Paper Money (AER)

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1 Monetary and Fiscal Policy are Related

1.1 Large Expansion of Fed’s Balance Sheet has not created inflation

- FTPL

\[ \frac{M_{-1} + (1 + i_{-1}) B_{-1}}{P_0} = E_0 \sum_{t=0}^{\infty} R_{0,t} s_t \]

- Open market swap of money for bonds does not affect the price level

- Large expansion of Fed’s balance sheet whereby it has purchased bonds for money has not created inflation
1.2 Raising Interest in Response to Inflation could create even more inflation

- Let’s say $P_0$ rises because the present-value of surpluses falls

- Fed responds by raising nominal interest rate

  - Fischer relation

    $$(1 + i_t) = (1 + r_t) E_t (1 + \pi_{t+1})$$

  - If real interest rate not affected by monetary policy then inflation and expected inflation both rise
• Money demand

\[ \frac{M}{P} = \frac{i}{1 + i}C \]

– To raise the nominal interest rate with \( P \) higher requires an even larger increase in \( M \) raising \( \frac{M}{P} \)

– Higher nominal interest rate raises inflation and inflationary expectations increasing \( P \) next period

– Monetary authority responds by raising \( i \) again, requiring larger increase in \( M \) to get \( \frac{M}{P} \) higher, setting off hyperinflation
2 Fiscal Backing for Fed Balance Sheet

- Fed could want to tighten so much that it wants to sell more assets than it has – requires fiscal backing to provide the assets

- Fed could print money to generate the revenue to recapitalize its balance sheet as it sells assets, but this is not contractionary

- Suppose Fed sells assets raising nominal interest rate, and creating capital loss on long-term assets – will Treasury recapitalize?

- During financial crisis, Fed took on considerable risk – had downside realized, what would Treasury do?
3 Benefits of Issuing Nominal Government Bonds

- Expected and actual inflation
  - Ex ante, nominal interest rate must compensate for expected inflation
  - Ex poste, actual inflation can deviate from expected
  - This deviation can offset fiscal shocks without need to adjust tax revenues or government spending

- Severe negative shock to expected future surpluses
  - If debt is nominal, price increase reduces its real value
– If debt is real or denominated in a currency of another country, lose this mechanism and must default

• Default
  – Inflation greater than expected is a type of default because the government pays less real value than originally expected
  – However, with inflation no negotiation over allocating loss
  – With actual default, deadweight losses are much greater due to uncertainty in allocation of loss
  – Evidence: interest rates in high-debt Euro countries are much higher than interest rates in high-debt countries with debt denominated in their own currencies due to differences in risk premia
4 Samuelson’s Consumption-Loan Model with Storage

- Equations for two-period overlapping-generations model with storage \((S)\) and government bonds \((B)\)
  
  - Utility for an agent born at time \(t\) with no discounting
    \[
    U(C_{1t}, C_{2t+1}) = \ln(C_{1t}) + \ln(C_{2t})
    \]
  
  - Budget constraints with one unit of endowment when young
    \[
    C_{1t} + S_t + \frac{B_t}{P_t} = 1
    \]
    \[
    C_{2t+1} = \frac{R_t B_t}{P_{t+1}} + \theta S_t
    \]
where $\theta < 1$ is rate of return on storage

- Inequality constraints

$$S_t \geq 0 \quad B_t \geq 0$$

- Goods market equilibrium

* current consumption by old

$$C_{2t} = \frac{R_{t-1}B_{t-1}}{P_t} + \theta S_{t-1}$$

* current consumption by young

$$C_{1t} = 1 - S_t - \frac{B_t}{P_t}$$
* economy’s resource constraint

\[ C_{2t} + C_{1t} + S_t = \theta S_{t-1} + 1 \]

* equilibrium

\[ C_{2t} + C_{1t} + S_t = \frac{R_{t-1}B_{t-1}}{P_t} + \theta S_{t-1} + 1 - \frac{B_t}{P_t} = \theta S_{t-1} + 1 \]

requiring

\[ R_{t-1}B_{t-1} = B_t \]

– Note similarity to government budget constraint where government must issue new bonds to pay interest on old

\[ R_tB_t = B_{t+1} \]

and where government can choose \( R_t \)
• First order conditions assuming perfect foresight

  – with respect to $C_1$

  \[ \frac{1}{C_{1t}} = \lambda_t \]

  – with respect to $C_2$

  \[ \frac{1}{C_{2t+1}} = \mu_{t+1} \]

  – with respect to $B$

  \[ \frac{\lambda_t}{P_t} = \frac{R_t\mu_{t+1}}{P_{t+1}} \]

  – with respect to $S$

  \[ \lambda_t = \theta \mu_{t+1} \]
• Euler equations

  – Bond Euler equation

\[
\frac{C_{2t+1}}{C_{1t}} = \frac{R_t P_t}{P_{t+1}}
\]

  – Storage Euler equation

\[
\frac{C_{2t+1}}{C_{1t}} = \theta \quad C_{1t} = \frac{C_{2t+1}}{\theta}
\]
Equations which must be satisfied in equilibrium

- Define the rate of return to savings as

\[ \rho = \frac{R_t P_t}{P_{t+1}} = \theta \]

* implying from Euler equations that

\[ C_{1t} = \frac{C_{2t+1}}{\rho} \]

- Define wealth as

\[ W_t = S_t + \frac{B_t}{P_t} \]
– Using the budget constraint for period-two consumption

\[ C_{2t} = \frac{R_{t-1}B_{t-1}}{P_t} + \theta S_{t-1} = \rho W_{t-1} \]

– Substitute into the budget constraint for period-one consumption,

\[ C_{1t} + W_t = 1 = C_{1t} + \frac{C_{2t+1}}{\rho} = 2C_{1t} \]

implying that saving must be half the endowment with consumption the other half

\[ C_{1t} = 0.5 \]
Equilibrium with no storage and valued nominal government debt ($P_t < \infty$)

- No storage implies

\[ C_{1t} = 0.5 \]

and since all savings is used to buy debt from the old

\[ C_{2t} = 0.5 \]

- Rate of return on savings

\[ \rho = 1 = \frac{R_t P_t}{P_{t+1}} \]

implying that the price level grows at rate $R$ for all $t$

\[ \frac{P_{t+1}}{P_t} = R \]
with real value of both newly issued debt and and maturing debt constant at 0.5

– Initial debt

\[ \frac{RB_0}{P_1} = 0.5, \]

implying

\[ P_1 = 2RB_0 \]
• Equilibrium with positive storage
  
  – Requires rate of return on storage and bonds to be equal
    \[ \rho \equiv \theta = \frac{R_t P_t}{P_{t+1}} \]
    implying that the price level grows at rate
    \[ \frac{P_{t+1}}{P_t} = \frac{R}{\theta} > R \]
  
  – Nominal debt grows at rate \( R \), implying that real debt shrinks over time
    \[ \frac{B_t}{P_t} = \theta \frac{B_{t-1}}{P_{t-1}} \]
– Initial values
  * Any value for

\[
\frac{B_1}{P_1} < 0.5
\]

yields equilibrium storage of

\[
S_1 = 0.5 - \frac{B_1}{P_1}
\]

– Over time, storage increases toward 0.5 as the value of real bonds fall toward zero

– Every price level satisfying

\[
P_1 > 2RB_0,
\]

including infinity is an equilibrium
* Price level is indeterminate

- Consumption
  * Initial consumption
    \[ C_{1t} = 0.5 \]
  * Resource constraint
    \[ C_{2t} + C_{1t} + S_t = \theta S_{t-1} + 1 \]
    \[ C_{2t} = 0.5 - S_t + \theta S_{t-1} \]

* In equilibrium \( S \) is either increasing or constant, implying that
  \[ C_{2t} < 0.5 \]

* Inferior equilibrium to one without storage
• Replace $B$ with $M$ and set $R = 1$ yields Samuelson’s model showing how fiat money can have value
5 Add Tax Backing for Government Debt

- Add lump-sum tax to government budget constraint

\[
\frac{B_t}{P_t} = R \frac{B_{t-1}}{P_t} - \tau
\]

- Consider equilibrium with both storage and government bonds where

\[
R \frac{P_{t-1}}{P_t} = \theta,
\]

implying that the government budget constraint becomes

\[
\frac{B_t}{P_t} = \theta \frac{B_{t-1}}{P_{t-1}} - \tau
\]
* Since $\theta < 1$, stable difference equation which we can solve forward

$$\frac{B_t}{P_t} = \sum_{s=1}^{t-1} -\tau \theta^s + \theta^t \frac{B_0}{P_0}$$

* Right-hand side grows smaller and becomes negative as $t$ increases

  * Eventually reach a point where government is trying to buy debt with $\tau$, but there is no debt to buy

  * Anyone foreseeing this would hold onto some debt to exchange at an extremely favorable price

  * Cannot be an equilibrium

  * Government is always willing to buy debt with $\tau$ implies that debt cannot become valueless
- Rules out equilibrium with storage once government backs debt with taxes

- Equilibrium with no storage and tax backing for government debt
  - Tax affects budget constraint when young
    \[ C_{1t} + \frac{B_t}{P_t} + \tau = 1 \]
  - Budget constraint when old
    \[ C_{2t+1} = \frac{RB_t}{P_{t+1}} \]
    solving for real bonds
    \[ \frac{B_t}{P_t} = \frac{C_{2t+1}P_{t+1}}{RP_t} \]
– From first order conditions

\[ R \frac{P_t}{P_{t+1}} = \rho = \frac{C_{2t+1}}{C_{1t}} \]

– Rewrite young-period budget constraint as

\[ C_{1t} + \frac{C_{2t+1}P_{t+1}}{RP_t} + \tau = 1 = 2C_{1t} + \tau \]

implying that consumption when young is

\[ C_{1t} = \frac{1 - \tau}{2} \]

– Consumption when old from resource constraint

\[ C_{2t+1} = 1 - \frac{1 - \tau}{2} = \frac{1 + \tau}{2} \]

yielding total utility less than max because consumption is not constant
over time
\[
\ln \left( \frac{1 - \tau}{2} \right) + \ln \left( \frac{1 + \tau}{2} \right) = \ln (1 - \tau) + \ln (1 + \tau) + 2 \ln(1/2) < 2 \ln(1/2)
\]

- Gross real interest rate

\[
\rho = R \frac{P_t}{P_{t+1}} = \frac{C_{2t+1}}{C_{1t}} = \frac{1 + \tau}{1 - \tau}
\]

- Government budget constraint

\[
\frac{B_t}{P_t} = \rho_{t-1} \frac{B_{t-1}}{P_{t-1}} - \tau
\]

- Old-period budget constraint

\[
C_{2t+1} = \frac{1 + \tau}{2} = \rho \frac{B_t}{P_t},
\]

then \( \frac{B_t}{P_t} \) is constant in equilibrium
Government budget constraint with constant real bonds is

\[ \frac{B}{P} = \frac{\tau}{\rho - 1} \]

Cannot take limit as \( \tau \to 0 \) without substituting for \( \rho = \frac{1+\tau}{1-\tau} \)

\[ \frac{B}{P} = \frac{\tau}{\rho - 1} = \frac{\tau}{\frac{1+\tau}{1-\tau} - 1} = \frac{\tau (1 - \tau)}{1 + \tau - 1 + \tau} = \frac{(1 - \tau)}{2}, \]

implying that real debt converges to \( 1/2 \), its value at the optimum

Initial price level uses government budget constraint and equilibrium real value of bonds with initial bonds and interest on them predetermined

\[ \frac{B_0}{P_0} = \frac{(1 - \tau)}{2} = \frac{R_0 B_0}{P_0} - \tau \]
* Solution for $P_0$

\[ P_0 = \frac{2R_{-1}B_{-1}}{1 + \tau} \]

* If $R_{-1}B_{-1} = 0$, then describing fiscal policy as constant $\tau$ does not make sense because the old have no bonds to be purchased with the $\tau$

* Cannot determine $P_0$ with no outstanding debt
5.1 Debt as a Fiscal Cushion

- Barro (1979) argues that distortionary taxes should not be raised to pay debt down after an increase due to large stochastic government spending
  
  – Costs of raising distortionary taxes by large amount to get debt down exceed cost of raising them by small amount necessary to service debt

  – Debt and taxes should be a random walk

  – Ignores fiscal limits – how high can distortionary tax rates get?

  – Alternative is to allow price surprises to revalue debt in the presence of spending shocks – but what if price surprises are costly too?
• FTPL equation with two-period debt (discount bonds)

\[
\frac{M_{-1} + B_{1,-1} + B_{2,-2} + \rho_{0,-1}B_{2,-1}}{P_0} = E_0 \sum_{t=0}^{\infty} R_{0,t \delta t}
\]

– \( B_{1,-1} \) is face value of one-period debt, issued in period \(-1\)
– \( B_{2,-2} \) is face value of two-period debt, issued in period \(-2\)
– \( B_{2,-1} \) is face value of two-period debt, issued in period \(-1\)
– \( \rho_{0,-1} \) is the period 0 price of two-period debt, issued in period \(-1\)

\[
\rho_{0,-1} = \frac{1}{1 + i_0} = E_0 \frac{1}{(1 + r_0) \frac{P_1}{P_0}}
\]

• If PV surpluses rise and have two period debt,
– adjustment can occur both with an increase in current price and with

– a fall in $\rho_{0,-1} = E_0 \frac{1}{(1+r_0) \frac{P_1}{P_0}}$ due to expected future inflation

– can spread the price increase over time