty} E_t d_{t+T} \frac{P_{t+T}}{P_t} \left(\frac{1}{1+i^*} \right)^{T+1} = 0,$$

or equivalently

$$\frac{P_{t-1}}{P_t} d_{t-1} = E_t \sum_{j=0}^{\infty} s_{t+j}^j \frac{P_{t+j}}{P_t} \left(\frac{1}{1+i^*} \right)^j.$$

We show below that the government's flow and intertemporal budget constraints are required in equilibrium.

The representative agent maximizes expected utility over an infinite horizon, subject to sequences of flow budget constraints and borrowing constraints

$$d_t \leq (1+i^*) \left(d_{t-1} \frac{P_{t-1}}{P_t} + y - \tau_t - c_t - \left(\frac{i^*}{1+i^*} \right) \frac{M_t}{P_t} \right), \quad t \geq 0,$$

$$d_t \geq -E_t \sum_{j=1}^{\infty} (y - \tau_{t+j}) \frac{P_{t+j}}{P_t} \left(\frac{1}{1+i^*} \right)^{j-1}, \quad t \geq 0,$$

where $d_{-1} = 0$. The borrowing constraints require that the agent's assets (d_t) have a lower bound given by the negative of the expected present value of future disposable income.

Maximization of expected utility yields the standard Euler equation and an implicit money demand equation, analogous to equations (12) and (13), as well as the requirement that the sequences of flow and intertemporal budget constraints hold with equality (Woodford

The Fiscal Theory of the Price Level and Initial Government Debt

1994). The sequence of intertemporal budget constraints can be expressed either as

$$\frac{P_{t-1}}{P_t} d_{t-1} = E_t \sum_{j=0}^{\infty} \left(\tau_{t+j} + c_{t+j} + \left(\frac{i^*}{1+i^*} \right) \frac{M_{t+j}}{P_{t+j}} - y \right) \frac{P_{t+j}}{P_t} \left(\frac{1}{1+i^*} \right)^j, \quad t \geq 0,$$

or as the sequence of transversality conditions

$$\lim_{T \rightarrow \infty} E_t d_{t+T} \frac{P_{t+T}}{P_t} \left(\frac{1}{1+i^*} \right)^{T+1} = 0, \quad t \geq 0. \quad (20)$$

The Euler equation, together with an assumption that the monetary authority pegs the gross interest rate at β^{-1} , yields

$$E_t \left(\frac{P_t}{P_{t+1}} \right) = \frac{1}{\beta(1+i^*)} = 1. \quad (21)$$

Defining

$$\gamma_t = \begin{cases} d_{t-1} \left(\frac{P_{t-1}}{P_t} - 1 \right) & d_{t-1} \neq 0 \\ \gamma_t = 0 & d_{t-1} = 0 \end{cases} \quad (22)$$

implies that monetary policy, together with the Euler equation, restrict γ_t according to:

$$E_{t-1} \gamma_t = 0, \quad t \geq 1. \quad (23)$$

Note that $\gamma_t \neq 0$ is equivalent to unexpected capital gains or losses on debt due to price surprises. In particular, $\gamma_t < 0$ represents capital losses on real outstanding debt due to unexpected inflation. The initial value for γ_t must be zero when there is no outstanding debt, requiring that feasible sequences for $\gamma = \{\gamma_t\}_{t=0}^{\infty}$ begin with $\gamma_0 = 0$.

We specify fiscal policy as a rule. However, the rule should be viewed as a simple way to describe a more complicated strategy as in Bassetto (2002). We assume the fiscal rule is

$$s_t = (1 - \alpha) s_{t-1} + \alpha \bar{s} + \eta_t, \quad 0 < \alpha < 1, \quad \bar{s} > 0, \quad t \geq 0, \quad (24)$$

The Fiscal Theory of the Price Level and Initial Government Debt

where

$$s_0 = \alpha \bar{s} + \eta_0, \tag{25}$$

under the assumption that the government which begins in period 0 has no history of surpluses ($s_{-1} = 0$). The surplus depends on its own lagged value and on a long-run target, \bar{s} . The innovation, η_t for $t \geq 1$, can be stochastic. If so, it is assumed to be i.i.d. around a possibly time-varying mean. Its behavior determines whether fiscal policy is Ricardian or non-Ricardian.

A Ricardian fiscal policy is a function which maps sequences for $\gamma = \{\gamma_t\}_{t=0}^{\infty}$ into sets of sequences $(\eta, d) = \{\eta_t, d_t\}_{t=0}^{\infty}$ such that the government's intertemporal budget constraint, equation (20), is satisfied for any feasible sequence γ . A non-Ricardian fiscal policy is a function which maps sequences for $\gamma = \{\gamma_t\}_{t=0}^{\infty}$ into sets of sequences $(\eta, d) = \{\eta_t, d_t\}_{t=0}^{\infty}$ such that the government's intertemporal budget constraint, equation (20), is not satisfied for some feasible sequence γ .

Equilibrium is defined as in the two-period economy except that sequences go to infinity instead of unity. Goods market equilibrium together with household optimization imply that the government's flow and intertemporal budget constraints must be satisfied along the optimal path. Therefore, equations which must be satisfied in equilibrium include: the government's flow budget constraints, given by equations (18) and (19); the agent's transversality condition, equivalently intertemporal government budget balance, given by equations (20); the agent's Euler equation combined with monetary policy, given by equation (23); and equations (24) and (25) for the evolution of the surplus. Equation (13) must also

The Fiscal Theory of the Price Level and Initial Government Debt

be satisfied, but this equation determines the equilibrium value of nominal money and does not restrict equilibrium price.

To derive equilibrium restrictions on the sequences γ and η under alternative types of fiscal policies, it is necessary to solve the dynamic system in the surplus and debt, given by equations (18) and (24), subject to initial conditions (19) and (25). Substituting the definition of γ_t into equation (18) yields an expression for the government's flow budget constraint as a linear difference equation in real debt and the real surplus

$$d_t = (1 + i^*) (d_{t-1} + \gamma_t - s_t). \quad (26)$$

Solutions for the surplus and debt can be represented as

$$s_t = \bar{s} + \Phi_{1,t} (1 - \alpha)^t \quad (27)$$

$$d_t = \frac{1 + i^*}{i^*} \bar{s} + \frac{1 + i^*}{\alpha + i^*} \Phi_{1,t} (1 - \alpha)^{t+1} + \Phi_{2,t} (1 + i^*)^{t+1}, \quad (28)$$

where

$$\Phi_{1,t} = \left[- (1 - \alpha) \bar{s} + \sum_{j=0}^t \left(\frac{1}{1 - \alpha} \right)^j \eta_j \right],$$

$$\Phi_{2,t} = \sum_{j=1}^t \left[\gamma_j - \frac{1 + i^*}{\alpha + i^*} \eta_j \right] \left(\frac{1}{1 + i^*} \right)^j - \left(\frac{1 + i^*}{\alpha + i^*} \right) \left[\eta_0 + \left(\frac{1 + i^*}{i^*} \right) \alpha \bar{s} \right].$$

Since the price level must be positive in equilibrium, the solution for real debt will violate the sequence of transversality conditions, given by equation (20), unless

$$\lim_{T \rightarrow \infty} E_t \Phi_{2,t+T} = 0, \quad t \geq 0. \quad (29)$$

Consider equilibrium restrictions on fiscal policy, expressed as equilibrium restrictions on sequences for $\eta = \{\eta_t\}_{t=0}^{\infty}$. Given that monetary policy fixes the gross interest rate at

The Fiscal Theory of the Price Level and Initial Government Debt

β^{-1} , any fiscal policy consistent with equilibrium must satisfy equation (23). Imposing $\lim_{T \rightarrow \infty} E_0 \Phi_{2,T} = 0$, as required by equation (29), together with equation (23), yields

$$\eta_0 = - \left(\frac{1+i^*}{i^*} \right) \alpha \bar{s} - E_0 \sum_{j=1}^{\infty} \eta_j \left(\frac{1}{1+i^*} \right)^j. \quad (30)$$

The initial real surplus must be determined such that the government is expected to balance its intertemporal budget. Since outstanding debt is zero in the initial period, the initial surplus can be negative only if the expected present value of future surpluses is positive.

We restrict attention to fiscal policies consistent with equilibrium and assume that η_0 is given by equation (30). Using the expression for η_0 , the sequence of transversality conditions for $t \geq 1$ can be expressed as

$$\lim_{T \rightarrow \infty} E_t \Phi_{2,t+T} = \sum_{j=1}^t \left[\gamma_j - \frac{1+i^*}{\alpha+i^*} (\eta_j - E_0 \eta_j) \right] \left(\frac{1}{1+i^*} \right)^j = 0, \quad t \geq 1, \quad (31)$$

where we have used the fact that $E_t \sum_{j=t+1}^{\infty} [\gamma_j - \frac{1+i^*}{\alpha+i^*} (\eta_j - E_0 \eta_j)] \left(\frac{1}{1+i^*} \right)^j = 0$ under the assumptions of the model. Since equation (31) must hold for all $t \geq 1$, it requires

$$\gamma_t = \frac{1+i^*}{\alpha+i^*} (\eta_t - E_0 \eta_t), \quad t \geq 1. \quad (32)$$

Therefore, intertemporal government budget balance requires both equations (30) and (32). This is equivalent to requiring that the coefficient on the unstable root in equation (28) be zero, yielding a saddlepath equilibrium.

Equilibrium conditions, potentially restricting the equilibrium price sequence, include: the government's initial flow budget constraint, given by equation (19); the agent's Euler equation combined with monetary policy, represented by equation (23); and equations (30) and (32), which impose intertemporal government budget balance.

The Fiscal Theory of the Price Level and Initial Government Debt

Consider, first, the equilibrium value for initial price. As in the two-period economy, initial price is indeterminate under any admissible fiscal policy. Equation (19) places no restrictions on P_0 in equilibrium. Given a value for s_0 , any value for P_0 is an equilibrium because D_0 can adjust to assure that equation (19) holds. No other equilibrium equations impose restrictions. Therefore, any positive value for P_0 is an equilibrium, as in the two-period model.

Now consider equilibrium restrictions on future prices by considering equilibrium restrictions on γ . Let Γ^* be the set of equilibrium sequences for γ under Ricardian fiscal policy. We show that the set of equilibrium sequences for γ under non-Ricardian policy is a proper subset of Γ^* .

By the definition of Ricardian policy, equation (32) imposes no restrictions on γ . Under Ricardian policy, equilibrium sequences for γ are restricted only by the requirement that $\gamma_0 = 0$ and by equation (23). Therefore, Γ^* contains all sequences for γ which begin with zero and satisfy equation (23).

We define two particular non-Ricardian policies, one non-stochastic, Ω_N , and one stochastic, Ω_S , as in the two-period economy. Under either policy, $\gamma_0 = 0$, and η_0 is given by equation (30). Under Ω_N , we assume that $\eta_t = 0$ for $t \geq 1$. With this policy, the only sequence γ satisfying equation (32) is $\gamma_t = 0$ for $t \geq 0$. Under Ω_S , we assume that η_t takes on a stochastic value around a mean of zero. Under this policy, the only sequence for γ which satisfies equation (32) is the sequence given by $\gamma_0 = 0$ and $\gamma_t = \frac{1+i^*}{\alpha+i^*}\eta_t$ for $t \geq 1$.⁶ By

⁶ In the event that debt were to return to zero at some future date, then the FTPL would place no restrictions on the price level at that date, and η_t must take on its expected value of zero. Price could jump without affecting γ_t since $\gamma_t \equiv 0$ when $d_{t-1} = 0$. In the following period, debt would adjust along the saddlepath,

The Fiscal Theory of the Price Level and Initial Government Debt

construction, equilibrium sequences for γ under both non-Ricardian policies satisfy equation (23). Therefore, under either non-Ricardian policy, the set of equilibrium sequences for γ is a proper subset of Γ^* .

The saddlepath equilibrium, equation (32), provides an intuitive classification of fiscal policy as Ricardian or non-Ricardian based on identification of the exogenous variable and the endogenously jumping variable. Under Ricardian policy, γ_t is free around a mean of zero and therefore moves first. The surplus innovation, η_t , is the variable which jumps in response to γ_t in order to keep the system on the saddlepath. Since γ_t is free, sunspot equilibria are possible. Under non-Ricardian policy, η_t is free around a mean consistent with equation (23), and thereby moves first, with γ_t being the endogenously jumping variable. Since γ_t must take on a unique value for any value of η_t , sunspot equilibria are not possible.

4 Conclusion

This paper has shown that requiring that a government begin with zero initial debt does not invalidate the FTPL as long as the government issues a restricted set of securities, containing standard nominal debt contracts and money. This holds in both a two-period economy and in an infinite-horizon economy. In particular, it is possible to define a non-Ricardian fiscal policy, consistent with the equilibrium conditions derived from intertemporal optimization by households, under which the set of equilibrium price sequences is a proper subset of those under Ricardian fiscal policy. Although the FTPL cannot determine price in a period in which outstanding government debt is zero, fiscal price level determination is consistent

placing the economy back within the FTPL framework.

The Fiscal Theory of the Price Level and Initial Government Debt

with equilibrium in subsequent periods after the government has issued nominal debt.

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