Investment necessary for growth is risky and often requires external financing. We present a model in which capital market imperfections separate countries into a safe credit club of industrial countries, with low interest rates and steady credit access, and a risky club of emerging markets, with high interest rates and volatile access. In an emerging market, a large negative productivity shock interacts with credit market imperfections to trigger a severe contraction in external lending. Domestic agents react with widespread default. We calibrate to South Korean parameters and argue that the 1998 financial crisis could have been the downside of risky investment financed in imperfect capital markets.
1 Introduction

Countries at different stages of development experience substantial differences in credit market characteristics. Underdeveloped countries have no access to credit; emerging markets have access, but loans are relatively risky, with interest rates reflecting that risk, and access is volatile; developed countries have reliable access with low interest rates. The Asian financial crises, characterized by high levels of non-performing loans and bankruptcies (Corsetti, Pesenti and Roubini, 1999) and high costs of recapitalizing the financial sector (Burnside, Eichenbaum and Rebelo, 2001), drew attention to the volatile nature of emerging market credit and the financial crises which can occur in these economies. A large quantity of literature has been devoted to understanding the cause of these crises with the objective of developing policy to prevent future crises.

The central hypothesis in this paper is that financial market imperfections, likely to characterize emerging markets, can magnify the effect of ordinary productivity shocks, creating sudden stops of capital flows and widespread default. We take seriously the idea that investment necessary for growth is risky. Business cycle research views the downside of that risk as a recession. With financial market imperfections, the most severe of these recessions are accompanied by financial crises. In contrast to much of the literature on international financial crises, we are not trying to explain all sudden stops and crises with one model. We are focused on those originating in the private sector as a consequence of productivity shocks, not those due to sovereign (mis)management of debt. Hence we are not explaining sovereign default. We seek to explain private-sector financial difficulties like those which occurred in the East Asian crisis in 1997-98.

The literature contains two dominant models of financial market imperfections, collateral constraints and costly state verification. The literature on financial crises in emerging markets has focused on collateral constraints. Mendoza and Smith (2006) study the quantitative effects of productivity shocks in models with collateral constraints, whereby
a negative productivity shock reduces the value of capital used as collateral. This forces
sale of the capital, creating current account reversals and Fisherian asset-price deflation when there are asset-trading costs. Arellano and Mendoza (2003) survey several
collateral-constraint models to assess the quantitative effects of suddenly binding collateral constraints, and Mendoza (2010) develops a full-scale DSGE model in which financial crises are nested within business cycles as rare events.¹ Collateral-constraints, combined with asset-trading frictions, can explain current-account reversals and asset-price deflation. They cannot explain default or why debt-finance was chosen over equity finance. These models are also silent on the issue of maturity mismatch, whereby the inability of firms to roll over short-term debt defines the crisis.

The second dominant model of financial market imperfections is Townsend’s (1979) model of costly state verification. It relies on asymmetric information between the firm and outsiders and endogenously generates debt finance as a less costly alternative to equity finance. Additionally, it implies debt ceilings, based on the expected value of the output of the project, and endogenous default for agents with productivity below a critical value. In equilibrium, agents with high enough productivity always repay. Bernanke and Gertler (1989) use this model to show that an exogenous shock to the entrepreneur’s inside assets, perhaps caused by a negative productivity shock, increases the cost of credit, creating a financial accelerator. This literature has focused on the cost, not the quantity, of credit in the propagation of business cycles.² The model has loans matched in maturity to the investment project and is therefore silent on the role of maturity mis-match in financial distress.

Maturity mis-match has been addressed primarily in the literature in which financial crises are one possible outcome in a model of multiple equilibria. Cole and Kehoe

¹Other models, which use shocks to collateral constraints, include Radalet and Sachs (1999), Caballero and Krishnamurthy (2001), Aghion, Bacchetta and Banerjee (2001), Chang and Velasco (2002).

²Other prominent contributors to this literature include: Bernanke, Gertler, and Gilchrist (1996), and Carlstrom and Fuerst (1997).
(1996, 2000) show how a sudden stop in lending can occur when debt is short-term, and borrowers need to roll over debt to repay current loans. A sudden stop occurs when agents exogenously coordinate on the expectation that no one will lend in the next period, implying that borrowers will not be able to repay, yielding no loans in equilibrium. Calvo (1998) discusses the implications of exogenous sudden stops of capital to emerging markets, and Chang and Velasco (2001) demonstrate how bank runs can cause a financial crisis. What these papers and others like them have in common is that financial market imperfections are responsible for multiple equilibria, and a bad equilibrium is a crisis equilibrium. However, crises in these models are not generated by productivity shocks or their expectations.

We develop a model with costly state verification and modify it to incorporate maturity mismatch in investment. We show that this set of financial market imperfections is an alternative to collateral constraints in implying that a negative productivity shock can cause a financial crisis. Additionally, the alternative model is able to explain some aspects of a financial crisis, which collateral constraint models cannot, including why agents optimally chose debt finance over equity finance, and why agents sometimes default. In the model, agents take on initial debt to finance the investment project and to acquire a hedge asset as a precaution against a sudden stop in lending prior to maturity of the investment project. This debt is risky when it is large enough that bad news about future output could cause lenders to reduce the ceiling on new debt below contractual debt repayments. Bad news about future output triggers a sudden stop of capital by reducing the debt ceiling, where the debt ceiling depends on expected future productivity. A large enough fall in expected future productivity reduces the debt ceiling enough to create widespread default and rescheduling – a financial crisis. The combination of costly state verification and maturity mismatch yields a critically important role for the debt ceiling in addition

\footnote{We offer a justification in the appendix for the maturity mismatch which can also explain why short-term debt is chosen over long-term debt to finance long-term projects.}
to the standard interest rate effect in the financial accelerator. Only severely negative productivity shocks create a financial crisis, implying the correlation between recession severity and financial crisis seen in the data (Calvo and Reinhart, 2000; Mendoza, 2002).

The financial market imperfections we consider are present in advanced economies, but are less severe, as reflected by different parameter values. We show that these different parameter values divide countries into credit clubs. Countries without access to technology, with low wealth, or with weak domestic capital markets have no access to international credit markets. Technology and a minimum level of initial resources, together with reasonably strong domestic credit markets, give a country access, but that access is volatile and interest rates carry default risk premia. A country acquires stable access to international credit with risk-free loans once resources and the strength of domestic credit markets cross thresholds.4

To address quantitative properties, we create a model with overlapping generations of agents with access to risky projects and agents with safe projects. We show that a model with productivity shocks, calibrated to match the standard deviation of HP-filtered log GDP for South Korea, and the probability of a financial crisis, calibrated to match the probability for non-Latin-American economies experiencing lending booms (Gourinchas, Valdes, and Landerretche, 2001), can generate large falls in the quantity of loans, large increases in interest rates, widespread default, and large output declines in response to negative productivity shocks. Financial crises require large negative productivity shocks, generating the empirical association between severe recessions and financial crises.5

The paper is organized as follows. The next section presents the model’s assumptions, including a characterization of the agent’s optimization problem. Section 3 determines equilibrium debt ceilings and interest rates for given levels of debt. Section 4 characterizes

4Other models in which financial crises are more likely in emerging markets include Aghion, Bacchetta and Banerjee (2004) and Martin and Rey (2006).

5This contrasts with the result in Mendoza and Smith (2006) in which negative productivity shocks have a single magnitude and therefore all create the same size recession, but create a financial crisis only if the collateral constraint is binding.
the general equilibrium for the model. Section 5 places the agents with risky investment projects into an aggregate economy and calibrates the model to match parameters for South Korea. Section 6 contains conclusions.

2 Assumptions: Small Emerging-Market Economy

2.1 Economic Environment

The domestic economy is small and open. The world interest rate \( r \) is fixed, and foreign creditors are risk-neutral. The world price of the single good is fixed and normalized at unity. The single good rules out changes in a relative price or the real exchange rate as a cause for crises in the model.\(^6\) There are three periods in the model, labeled 0, 1, and 2.

In period 0 agents choose between the risky two-period investment project and the safe international bond with the objective of maximizing the expected utility of consumption in the next two periods. There are two types of agents, high-productivity and low-productivity, and agents do not learn their identity until period 2 when the project matures. If an agent chooses risky investment, then he must obtain external financing. We assume that agents cannot commit to repay and that information on agent type, which the agent learns in period 2 as the project matures, is private and accessible to the external financier only after payment of a state-verification fee (Townsend, 1979).

In contrast to the standard costly-state-verification model, we assume that payment of the fee entitles the external financier to \( \eta < 1 \) of the value of resources from investment. The assumption that \( \eta < 1 \) reflects our assumption that protection for the external financier is weaker in emerging markets. Additionally, we assume that output of the low-productivity agent is so small that external financiers receive no net payout from low-

\(^6\)These changes undoubtedly occur and are emphasized in the collateral constraint models. We rule them out here to focus on aggregate productivity shocks.
productivity agents.\footnote{This is a simplifying assumption, and as long as the net payout is small, it could be of either sign, and the results should go through.} We also assume that multi-period debt contracts are not available and justify this using moral hazard in the appendix.

There is no role for government spending in the model, so we assume that spending is zero. Additionally, since the model is about private financial crises, not sovereign crises, we assume that the government cannot borrow internationally, implying that there is no sovereign debt. Finally, since productivity across agents is stochastic and the model is not about risk-sharing across agents, we assume that the government redistributes income using the tax system in period 2 to achieve perfect risk-sharing across agents.\footnote{This requires that the government can learn the identity of agents without paying the state-verification fee. The assumption of perfect risk-sharing across agents with a given type project greatly simplifies the analysis.} Aggregate period-2 taxes ($\tau_2$) are allowed to be non-zero to facilitate placing the model in a more general context for calibration.

\section*{2.2 Agent’s Problem}

In period 0, a unit mass of agents has access to an endowment $Y$ and to a technology for a risky investment project with expected returns substantially greater than the world interest rate. The risky investment project requires each agent’s full labor together with a particular fixed size of capital ($K$), exceeding the agent’s endowment ($Y$) such that $K > Y$. The investment project does not mature until period 2. Each agent chooses whether or not to invest in the risky project to maximize expected utility given by

$$\int_{\rho_l}^{1} [\ln c_1 + \beta(\rho \ln c_{2h} + (1 - \rho) \ln c_{2l})] f(\rho) d\rho. \quad (1)$$

In equation (1), $\beta$ is the discount factor, assumed to equal the inverse of the world gross interest rate ($\beta = (1 + r)^{-1}$), $\rho$ is the number of $h$-agents, $f(\rho)$ is the density function for the number of high-productivity agents with a lower support of $\rho_l$, $c$ denotes consumption.
subscripts 1 and 2 denote periods, and subscripts h and l denote the high-productivity and low-productivity agent respectively.

In period 0, agents use their endowment \((Y)\), together with foreign financing \((D_0)\), to buy capital for the investment project of fixed size \((K)\) and to acquire a safe international bond \((B_0)\). Agents cannot issue safe international bonds, implying a non-negativity constraint on \(B_0\). Period 0 is a planning period in which they do not consume. The budget constraint is expressed as

\[
Y + D_0 = K + B_0 \quad B_0 \geq 0. 
\] (2)

In period 1, each agent receives an identical value of output plus depreciated capital, proportional to the aggregate value in period 2. We assume that the factor of proportionality is small, subject to \(\lambda < \frac{1}{1+r}\), consistent with the assumption that the investment project does not fully mature until period 2. Period 1 output yields information on the realization of \(\rho\), but agents do not learn their own identity since all receive identical output. The project yields output plus depreciated capital equal to \([\rho H + (1 - \rho) L] \lambda K\). The agent must reinvest \(\lambda K\) to assure a capital stock of size \(K\) in period 2, implying that net output is

\[
Y_1 = [\rho H + (1 - \rho) L - 1] \lambda K, 
\]  

We assume that \(H\) is sufficiently larger than \(L\), such that \(\rho H > L\) for all possible values for \(\rho\). Letting \(D_1\) be external financing in period 1 and \(D_0'\) be payment in period 1 for external financing of \(D_0\), the agent’s period-1 budget constraint is given by

\[
c_1 = Y_1 + D_1 - D_0' + (1 + r) B_0 - B_1. \] (3)

In period 2, agents receive different values of output, thereby revealing their identity.
The $h$-agent receives output of $HK$ and pays $D_{1h}'$ for the use of $D_1$.

$$c_{2h} = HK - D_{1h}' + (1 + r) B_1 - \tau_{2h}. \quad (4)$$

The $l$-agent has an analogous budget constraint according to

$$c_{2l} = LK - D_{1l}' + (1 + r) B_1 - \tau_{2l}. \quad (5)$$

Agents understand that the government chooses income-specific tax and transfer rates to equalize income net of debt repayments, subject to a constraint that aggregate taxes satisfy

$$\rho \tau_{2h} + (1 - \rho) \tau_{2l} = \tau_2.$$ 

This implies that period-2 budget constraints for agents are equivalent, whether they have high or low productivity.

$$c_2 = \rho \left( HK - D_{1h}' \right) + (1 - \rho) \left( LK - D_{1l}' \right) + (1 + r) B_1 - \tau_2. \quad (6)$$

We assume that expected returns on the risky project are sufficiently high that all agents prefer investment to the risk-free bond.

### 3 Equilibrium

#### 3.1 Costly State Verification

Models with costly state verification imply that external financing takes the form of debt contracts instead of equity, thereby minimizing payment of verification fees.\(^9\) Therefore, $D_j$ for $j \in (0,1)$ takes on the interpretation of new debt, and the contractual

\(^9\)See Romer (1996) for a presentation of the model and its solution.
value of repayments is given by $D_j' = (1 + r_j) D_j$, where $r_j$ denotes the interest rate. We define default as failure to repay the contractual amount, implying that the value of $\eta$ represents creditor protection in bankruptcy. The absence of multi-period debt contracts implies that all debt will be single-period, yielding maturity mismatch between debt and investment projects.

We solve for the equilibrium with investment in two stages. First, we define an equilibrium in international credit markets, conditional on values for $D_0$ and $D_1$. This yields equilibrium values for interest rates and upper bounds on debt in both periods. Second, we solve for equilibrium values for $D_0$ and $D_1$.

### 3.2 Equilibrium in International Credit Markets

An equilibrium in credit markets is defined for given values of $D_0$ and $D_1$, as interest rates in each period, $\{r_0, r_1\}$ and debt ceilings in each period $\{D_0, D_1\}$ such that risk-neutral international creditors willingly provide loans when they expect to receive the risk-free interest rate, and consumers choose between repayment or surrender of bankruptcy awards in order to maximize utility.

#### 3.2.1 Period-1 interest and debt ceiling

Working backwards, consider the equilibrium values for the debt ceiling and interest rate in period 1. At the beginning of period 1, the market learns the number of productive agents ($\rho$), but agents do not learn their own identity. Failure to repay the contractual amount triggers state-verification, allowing creditors to seize $\eta$ of the agent’s resources, interpreted as the bankruptcy settlement in default.

Agents with low productivity, $l$-agents, will repay their period-1 debt with interest in period 2 when their debt obligations $(1 + r_1) D_1$ are less than the bankruptcy settlement $(\eta L K)$. When $l$-agents repay period-1 debt in all states, debt is completely safe since no
agent ever defaults. When period-1 debt is perfectly safe, \( r_1 = r \), and the criterion for repayment is expressed according to

\[(1 + r) D_1 \leq \eta LK.\]

This implies that the maximum value of debt for which \( l \)-agents repay is given by the present-value of the \( l \)-agent’s bankruptcy settlement,

\[D_1 = \frac{\eta LK}{1 + r} \quad \text{all agents repay.}\]

Creditors are willing to offer debt which exceeds the bankruptcy settlement for \( l \)-agents as long as it does not exceed the bankruptcy settlement for \( h \)-agents and they are compensated with a higher interest rate for default by \( l \)-agents. The requirement that debt with interest not exceed the bankruptcy payment for \( h \)-agents can be expressed as

\[(1 + r_1) D_1 \leq \eta H K,\] (7)

Therefore, when only \( h \)-agents repay, the debt ceiling is

\[D_1 = \frac{\eta H K}{1 + r_1} \quad \text{only } h \text{-agents repay.}\] (8)

When only \( h \)-agents repay, period-1 debt is risky, and the equilibrium interest rate must be higher to compensate creditors for default by \( l \)-agents.

Assuming that loans to the emerging economy’s agents can be pooled to yield a risk-free asset, arbitrage requires that the period-1 interest rate equate the payments from loans to agents in the emerging market with the payments on the same loans in the risk-free international bond market. Given the debt ceiling in equation (8), the \( \rho \) agents of type \( h \) always pay \( 1 + r_1 \) on \( D_1 \). The \( 1 - \rho \) agents of type \( l \) pay \( \eta L K \), and the international
creditor surrenders the state-verification fee of \( \varpi \) on loans to \( l \)-agents. Arbitrage requires

\[
(1 + r) D_1 = \rho (1 + r_1) D_1 + (1 - \rho) (\eta L K - \varpi).
\] (9)

The simplifying assumption that creditors receive no net payout from \( l \)-agents, implies \( \eta L K = \varpi \), yielding the period 1 gross interest rate on risky debt as

\[
1 + r_1 = \frac{1 + r}{\rho}.
\] (10)

When period-1 debt is risky, a bad signal about the number of productive agents, represented by a low value for \( \rho \), raises the interest rate because only agents of type \( h \) will repay. Therefore, the interest rate is rising as the fraction of \( h \)-agents, given by \( \rho \), falls.

When period-1 debt is risky, equations (8) and (10) can be solved for the period-1 debt ceiling \( \bar{D}_1 \) to show that it is increasing in both in the number of productive agents \( \rho \) and in the fraction of output that creditors can claim in the event of default \( \eta \),

\[
\bar{D}_1 = \frac{\rho \eta HK}{1 + r} \quad \text{only } h \text{-agents repay.}
\] (11)

The upper bound on debt is therefore the maximum of

\[
\bar{D}_1 = \max \left\{ \frac{\rho \eta HK}{1 + r}, \frac{\eta L K}{1 + r} \right\} = \frac{\rho \eta HK}{1 + r},
\] (12)

where the equality follows from the assumption that \( \rho H > L \).

Therefore, in equilibrium, period-1 debt can take on values up to \( \bar{D}_1 \). When \( D_1 \leq \eta L K / (1 + r) \), period-1 debt is safe and the interest rate equals the risk-free rate. This is because all agents, including those with low productivity, will repay in period 2. For these low values of debt, the debt ceiling is never binding. When period-1 debt is larger, \( l \)-agents do not repay. Debt is risky, and the interest rate exceeds the risk-free rate from
equation (10).

3.2.2 Initial period interest and debt ceiling

Now, consider the market for period-0 debt \( D_0 \), beginning with the default decision in period-1 on period-0 debt. In period 1, agents in the economy and creditors receive information on \( \rho \), giving the number, but not the identity, of productive agents. Since agents have no information on their own identity, they continue to make the same decisions in equilibrium. Each agent must choose period-1 debt, \( D_1 \), subject to the debt ceiling, and whether or not to default on debt taken out in period 0, \( D_0 \). We consider the case for which \( D_1 > \eta L K \), so that \( l \)-agents will always default on \( D_1 \), consistent with the costly-state-verification literature in which bad outcomes yield default. The decision to default on \( D_0 \) depends on the penalties the creditor imposes.

Following a default, the creditor has two choices. He could claim bankruptcy awards in period 1 from the unit mass of defaulting agents, equal to \( \eta (\rho H + (1 - \rho) L) \lambda K \), and pay the costly state verification fee of \( \varpi \). Alternatively, he could roll over the loan, effectively offering period-1 debt equal to \( (1 + r_0) D_0 \). We show in Lemma 1 below that in default, \( (1 + r_0) D_0 > \bar{D}_1 \). A roll-over would give the agent a period-2 choice of repaying \( (1 + r_1) (1 + r_0) D_0 \), or surrendering \( \eta HK \), if he is an \( h \)-agent, or \( \eta LK \), if he is an \( l \)-agent. Since bankruptcy awards would be less than debt with interest (since debt exceeds the ceiling), both types of agents would declare bankruptcy in period 2. The international creditor would verify output and pay state verification fees, yielding a net present value of awards of \( (\eta (\rho H + (1 - \rho) L) K - \varpi) / (1 + r) \). The assumption that \( \lambda < (1 + r)^{-1} \) is sufficient to assure that the creditor prefers to roll over debt following default in period-1.

The decision to default on \( D_0 \) is based on the value of initial debt repayments and the realization of a value for \( \rho \). When agents repay period-0 debt, the expected present value
of net debt repayments over the life of the project is given by

\[(1 + r_0) D_0 - D_1 + \frac{\rho (1 + r_1) D_1}{1 + r} + \frac{(1 - \rho) \eta L K}{1 + r} = (1 + r_0) D_0 + \frac{(1 - \rho) \eta L K}{1 + r},\]

where the equality uses equation (10) to substitute for the interest rate on period-1 debt.

Alternatively, when agents default on period-0 debt and creditors react by rolling over debt with plans to claim bankruptcy awards in period 2, effectively rescheduling, the expected present value of net debt repayments becomes

\[\eta \left(\rho H + (1 - \rho) L\right) K \frac{1}{1 + r}.\]

The expected present value of debt repayments are lower under default and rescheduling when the realization of \(\rho\) is low such that

\[\rho < \rho^d = \frac{(1 + r) (1 + r_0) D_0}{\eta H K}.\]

Additionally, equation (12) can be used to show that period 1 net resources from borrowing, when an agent repays, are bounded by

\[D_1 - (1 + r_0) D_0 \leq \frac{\rho \eta H K}{1 + r} - (1 + r_0) D_0.\]

Therefore, when \(\rho < \rho^d\), and an agent does not default, his period-1 resources from borrowing are negative because he must repay more in interest and principle than he can borrow; equivalently, he cannot rollover debt with new loans.

**Lemma 1.** Given a value for initial debt with interest, \((1 + r_0) D_0\), there is a critical value of \(\rho = \rho^d\), below which agents choose default and above which they choose repayment.

All proof are in the appendix. When \(\rho < \rho^d\), default increases total resources and the
increase in resources comes in period 1 when the agent could want the increased resources to smooth consumption. Agents optimally default when the sudden stop in lending is so large that they cannot roll over interest and principle on debt.

Since \( \rho < \rho^l \) is impossible, we can define \( \rho^d \) more formally as

\[
\rho^d = \max \left[ \frac{(1 + r)(1 + r_0)D_0}{\eta HK}, \rho^l \right].
\]

When \( \rho^d \) takes on the value of its lower support, given by \( \rho^l \), there is no value of \( \rho \) for which agents would default, and period-0 loans are perfectly safe.

Since \( \rho^d \) is increasing in \( D_0 \), there is a critical value for initial debt \( (D_{0R}) \), above which \( D_0 \) is risky and below which \( D_0 \) is safe. Setting \( r_0 = r \) and \( \rho^d = \rho^l \) and solving equation (13) for safe debt yields

\[
D_{0R} = \frac{\rho^l \eta HK}{(1 + r)^2}.
\]

When \( D_0 \leq D_{0R} \), \( D_0 \) is perfectly safe since no value for \( \rho \) could elicit default.

Now, consider the equilibrium interest rate on loans made in period 0 \( (D_0) \) when there is uncertainty regarding \( \rho \). If agents choose default in period 1, creditors claim \( \eta \) of second period output at the cost of the verification fee of \( \varpi \). Given risk-neutral creditors, the value of expected debt repayments on the risky loan must equal the value of debt repayments on a safe international loan. With the simplifying assumption that \( \eta LK = \varpi \), the period-0 interest rate must satisfy

\[
(1 + r) D_0 = (1 + r_0) D_0 \int_{\rho^d}^{1} f(\rho) d\rho + \int_{\rho^d}^{\rho^d} \left( \frac{\eta(HK - LK)}{1 + r} \right) \rho f(\rho) d\rho,
\]

where the first integral represents the probability that agents repay in period 1, and the second represents the expected present value of net repayments in period 2, arising from default and rescheduling in period 1. Solving for contractual debt repayments \((1 + r_0) D_0\)
as a function of $D_0$ and $\rho^d$ yields

$$(1 + r_0) D_0 = \frac{(1 + r) D_0 - \int_{\rho^d}^{\rho_l} \left( \frac{H K - L K}{1 + r} \right) \rho f(\rho) d\rho}{\int_{\rho^d}^{\rho_l} f(\rho) d\rho}. \quad (15)$$

Equations (13) and (15) constitute a pair of non-linear equations, which can be used to determine equilibrium values for $r_0$ and $\rho^d$ for a given value of initial debt, $D_0$. We characterize the solution graphically.

In Figure 1, the contractual value of debt obligations, given by $(1 + r_0) D_0$, is on the vertical axis, and the value of $\rho$ which elicits default, given by $\rho^d$, is on the horizontal axis. The kinked line labeled $OD$, represents the agent’s optimal default decision, relating $\rho^d$ and $(1 + r_0) D_0$ from equation (13). Since $\rho^l$ is the lower support of the distribution, there is a range of values for $(1 + r_0) D_0$ for which no value of $\rho$ would elicit default, implying
that for low contractual debt obligations, $OD$ is vertical at $\rho'$. As $(1 + r_0) D_0$ increases, equation (13) implies a positive linear relationship between $(1 + r_0) D_0$ and $\rho^d$.

The curves labeled $AA$ plot the arbitrage relationship in equation (15) between $\rho^d$ and $(1 + r_0) D_0$, for different values of period-0 debt ($D_0$). For equilibrium values of $\rho^d \leq \rho'$, there is no risk of default and the interest rate equals the world rate, implying that the $AA$ curves have intercepts at $(1 + r) D_0$ and that they are horizontal for $\rho^d \leq \rho'$. Higher initial debt implies higher intercepts. For $\rho^d > \rho'$, equation (15) implies that $(1 + r_0) D_0$ rises at an increasing rate in $\rho^d$, requiring that the $AA$ curves have an increasing slope.

The first intersection of a particular $AA$ curve with the $OD$ curve is a stable equilibrium and gives the equilibrium values for $\rho^d$ and $r_0$ for a given level of $D_0$. For $D_0 = D_{0A}$, equilibrium occurs at point $A$, with $\rho^d = \rho^d_A$ and with $r_0 > r$. Larger values of initial debt, represented by $AA$ curves with higher intercepts, have higher equilibrium values for $r_0$ and $\rho^d$. The largest level of risk-free debt is given by $D_{0R}$, and the debt ceiling is $\bar{D}_0$. For larger levels of debt, there is no intersection with the $OD$ curve. Since the $AA$ curves have slopes increasing at an increasing rate, equilibrium values for $\rho^d$ and $r_0$ are rising in $D_0$ at increasing rates.

Consider the effect of credit market strength, represented by $\eta$, on the size of the debt ceiling. Equating the values for $\rho^d$ and $D_0$ in equations (13) and (15) and setting their slopes equal yields an implicit solution for the value of $\bar{\rho}$, given by

$$\bar{\rho}^d f (\bar{\rho}^d) L - 1 \int_{\bar{\rho}^d}^{1} H f (\rho) d\rho = 0.$$  

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10To demonstrate stability, assume that the level of debt is given by $D_{0A}$. At $\rho^d = \rho'$, the arbitrage value of interest lies directly above $\rho'$ along $AA_A$ and is given by $r_0 = r$. However, for an interest rate of $r_0 = r$, the household’s optimal choice of $\rho^d$ is higher, rightwards along $OD$. For the higher value of $\rho^d$, the arbitrage value of $r_0$ is higher, upwards along $AA_A$. The pattern continues in ever smaller steps until the equilibrium at point A is reached.
Defining
\[ \Gamma = \bar{\rho}^d \int_{\bar{\rho}^d}^{1} f(\rho) \, d\rho + \int_{\bar{\rho}^d}^{\rho^d} \left( \frac{H - L}{H} \right) \rho f(\rho) \, d\rho, \]
solutions for \( \bar{D}_0 \) and \( \bar{r}_0 \), as functions of \( \bar{\rho}^d \) and \( \Gamma \) can be expressed as
\[ \bar{D}_0 = \frac{\eta HK \Gamma}{(1 + r)^2}, \]
\[ 1 + \bar{r}_0 = \frac{\bar{\rho}^d (1 + r)}{\Gamma}. \]
Note that both \( \bar{\rho}^d \) and \( \bar{r}_0 \) are independent of \( \eta \), while \( \bar{D}_0 \) is increasing in \( \eta \). An increase in the proportion of output awarded in bankruptcy, given by \( \eta \), flattens the AA curve and increases the slope of the \( \rho^d \) curve, implying that the tangency occurs for a higher value of debt. Stronger credit markets are associated with higher debt ceilings.

To summarize, the resulting equilibrium in credit markets has characteristics of credit markets in emerging economies. Creditors offer single-period debt contracts to finance longer-term investment, yielding maturity mismatch. For risky projects, equilibrium interest rates offered at the beginning of the project are increasing in the magnitude of the loan. Creditors impose endogenous credit ceilings, conditional on awards they expect in bankruptcy court in the event of default. The period-1 credit market is characterized by potentially binding debt ceilings which fluctuate with news on the proportion of high-productivity agents. The modification of the costly-state-verification model for maturity-mismatch implies that shocks have significant effects on the availability of credit, in addition to their financial accelerator effects on the cost of credit.

### 3.3 First Order Conditions

In making decisions, agents take taxes and interest rates as given. The problem is solved backwards, beginning with choices made in period 1, which together with the
realization of an agent’s identity in period 2, determine period 2 outcomes. A time line for the economy is given below.

**Time Line**

<table>
<thead>
<tr>
<th>Period 0</th>
<th>Period 1</th>
<th>Period 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>agents choose to invest</td>
<td>learn ρ and receive $Y_1$</td>
<td>agents learn identity</td>
</tr>
<tr>
<td>creditors choose $r_0$, $\bar{D}_0$</td>
<td>creditors choose $r_1$, $\bar{D}_1$</td>
<td>creditors pay verification fees</td>
</tr>
<tr>
<td>agents choose $D_0$, $B_0$</td>
<td>agents make default decision</td>
<td>l-agents receive $LK$</td>
</tr>
<tr>
<td>agents choose $c_1$, $D_1$, $B_1$</td>
<td>l-agents pay min $[\eta LK, (1 + r_1)D_1]$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>h-agents receive $HK$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>h-agents pay min$[\eta HK, (1 + r_1)D_1]$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>agents consume $c_2$</td>
</tr>
</tbody>
</table>

Consider the agent’s choice for first-period debt. The Euler equation has an inequality depending on whether or not the debt ceiling in period 1 is binding

$$\frac{1}{c_1} \geq \frac{\beta \rho (1 + r_1)}{c_2} = \frac{\beta (1 + r)}{c_2} \quad D_1 \leq \bar{D}_1, \quad B_1 \geq 0, \quad (17)$$

where the equality uses equation (10) to substitute for $(1 + r_1)$. Given the assumption that $\beta = (1 + r)^{-1}$, equation (17) implies that, when the debt ceiling does not bind, agents choose equal consumption across periods. If the debt ceiling binds, then second-period consumption exceeds first-period consumption with debt given by the ceiling in equation (11). When the ceiling on $D_1$ is binding, $B_1 = 0$. When it is not binding, agents are indifferent between increases in $D_1$ to hold more $B_1$ and setting $D_1$ at a value for which $B_1 = 0$. We assume they choose the later.

Now, consider the choices made in period 0 for an agent who chooses the risky investment project. In choosing initial debt and bonds, he takes interest rates as given, but he
understands that his decision to default in period 1 can change with his choice of $D_0$.

The first-order condition on $D_0$ is given by

$$
\left\{ \int_{\rho^d}^{\rho^l} \left( \frac{1 + r}{c_1(\rho)} \right) f(\rho) \, d\rho + \frac{1 + r_0}{c_1(\rho^d)} \frac{\partial \rho^d}{\partial D_0} \right\} - \left\{ \int_{\rho^d}^{1} \left( \frac{r_0 - r}{c_1(\rho)} \right) f(\rho) \, d\rho \right\} \geq 0, \tag{18}
$$

where $D_0 \leq \bar{D}_0$, $B_0 \geq 0$. The first term in brackets in equation (18) is the marginal benefit of additional debt above that necessary to finance the investment project, and the second is the marginal cost. The marginal benefit of debt is the utility from additional consumption an agent receives in states in which debt is not repaid. The marginal cost is the utility of the consumption reduction, due to the excess of the risk-adjusted interest rate above the risk-free rate, in states for which debt is repaid. The agent realizes that if he borrows more, he could increase states in which his debt repayments exceed the ceiling on new debt, increasing the space of default states $(\partial \rho^d / \partial D_0 \geq 0)$. However, the agent is a price-taker with respect to interest rates and does not account for the effect of his decision to borrow on the interest rate that creditors charge on this class of loans.

For $D_0 < D_{0R} \left( \rho^d = \rho^l, r_0 = r, \partial \rho^d / \partial D_0 = 0 \right)$, both marginal costs and marginal benefits are zero. These values of debt are safe, and if they are large enough to finance investment, there is no incentive to increase debt further.\(^\text{11}\) However, if values of $D_0 < D_{0R}$ are insufficient to finance the risky project, then debt must be larger. If debt necessary to finance investment equals $D_{0R}$, implying that $\partial \rho^d / \partial D_0 > 0$, with $\rho^d = \rho^l$ and $r_0 = r$, the marginal benefit of additional debt becomes positive while the marginal costs remain zero. The agent chooses to borrow more than necessary to finance investment. As debt increases such that $\rho^d > \rho^l$ and $r_0 > r$, both marginal benefits and marginal cost rise at increasing rates in debt with marginal benefits rising faster.\(^\text{12}\) This implies that in

\(^{\text{11}}\) However, we cannot absolutely rule out an increase in debt from these levels since agents are indifferent between borrowing more at the risk-free rate and using this to hold the risk-free asset. Thus, even rich countries could see debt driven to upper bounds. We assume that agents coordinate on the safe equilibrium when it exists.

\(^{\text{12}}\) We verify this in calibration exercises.
equilibrium, if agents must borrow at least $D_{0R}$, then debt is driven to its upper bound. Given $D_0$, the value for $B_0$ is determined by the budget constraint, given by equation (2).

Consider the intuition behind these results. The investment project yields little output in the first period, and agents know that a low value for $\rho$ could constrain new loans. Therefore, agents choose initial debt, both to finance investment and to hedge against a sharp drop in period-1 consumption. This extra debt is beneficial only in states for which agents do not repay period 1 debt. In states for which they do repay, the interest rate is greater than the risk-free rate, making such borrowing costly. When debt necessary to finance investment is large enough that there are some states in which agents do not repay, then debt increases beyond that necessary for investment due to the precautionary motive. This additional debt increases the probability of default by reducing the number of states in which debt is repaid. The higher default probability further raises the marginal benefits of debt, driving debt to its upper bound.

4 General Equilibrium

We consider an equilibrium for parameter values such that all agents choose the risky two-period investment project.

4.1 Definition

Given a value for $\eta$, the distribution of $\rho$, and a realization for $\rho$ in period 1, equilibrium is defined as the set of values for consumption, debt, and the risk-free bond in each period, interest rates, debt ceilings, and a value for $\rho^d$, \{ $c_1, c_2, D_0, D_1, B_0, B_1, r_0, r_1, D_0, D_1, \rho^d$ \}, for which arbitrage conditions on interest rates (equations 10 and 15) and the period-1 debt ceiling (equation 11) are satisfied, agent budget constraints (equations 3 and 6) and first order conditions (equations 17 and 18) hold, agents choose default optimally (equation 13), and expectations are rational.
4.2 Equilibrium Debt and Risk

Consider the effect of the upper bound on debt for an agent’s access to international credit. In an equilibrium with investment

\[ D_0 = K - Y + B_0 \leq \bar{D}_0. \]  

(19)

**LEMMA 2.** A country for which the size of the investment project is large relative to an agent’s endowment, \( K - Y > \bar{D}_0 \), does not have access to international financial markets. Access requires either stronger capital markets, modeled by a larger value for \( \eta \) which raises \( \bar{D}_0 \), or a larger endowment, \( Y \).

Given \( \bar{D}_0 \), countries whose agents have too few resources, relative to the size of the investment project, cannot access credit markets because the risk of default would be too high and the return in default too low. Since \( \bar{D}_0 \) is increasing in \( \eta \), a stronger credit market allows access for a given value of \( K - Y \).

Equations (13) and (15) allow characterization of \( (1 + r_0)D_0 \) and \( \rho^d \) as a function of the agent’s choice for \( D_0 \). These equations also imply an upper bound on initial debt. Equation (18) characterizes the agent’s optimal choice for initial debt.

**LEMMA 3.** Under the assumption that agents coordinate on the safe equilibrium value for \( D_0 \) if it exists, then if \( D_{0R} < K - Y \), equilibrium implies \( D_0 = K - Y \). Alternatively, if \( D_{0R} \geq K - Y \), then equilibrium requires \( D_0 = \bar{D}_0 \).

Note that this Lemma implies that countries either hold debt low enough to be safe, or they hold the maximum amount. A country which requires debt large enough to generate a small probability of default at the risk-free interest rate has the incentive to borrow to hold the international bond as a precaution against a default state. The increase in debt increases the probability of default, yielding incentives to increase debt further until values are driven to their upper bounds.
We are now in a position to show that countries with identical technologies for the risky projects, but with different values for initial endowments and the strength of capital markets, belong to different credit clubs.

**PROPOSITION 1.** Given parameters characterizing the inherent riskiness of the project, countries with high values for \((K - Y) / \eta K\) have no access to credit for financing profitable, risky projects, countries with lower values have access which is volatile, and countries with even lower values have safe, stable access.

Under the assumption that markets coordinate on the safe equilibrium when it exists, a country has risk of a debt crisis if values of \(Y\) and \(\eta\) are low relative to an advanced country, so that the safe equilibrium does not exist, but high relative to an underdeveloped country, so that creditors are willing to lend. The division of countries into credit clubs with different access to international credit markets occurs because, if a country needs debt greater than the safe amount, debt is driven to its upper bound, maximizing risk. Hence, a country with access either has no risk or maximum risk, with corresponding implications for interest rate premia. This differs from the general result in Townsend’s (1979) costly state verification model, in which the risk of default is a continuous function of the agent’s initial endowment.\(^{13}\)

### 4.3 Debt Crises

A debt crisis is triggered by a low value for the number of productive agents.

**PROPOSITION 2.** In period 1, a value for \(\rho < \bar{\rho}^d\) triggers a debt crisis in which all agents choose not to repay period-0 debt.

In equilibrium, international creditors impose a ceiling on period-1 debt, given by \(\rho \eta HK / (1 + r)\), to assure that they receive the expected risk-free rate of return. A small

\(^{13}\)This is because in Townsend’s (1979) model, agents cannot borrow more than is needed for the investment project, net of own resources.
realization for $\rho$ implies a low credit ceiling and has the interpretation of capital flight. Agents choose to default on their debt whenever new loans offered are insufficient to rollover debt. This default is widespread, affecting more agents than those who will ultimately have low productivity.

The low value of $\rho$ also implies low output. Since only the lowest values of $\rho$ trigger financial crises, only severe recessions are accompanied by financial crises.

5 Calibration and Simulations

In this section, we calibrate the model to demonstrate its quantitative implications for financial crises like those which occurred in Southeast Asia in 1997-98. Crises in Indonesia, Malaysia, South Korea, the Philippines, and Thailand were all preceded by strong GDP growth and current account deficits, and were accompanied by widespread default, severe recessions, and current account reversals.\footnote{Corsetti, Pesanti, and Roubini (1999) document a large share of non-performing loans (Table 22) for these five countries as well as defaults in corporate and private sectors (p.5). They also document large current account deficits and high output prior to the crisis. Moreno (2008) documents the large current account reversals and severe recessions for these countries, excluding South Korea.} We choose South Korea as a representative country to which to calibrate. Since the world is not a three-period economy, we must place the agents in this model into a broader economy. We specify a very simple structure to illustrate the quantitative features of the model. The aggregate model is not a fully-articulated DSGE model and should not be judged on its general ability to match moments. The purpose of the calibration is to demonstrate that the model can replicate quantitative features of the crisis in South Korea, as well as the qualitative feature of widespread default. Since the crises in other Southeast Asian countries were similar to those in South Korea, the calibration implies that the model applies more generally to crises like those which occurred in Southeast Asia in 1997-98.

We assume that the economy has a risky sector, comprised of overlapping generations.
of entrepreneurs with access to the risky project, and a safe sector with less risk and lower productivity. Overlapping generations are structured such that the new generation is planning, while the old generation is enjoying its period 2 output and the middle generation is receiving its period 1 output.\footnote{We calibrate each period as a year, so we should think in terms of agents having more than a single project over a lifetime.} The new generation’s endowment ($Y$) equals the old generation’s period 2 taxes ($\tau_2$). Since two generations are producing in the risky sector in each period, total GDP for the risky sector is the sum of period-1 GDP for the middle generation and period-2 GDP for the old generation.

Although this set-up could be interpreted as a steady state, we do not use this interpretation. Proposition 1 states that once the endowment is large enough, the country enters the club with safe credit. With any linkages between the generations of entrepreneurs, this will imply strong precautionary savings motives. Therefore, the period for which countries belong to the risky club is temporary if additional market imperfections do not hinder natural precautionary savings. We view the model as applying at a particular transient stage of development, in which agents have sufficient resources to access the risky profitable technology, but insufficient resources to limit debt to the safe level.

Additionally, since the aggregate model is simply an aggregation of different types of agents who act independently of each other, there are no spill-overs from one type of agent to another. In particular, there are no spillovers of widespread default to agents who did not default. Interaction among overlapping generations of agents is not part of the theoretical model, and addition at this stage would be ad hoc.\footnote{Mendoza and Yue (2008) and Mendoza (2010) construct models with feedback from the financial crisis to the broader economy.} We do not explicitly account for growth. Therefore, the model is a business cycle model, detrended for growth.

Table 1 contains parameter values set according to standard values as well as a normalization of the capital stock to unity. Table 2 contains inequality constraints, necessary for a crisis to be possible. The distribution of $\rho$ is given by a discrete approximation to a
bounded normal with twenty-two values.\footnote{We use Gaussian quadrature.}

Calibrated parameter values are given in Table 3, where $y_3$ is the per capita value of output in the riskless sector, and $\bar{\rho}$ is the mean for $\rho$. We choose the sequence of values for $\rho_t$ in the years around the crisis to match the percent deviation of GDP from trend in the data, given by $\% \Delta GDP$ for each year and present this in Table 4. Calibration targets are in Table 5. We denote the standard deviation of the HP-filtered logarithm of annual real GDP for South Korea\footnote{We use IFS data.} from 1970 to 2006 by $\sigma_{GDP}$. Gourinchas, Valdes and Landerretche (2001) report the probability of a banking crisis following a lending boom in non-Latin-American emerging markets as between 7 and 10%. Our debt crisis can be viewed as a banking crisis under the assumption that banks were intermediating the loans. Therefore, we choose parameters so that the probability of receiving a value for $\rho$ less than its smallest safe value, $\rho^d$, is 8.1%. We choose the size of the risky profitable sector to be consistent with evidence reported in Zin (2005) that 20% of South Koreans have 38% of the income. We calibrate to place 20% of the agents in the risky profitable sector and give them 38% of the income. We choose the capital stock in the riskless sector such that the overall ratio of capital to GDP is unity,\footnote{Using OECD data, we compute the capital stock using the perpetual inventory method. We use the value of the capital output ratio prior to the crisis.} and we assume that the endowment in the riskless sector is just large enough to finance the investment without external borrowing.

A comparison of the calibration targets in the data and the model in Table 5 indicates that we have successfully calibrated the model to match important features of the data. Additionally, this calibration yields a value for $\rho^d = .649$, indicating that the model produces a financial crisis with widespread default in 1998, a year with severe recession. Corsetti, Pesanti and Roubini (1999) provide evidence of widespread default in South Korea.\footnote{They report that local banks' non-performing loans as a fraction of assets at the end of 1997 was 16% and that as many as 7 of 30 of the largest conglomerates were effectively bankrupt by the end of 1997.} The IMF (2000) reports an agreement with foreign banks in early 1998 to
extend the maturity of short-term loans, consistent with default and rescheduling.

If we consider the characteristics of the risky investment project to be globally determined, then there are two key domestic parameters responsible for the generating a positive crisis probability, \( \eta \) and \( Y \). Given other parameter values, the critical value for \( Y \) is 0.602, and for \( \eta \) is 0.38. We calibrate a model which has a safe equilibrium, in the sense that there is never widespread default and rescheduling in the middle period. In this "safe" equilibrium, low-productivity agents do default when they realize their output in the final period. To calibrate a safe equilibrium in which agents are not wealthier, we retain the value for the endowment and improve capital markets by raising \( \eta \) to yield: \( \eta = .45 \). We also retain the value for the verification fee \( \overline{\omega} = .28 \), such that the higher \( \eta \) implies that creditors receive a positive net payout from \( l \)-agents. In the safe equilibrium, there is no widespread default with the realization of a low value for \( \rho \) in 1998, but all model-generated values in Table 4, except the probability of a crisis, are unchanged.

To compare the model results in a safe equilibrium with those in a risky equilibrium and with data, we solve both models for the percent deviation of consumption from trend, for the current account as a fraction of GDP, and for the average interest rate. The current account is the change in debt, whether the change is due to repayment or to forgiveness. Period-0 agents borrow, yielding

\[
CA_{0t} = -(D_0 - B_0) = -(K - Y).
\]

Period-1 agents repay initial debt and take out new debt based on the realization of the productivity shock, yielding

\[
CA_{1t} = K - Y - D_{1t}(\rho_t).
\]

\[\text{If middle-period agents default and reschedule, their debt is rescheduled to equal unrepaid initial debt with interest, implying a change in debt, given by initial debt of } D_0 - B_0 = K - Y \text{ less new debt of } D_1 = (1 + r_0)D_0. \text{ Therefore, the current account of middle agents has the same form whether they repay or not.}\]
Period-2 agents eliminate their debt, given by $D_{t-1} \left( \rho_{t-1} \right)$, whether they repay or default, allowing creditors to take $\eta$ of output and have the remainder forgiven. Therefore,

$$CA_{2t} = D_{t-1} \left( \rho_{t-1} \right).$$

We assume that the riskless sector makes no contribution to the current account.

To aggregate activity in the risky sector, we must consider how to count period-0 agents, who have no output and no consumption, but who contribute to the current account. We choose to view them as the same people as the old agents. Therefore, old agents are receiving output and consuming from a previous project at the same time that they are acting like new agents and planning a new project. Their net current account is the sum of the period-0 and period-2 current accounts, and they comprise half of agents in the risky sector. Therefore, the current account for the risky sector, is the sum of the current accounts and is given by\(^\text{22}\)

$$CA_t = D_{t-1} \left( \rho_{t-1} \right) - D_t \left( \rho_t \right).$$

Figure 2 plots values for $D_1$ as a function of $\rho$ in both the risky and safe equilibria, where $D_1$ and $D_1S$ denotes debt in the risky and safe equilibrium, respectively.

\(^{22}\)To compute the current account relative to GDP, we compute per capita current account and divide that by per capita GDP.
In the safe equilibrium, debt rises linearly with $\rho$, while in the risky one, debt is flat in $\rho$ over the default region and then rises more steeply. A current account surplus is caused by a fall in $\rho$. The surplus in the risky equilibrium can be as much as 48% higher than that in the safe equilibrium. This occurs when $\rho$ initially takes on its largest value and falls to $\rho^d = 0.649$, where the difference between values for $D_1$ in the safe and risky equilibria is highest.

Values in the data and in the simulations based on the calibrated sequence of $\rho_t$ are given in Table 6. Consumption is expressed as a percent deviation from trend, and the current account as a fraction of GDP is expressed as a percent. The pattern in both models for consumption and the current account are similar and broadly fit the data. The period begins with strong consumption and a current account deficit in both models and in the data. Interest rates in the risky equilibrium and in the data are very similar. By 1998, both the consumption boom and current account deficit have been reversed.

---

23Calculation of consumption requires that we specify consumption in the risk-free sector, where $c_3 = y_3 - \delta k_3$. We choose $k_3 = .293$ to match the calibration target that the aggregate capital output ratio is unity.
With the fall in productivity, interest rates rise sharply in the data and more sharply in
the model.\textsuperscript{24} The current account reversal begins in 1997 in the model, while in the annual
data, it begins in 1998 (the fourth quarter of 1997 in quarterly data). The 1997 reversal
is much larger in the risky equilibrium because the low value for $\rho_{1997}$ restricts borrowing
much more in the risky equilibrium than agents choose to limit it in the safe equilibrium.
The further fall in $\rho$ in 1998 implies no reduction in debt in the risky equilibrium, since
it is already at its minimum, while in the safe equilibrium, agents choose to reduce debt
further. The end values of debt are very similar, such that the two-year cumulative current
account surplus in the risky model is only about 7\% percent higher than in the safe model.
The cumulative current account reversal in both models falls short of that in the data.
A reversal in the model large enough to match the data would probably require explicit
consideration of spillovers of default. As an example, if all new borrowing were eliminated
in the period in which the default occurs, the surplus in the default year of 1998 would
rise to over 10\% of GDP, yielding a cumulative surplus of 14.8\% in the risky model.

The calibrations demonstrate that in the presence of financial market imperfections, an
empirically meaningful and low realization for productivity can create a severe recession
with a sudden stop of capital flows and widespread default. We have calibrated the model
to South Korea, but the results are more generally applicable to private-sector lending
crises in emerging-market countries, whose financial market imperfections are more severe
than those in developed countries. The model has the potential to explain many of the
Southeast Asian crises in 1997-98 and emerging-market crises going forward.

\textsuperscript{24}In the risky equilibrium, there is no borrowing at the period-1 interest rate in the risky equilibrium,
so we do not report an interest rate.
6 Conclusion

The literature on financial market imperfections contains two dominant models, collateral constraints and costly state verification. The implications of collateral constraints for financial crises in emerging markets have been widely explored. This paper offers a model of financial crises with the alternative dominant model of financial market imperfections, costly state verification. When we add maturity mismatch between debt and investment to a model with costly state verification, we show that financial crises with default and recession are the downside of the interaction of profitable risky investment with capital market imperfections. Additionally, the financial market imperfections we impose have credit-market equilibria which look like those we observe. Investment and loans are mismatched in maturity; interest rates increase endogenously with debt; default and rescheduling occur in equilibrium with the fraction of loans defaulting occasionally large; and debt ceilings on loans fluctuate with news about productivity. Once we add a micro-founded justification for maturity mismatch, like moral hazard, all of these characteristics are endogenous to the fundamental capital market imperfections.

The model also demonstrates that the world can be divided into three credit clubs, those with no access, those with stable access at low interest rates, and those with volatile access at interest rates reflecting risk. Countries with very little wealth or very weak capital markets have no access to international credit. Countries with high wealth and strong credit markets enjoy high credit ceilings relative to their needs to borrow. For these countries, desired debt at the risk-free interest rate is so low that agents always prefer repayment to sacrificing the large fraction of output implied by the strong credit market. These are the advanced economies who have stable access to international credit at low interest rates. Countries in between, those with some wealth and a reasonably strong credit market, are driven to the risky credit club.

Finally, we demonstrate the quantitative ability of the model to explain financial crises
by placing the agents in the model into a broader economy with overlapping generations of entrepreneurs. We calibrate the model economy to match parameters for South Korea and show that a sequence for productivity, calibrated to match GDP, generates a recession with widespread default. Additionally, the patterns for consumption, the current account, and interest rate, implied by the calibrated sequence of shocks, are broadly similar to those in the data. Other countries in Southeast Asia were affected by similar crises. We view the model as a general explanation of the type of crisis which occurred in Southeast Asia in 1997-98. Emerging market financial crises with private sector default can be the downside of risky investment in the presence of capital market imperfections.
Table 1: Standard parameter values and normalization

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<thead>
<tr>
<th></th>
<th>r</th>
<th>β</th>
<th>δ</th>
<th>K</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>0.04</td>
<td>1/10</td>
<td>0.1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2: Inequality constraints

\[ D_0 R < K - Y < \bar{D}_0 \quad \lambda < \frac{1}{1+r} \quad D_1 < \eta L K \quad \rho H < L \]

agents prefer investing to holding bonds

Table 3: Parameter values

<table>
<thead>
<tr>
<th></th>
<th>Y</th>
<th>y_3</th>
<th>H</th>
<th>L</th>
<th>η</th>
<th>λ</th>
<th>\bar{p}</th>
<th>\rho^l</th>
<th>σ_ρ</th>
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<tr>
<td></td>
<td>0.569</td>
<td>0.337</td>
<td>2.4</td>
<td>0.8</td>
<td>0.35</td>
<td>0.5</td>
<td>0.75</td>
<td>0.513</td>
<td>0.115</td>
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Table 4: Calibrated values for the time series for ρ

<table>
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<th></th>
<th></th>
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<tbody>
<tr>
<td></td>
<td>.806</td>
<td>.919</td>
<td>.649</td>
<td>.535</td>
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Table 5: Calibration targets

<table>
<thead>
<tr>
<th>target</th>
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<tr>
<td>$\sigma_{GDP}$</td>
<td>0.031</td>
<td>0.033</td>
</tr>
<tr>
<td>$%\Delta GDP$ (1996)</td>
<td>5.3</td>
<td>5.2</td>
</tr>
<tr>
<td>$%\Delta GDP$ (1997)</td>
<td>4.5</td>
<td>4.4</td>
</tr>
<tr>
<td>$%\Delta GDP$ (1998)</td>
<td>-7.7</td>
<td>-7.7</td>
</tr>
<tr>
<td>Pr crisis</td>
<td>7% ≤ Pr ≤ 10%</td>
<td>8.1%</td>
</tr>
<tr>
<td>income distribution</td>
<td>20% of agents have 38% of income</td>
<td>20% of agents have 38% of income</td>
</tr>
<tr>
<td>capital/GDP</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 6: Consumption, current account, interest rate

<table>
<thead>
<tr>
<th></th>
<th>$C$</th>
<th>$\frac{CA}{GDP}$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>data</td>
<td>risky</td>
<td>safe</td>
</tr>
<tr>
<td>1996</td>
<td>12.6</td>
<td>8.6</td>
<td>7.7</td>
</tr>
<tr>
<td>1997</td>
<td>10.1</td>
<td>0.3</td>
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<tr>
<td>1998</td>
<td>-7.3</td>
<td>-9.5</td>
<td>-10.7</td>
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</tbody>
</table>
A Maturity Mismatch and Moral Hazard

We motivate maturity mismatch with moral hazard. However, since the particular motivation is not important to the model, this discussion has been relegated to the appendix.

Our modification follows Atkeson (1991), who creates moral hazard with the assumption that creditors cannot perfectly observe the use of the borrowed funds. Atkeson assumes that they cannot observe whether funds are used for consumption or investment. Additionally, Atkeson’s optimal contract is designed such that it does not aid in intertemporal consumption smoothing, convincing agents to use the borrowed funds for investment.

We assume that agents must provide credible evidence of an investment project to enable borrowing, assuring that funds are used for investment. However, we also assume that an agent with access to the risky investment project also has access to a safe one-period investment project. Both investment projects require his full labor endowment, implying that he can choose only one. The one-period project has a safe period-1 income \( Y_{1s} \), yielding a return greater than the world interest rate, but yields no output in period 2. The risky two-period project has sufficiently greater yield that agents, who expect to repay initial loans, prefer the risky project.

If two-period loans were available, agents would experience moral hazard with respect to the choice of investment projects. An agent could finance the one-period project with a two-period loan and completely escape debt obligations since there would be no period-2 output to support repayment when the loan matures. When utility with period-1 income from the safe project with default exceeds utility with expected net income from the risky project, the agent plans to finance the single-period project with a two-period loan and walk away.

The moral hazard problem can be solved with single-period debt contracts. If agents
use single-period loans to finance the safe one-period project and fail to repay, creditors can claim $\eta Y_s$, leaving agents with income of $Y_s^* (1 - \eta)$. These agents identify themselves by failing to provide evidence of an investment project for period 2, necessary to roll-over their one-period loan. Under the assumption that utility with income of $Y_s^* (1 - \eta)$ is less than expected utility with income from the risky two-period project, the moral hazard problem is solved when creditors offer only single-period loans. Single-period loans lead agents to prefer the risky two-period investment project with higher expected returns.

B  Proofs

B.1  Proof of Lemma 1

If $\rho < \rho^d$, default increases total resources. The increase in resources occurs in period 1. If agents are debt-constrained, default increases the ability to smooth consumption, whereas if agents are not debt-constrained, default has no effect on this ability because agents use the risk-free bond to transfer resources forward. Utility is higher under default because resources are higher and available when they could help smooth consumption.

If $\rho > \rho^d$, default reduces total resources. The reduction in resources occurs in period 1. If agents are debt-constrained, default reduces the ability to smooth consumption, whereas if agents are not debt-constrained, default has no effect on this ability because agents use the risk-free bond to transfer resources forward. Utility is higher under repayment because resources are higher and available when they could help smooth consumption.

B.2  Proof of Lemma 2

Since $B_0 \geq 0$, equation (19) implies that $K - Y$ must be less than the upper bound on debt for creditors to be willing to lend. An increase in $\eta$ raises $\bar{D}_0$ from equation (16).
B.3 Proof of Lemma 3

Equation (2) requires $D_0 \geq K - Y$. From equation (18), there is a safe equilibrium value for $D_0$ if there is a value of debt, satisfying the above inequality, such that debt is below its critical value, given in equation (14), requiring $\rho^l \eta HK / (1 + r)^2 - (K - Y) \geq 0$. Otherwise, debt is driven to its upper bound from equation (18).

B.4 Proof of Proposition 1

Access requires $K - Y < \bar{D}_0$. Substituting from equation (16), access requires $(K - Y) / \eta K < \Gamma H / (1 + r)^2$. A necessary condition for access is a positive $\eta$.

Safe initial debt requires $K - Y < D_{0R}$. Substituting from (14), for initial debt to be safe, is it necessary that $(K - Y) / \eta K < \rho^l H / (1 + r)^2$. The assumption $\rho^l H < L$ assures $\Gamma > \rho^l$. Therefore, a country can have access for a larger value of $(K - Y) / \eta K$ than is required for debt to be safe. Therefore, as $Y$ and/or $\eta$ increase, countries progress from having no access to having risky access. As $Y$ and/or $\eta$ increase further, a country progresses from risky access to safe access.

B.5 Proof of Proposition 2

Lemma 3 implies that debt is driven to its upper bound, driving $\rho^d$ to $\bar{\rho}^d$. When the stochastic realization of $\rho < \rho^d = \bar{\rho}^d$, agents default by Lemma 1.

References


