Household Consumption and Saving - National Saving
1. Household consumption and saving

2. Theory of aggregate consumption and saving

3. Extensions

4. Taxes, consumption, and saving

5. Desired national saving
1 Household consumption and saving

We will start with a simple two-period model in which the consumer cares only about consumption today $c$ and consumption in the future $c^f$. To simplify, we are not explicitly letting the household care about leisure, but we can add this back after we understand the consumption and saving decision. As before, to build the model we will make assumptions about 1) consumer preferences, 2) consumer’s budget constraint, and 3) consumer behavior. We will use these assumptions to develop a theory of household saving and consumption.

- Consumer preferences - consumption today $(c)$ and consumption in the future $(c^f)$ both yield utility

\[ U = u(c, c^f) \] with \[ \frac{\partial u(c, c^f)}{\partial c} > 0; \quad \frac{\partial u(c, c^f)}{\partial c^f} > 0 \]
• Indifference curves - combinations of $c$ and $c^f$ for which the agent receives the same level of utility

  – slope downward from left to right - to be willing to give up current consumption must receive more future consumption

  – higher indifference curves represent higher levels of utility - holding one good constant and increasing the other raises utility

  – bowed toward the origin due to the law of diminishing marginal utility

\[
\frac{\partial^2 u (c, c^f)}{\partial c^2} < 0 \quad \frac{\partial^2 u (c, c^f)}{\partial (c^f)^2} < 0
\]
Present-value budget (resource) constraint - requires assumption that the agent can borrow or lend as much as he wants at the real interest rate $r$ subject to this constraint. Requires the present-value of lifetime consumption (PVLC) equal present-value lifetime resources (PVLR).

$$c + \frac{c^f}{1 + r} = y + \frac{y^f}{1 + r} + a$$

where $y \ (y^f)$ represents the household’s current (future) income and $a$ represents initial wealth (assets).

- Graph $c^f$ as a function of $c$.

$$c^f = (1 + r)(y + a - c) + y^f$$

* slope is $-(1 + r)$

* intercept is $(1 + r)(y + a) + y^f$
Utility maximization subject to the budget constraint

- Graph

- Math

* Let

\[ u(c, c^f) = \ln c + \beta \ln c^f. \]

Substitute for \( c^f \) from the budget constraint to yield:

\[ \ln (c) + \beta \ln \left( (1 + r)(y + a - c) + y^f \right) \]

\( \beta \) has an economic interpretation as the agent’s degree of impatience.

* To maximize, take the derivative with respect to \( c \) and set it to 0.

\[ \frac{1}{c} - \beta \left( (1 + r) \frac{1}{c^f} \right) = 0 \]
or rearranging

\[ c^f = \beta (1 + r) c. \]

- Consumption smoothing and time path of consumption

- Solve this equation simultaneously with the budget constraint to yield current and future consumption.

\[ c^* = \frac{1}{1 + \beta} \left( y + \frac{y^f}{1 + r} + a \right) = \frac{1}{1 + \beta} PVLR \]

\[ c^{f*} = \frac{\beta (1 + r)}{1 + \beta} \left( y + \frac{y^f}{1 + r} + a \right) = \frac{\beta (1 + r)}{1 + \beta} PVLR \]

note that if \( \beta (1 + r) = 1 \), \( c^* = c^{f*} \).
Saving = income minus consumption

\[ s^* = y - c^* = y - \frac{1}{1 + \beta} \left( y + \frac{y^f}{1 + r} + a \right) \]

\[ = \frac{1}{1 + \beta} \left( \beta y - \frac{y^f}{1 + r} - a \right) \]

With income constant, an increase in current consumption is equivalent to a reduction in current saving.
2 Theory of aggregate consumption and saving

Add up all individual household consumption and saving functions and get desired aggregate consumption and saving.

- How does an increase in the real interest rate affect desired aggregate current consumption?
  - Graphically - budget line becomes steeper - give up less c to get one more $c^f$ - price of $c^f$ in terms of c has fallen
  - Substitution effect - holding utility constant, a reduction in the price of $c^f$ (increase in the price of c) will cause agents to substitute out of c and into $c^f$. As c falls, saving rises.
Income effect - an increase in the real interest rate increases resources for a saver and decreases them for a borrower. Therefore the income effect for a saver of an real interest rate increase tends to increase consumption and reduce saving and have the opposite effect for a borrower.

Generally believe the substitution effect dominates, even when household is a saver.

* mathematically - functional form is important. We have a functional form for which the substitution effect dominates (for \( y^f > 0 \)) regardless of whether the agent is a borrower or a saver.
- Increase in **current income**?

  - * graphically - both intercepts on budget line increase, shifting it rightward parallel to original line. Current and future consumption both increase. Therefore, saving must also increase

    - mathematically

  - Increase in expected **future income**?

    - graphically

      - both intercepts increase, shifting budget constraint outward

      - current and future consumption both increase

      - since current income did not change, saving must fall
* mathematically

- Increase in **wealth**? - similar to increase in current income

- To summarize, **desired saving** is an **increasing** function of the **real interest rate** and **current income** and a **decreasing** function of **wealth** and expected **future income**.
3 Extensions

- Permanent income theory - many horizon model - $\beta(1 + r) = 1$ so agents want a flat consumption profile
  - consumption depends on present value of expected future lifetime earnings
  - permanent increase in income affects consumption one for one
  - temporary increase in income has small effect on consumption

- Life-cycle model - three periods correspond to youth when agents begin work, middle-age when agents reach peak income-earning years, and old age when agents retire.
– How does age affect saving behavior?

– How does a bequest motive affect saving behavior?

– How would borrowing constraints in youth affect aggregate saving?

– Compare saving behavior across countries with well-functioning financial markets and those without.

• Why do stock market booms and consumption booms often go together?

• Why is the US saving rate so low compared to that in other countries?
4 Taxes and Consumption and Saving

4.1 Lump-Sum Taxes

- Amend the two-period model above to have a government.
  - The household present-value budget constraint should contain current and future disposable income instead of income.

\[
\begin{align*}
    c + \frac{c^f}{1+r} &= y - T + \frac{y^f - T^f}{1+r} + a \\
\end{align*}
\]

- Add a government present-value budget constraint.

\[
\begin{align*}
    G + \frac{G^f}{1+r} &= T + \frac{T^f}{1 + r}.
\end{align*}
\]
Household consumption will be given by

\[ c = \frac{1}{1+\beta} \left( y - T + \frac{y^f - T^f}{1+r} + a \right) = \frac{1}{1+\beta} PVLR. \]

Household savings is

\[ s = y - T - c = \frac{1}{1+\beta} \left( \beta (y - T) - \frac{y^f - T^f}{1+r} - a \right) \]

How does a tax cut affect individual consumption and saving?

4.2 Distortionary taxes

- tax nominal interest income
expected real after-tax interest rate

\[ r_{a-t} = i(1 - t) - \pi^e \]

where \( \pi^e \) is expected inflation

- budget constraint (slope of \( -(1 + r_{a-t}) \)) becomes flatter - get less future consumption for one unit of current consumption because must give up some interest earnings to government as taxes

- substitute out of future consumption into current consumption

- saving falls as consumption rises

- when returns to saving are taxed, saving falls

- other effects depending on how the government budget constraint adjusts with the extra tax revenue
5 Desired National Saving

- Desired saving is the sum of desired private saving plus desired public saving.

\[ S^d = (Y - T - C^d) + (T - G) = Y - C^d - G \]

- The effect of a tax increase on national savings

  - What adjusts in government budget constraint?

  - Effect is determined by the effect on \( C^d \) and \( G \) and possibly \( Y \).

- The effect of an increase in government spending on national saving
- direct effect reducing national saving
- indirect effect depending on how the government budget constraint is met and its effect on consumption