Asymmetries in Solow Residuals

Estimation with Stochastic Frontier Models

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Interdisciplinary Paper

- Adapt micro models of stochastic frontiers
- Address macro issues on aggregate productivity

Macro Problem

- Solow Residual is a measure of TFP for aggregate economy
- Letting factors of production be capital (K) and labor (N), output is given by a Cobb-Douglas production function

$$Y = AK^{\alpha}N^{1-\alpha} \qquad 0 < \alpha < 1$$

- Total factor productivity and Solow residual are given by

$$A = \frac{Y}{K^{\alpha}N^{1-\alpha}}$$

Two possibilities for fundamental determinantes of the Solow Residual

- Technology
  - Positive and negative shocks around a deterministic trend
  - If no technological regress, negative shocks are bounded by the trend
  - No reason to expect negative asymmetries
  - Recession when small negative technology shock pulls K and N down reducing output
- Capacity utilization
  - Costs of adjusting factors of production implies that efficient capacity utilization on average less than 100%
  - Positive demand shock raises capacity utilization, subject to upper bound
  - Negative demand shock decreases capacity utilization, with no lower bound
  - Possible asymmetry when shocks are large

Controversy 1: What are Solow residuals?

- Do Solow residuals measure
  - Exogenous shocks to technology
  - Or endogenous changes in capacity utilization?
- Negative asymmetries could imply a role for capacity utilization
- Absence of negative asymmetries could imply
  - A reflects small symmetric technology shocks
  - A reflects small changes in capacity utilization

Controversy 2: Are recessions fundamentally different from booms?

- Are recessions caused by
  - A negative draw from a normal distribution
  - Or a negative asymmetric error with no positive counterpart?
- Linear RBC models assume symmetry implying that booms and recessions are caused by same fundamentals
- Evidence of asymmetries implies causes can be different
  - What creates the asymmetric error?
  - Possibly asymmetric changes in endogenous capacity utilization
  - Remaining question: What is the exogenous shock?

Stochastic Frontier Models

- Stochastic frontier models decompose error term into a symmetric and an asymmetric component
- Eleven OECD countries with quarterly data beginning at least in the early 1980's
  - Compute time series of Solow residuals from data on capital, employment, and output for each country
- Time series data on output and other macro aggregates
  - Is not stationary
  - Has strong persistence
  - Is not measured in comparable units across countries
- Deal with non-stationarity by detrending with HP filter
- Deal with persistence by modeling detrended Solow residual as an AR process
- MLE estimation of AR model with composite error for the HP-filtered Solow residuals for each country

Empirical Results

- Eight of eleven countries have significant evidence of asymmetries (one-sided error)
  - Evidence that asymmetric response of capacity utilization to an exogenous shock is important in some countries
- What is the exogenous shock?
  - Countries without asymmetries are significant producers of oil
    - Hamilton argues that large increases in oil prices are responsible for all but one post-WWII US recession
    - Oil price increase deteriorates the terms of trade, reducing demand and capacity utilization
  - Reestimate model conditioning on oil prices, allowing for non-linearities
    - Reduces but does not eliminate evidence for asymmetries

Implications of Empirical Results

- For countries which produce little oil
  - Solow residuals are asymmetric, possibly reflecting asymmetric changes in capacity utilization
  - Fundamentals creating recessions are different from fundamentals creating booms
    - Large oil prices receive support as one fundamental exogenous shock creating recessions
- For countries which produce a lot of oil
  - Solow residuals do not reflect asymmetric changes in capacity utilization
  - Recessions are not caused by different fundamentals from booms
    - Oil price increase is a much smaller change in the term of trade
    - Does not create asymmetric response of capacity utilization

Model Specification

- Log of the Solow residual has three components
  - s_t is a deterministic trend component, capturing trend and seasonal components
  - μ_t is the conditional mean of the detrended series
  - w_t is a mean zero error term

$$\log(A_t) = s_t + \mu_t + w_t, \quad (1)$$

- Error term has two components
  - $u_t \geq 0$ is distributed exponentially with parameter μ_u
  - $v_t \sim N(E(u), \sigma_v)$ is a normal stochastic error component

$$w_t = v_t - u_t$$

Model Specification (cont)

- Use HP filter to eliminate s_t

$$y_t = \log(A_t) - \hat{s}_t \tag{2}$$

- Mean dynamics μ_t of y_t are modeled as AR(p)

$$y_t = \alpha + \sum_{i=1}^{p} \beta_i y_{t-i} + w_t \tag{3}$$

Model Evaluation

- MLE estimation of detrended Solow residuals allowing AR dynamics and SF model of error
- Compare MLE estimation of model with $u_t = 0$ with SF model and compute likelihood ratios
- The relative importance of u_t expressed by estimated variance ratio (VR) of the two error components

$$VR = \frac{\mu_u^2}{\sigma_v^2} \tag{4}$$

Table 1A
Countries with Large Variance Ratios

	AUS	FR	GER	ITA	JAP	KOR	SWI
LL s	-204.23	-84.13	-174.28	-100.11	-121.07	-120.86	-148.95
LL as	-201.19	-83.35	-170.79	-97.93	-116.34	-114.14	-148.28
LR	6.086	1.546	6.975	4.353	9.470	13.439	10.959
p	0.0068	0.1069	0.0041	0.0185	0.001	0.0001	0.0005
VR	0.5379	0.3887	0.6039	0.5915	0.7182	0.6566	0.5883
p(O)	0.0067	0.4463	0.0368	0.1003	0.0552	0.0011	0.0004
VR(O)	0.526	0.0715	0.4902	0.4541	0.5738	0.5799	0.7001

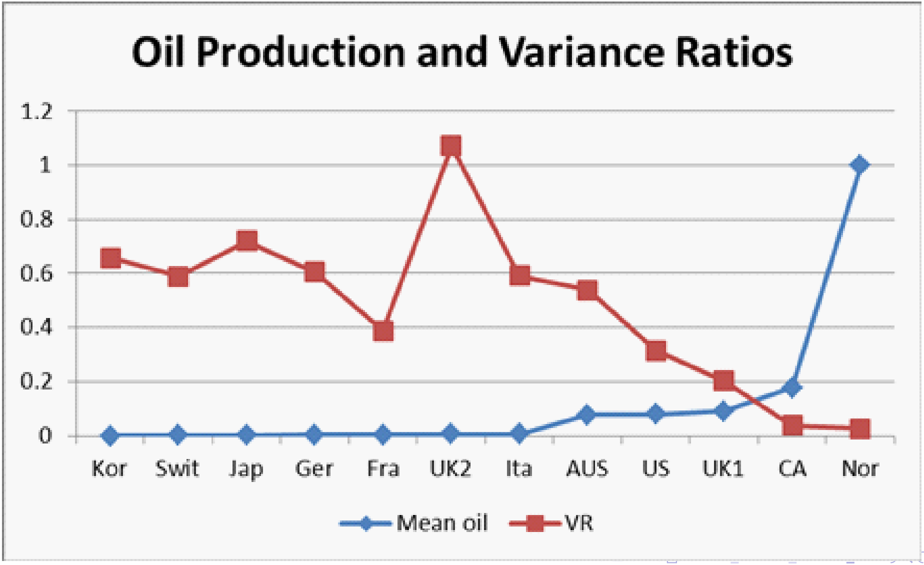
Table 1B
Countries with Small Variance Ratios and Split UK (1994)

	CA	NOR	US	UK	UK1	UK2
LL s	-182.862	-148.953	-198.504	-147.71	-96.024	-29.899
LL as	-182.957	-148.284	-197.452	-158.88	-97.970	-26.102
LR	-0.19	1.337	2.104	-22.34	-3.892	7.594
p	0.5	0.124	.074	0.5	0.5	0.003
VR	0.0386	.0264	.3148	0.2925	0.203	1.072
p(O)	0.3332	–	0.1096	0.5	0.5	0.3427
VR(O)	0.1326	–	0.2945	0.664	0.1347	0.1902

What Do Countries without Asymmetries Share?

- Oil production
- Three groups of countries
  - High oil production per worker
    - Canada, Norway
  - Intermediate oil production per worker
    - UK pre-1994, Australia, US
  - Negligible oil production per worker
    - All others

Oil Production and Variance Ratios



Re-estimate: Non-Linear Conditioning on Real Oil Price

- Break UK sample at 1994 to allow different response to oil prices when oil production is high versus low
- All countries (ex Norway) had significantly negative coefficients
  - Linear oil price term (France and Korea)
  - or Lag in oil price $<$ -1.5 (all remaining countries)
  - Coefficient on non-linear term for UK2 is four times as large as for UK1
- P-Values for the null of no asymmetries generally rise (exceptions Canada and Australia)
  - Asymmetries eliminated at 10% for US, UK2, Italy, and France
  - Asymmetries remain for Australia, Germany, Japan, Korea, and Switzerland
- Variance ratios fall for all countries except Switzerland and Canada
  - Canada is high oil and no asymmetries in either specification
  - Switzerland is a puzzle

Conclusions

- Solow residuals for countries with negligible and moderate oil production per worker have negative asymmetries
- For these countries, recessions have different fundamentals than booms, missed by standard linear macro models
- And Solow residuals could be reflecting asymmetric movements in capacity utilization in response to an exogenous shock
- Large negative oil price shocks are responsible for all asymmetries in some countries and for some asymmetries in others, excepting Switzerland
  - Large increase in oil price deteriorates terms of trade reducing demand and thereby capacity utilization
  - Symmetric large increase in oil price has smaller effect because capacity utilization is bounded upwards
  - For countries with larger oil production, an oil price increase has a smaller effect on the terms of trade, allowing symmetry
  - Economies react differently to oil prices due to structural differences