Private Sector Risk and Financial Crises in Emerging Markets*

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Abstract

Investment necessary for growth is risky and often requires external financing. For an emerging market, access to international credit markets is volatile and interest rates reflect risk of default. We present a theoretical model in which emerging market agents have access to a profitable two-period investment project of fixed size greater than their endowment. Credit market imperfections can magnify a small solvency problem into a financial crisis with widespread default and/or currency devaluation. In equilibrium, creditors offer single-period debt up to a ceiling based on expected future output. News about a negative productivity shock reduces the debt ceiling imposed by creditors, creating a sudden stop of capital flows. The sudden stop can be severe enough to trigger a debt crisis, when agents prefer default over debt repayment, and/or a currency crisis, as agents attempt to maintain desired consumption by swapping domestic currency for foreign currency to purchase goods. We also show that there are critical thresholds for parameters governing credit market imperfections that separate countries into a safe credit club with low interest rates and steady access and a risky club with high interest rates and volatile access.

- Key Words: exchange rate crisis, financial crisis, debt crisis, twin crises, liquidity crisis, solvency crisis, sudden stop, default, capital flight, foreign exchange reserves

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1 Introduction

Countries at different stages of development experience substantial differences in credit market characteristics. Underdeveloped countries have no access to credit; emerging markets have access, but loans are relatively risky, with interest rates reflecting that risk, and access is volatile; developed countries have reliable access with low interest rates. The Asian financial crises, characterized by high levels of non-performing loans and bankruptcies (Corsetti, Pesenti and Roubini 1999) and high costs of recapitalizing the financial sector (Burnside, Eichenbaum and Rebello 2001), drew attention to the volatile nature of emerging market credit and the financial crises which can occur in these economies. A large quantity of literature has been devoted to understanding the cause of these crises with the objective of developing policy to prevent future crises.

The literature attributes financial crises in emerging markets to three distinct causes. One is financial market imperfections which create multiple equilibria. Calvo (1998) discusses the implications of exogenous sudden stops of capital to emerging markets; Chang and Velasco (2001) demonstrate how bank runs can cause a financial crisis and possibly a currency crisis as well; and Chang (2007) shows how self-fulfilling expectations can raise interest rates, increasing debt and generating a crisis. What these papers and others like them have in common is that financial market imperfections are responsible for multiple equilibria, and a bad equilibrium is a crisis equilibrium. Crises are not generated by macroeconomic fundamentals.

Krugman (1998) and Corsetti, Pesenti, and Roubini (1999) offer an alternative explanation, attributing crises to the moral hazard caused by government bailout policy. Explicit or implicit policy to guarantee foreign loans creates moral hazard and leads to inefficiently high risk-taking, raising crisis probability. Ranciere, Tornell, and Westermann (2008) also focus on the role of government bailouts in generating financial crises, but in their setup, the interaction of bailout policy with capital market imperfections can lead to higher growth. The bailout policy can correct inefficiently high interest rates and low borrowing constraints. Hence their model combines policy with capital market imperfections creating borrowing constraints to show that financial crises could be a side-effect of policy chosen to stimulate growth. These models cannot explain default unless anticipated bailouts do not materialize.

The third type of model focuses on shocks to relative prices in the presence of financial market imperfections which create collateral constraints in borrowing.[1] In these models, the collateral constraint on loans is denominated in something other than the borrowing firm's output, and the risk is that the price of collateral relative to the price of the firm's output will fall. This would tighten the constraint and force the firm to sell collateral in order to repay the debt exceeding the tighter constraint. However, this sale further reduces the collateral price, creating Fisherian debt-price deflation. These models can explain sudden stops of capital inflows, current account reversals, and large relative price changes. They cannot explain default or why some countries face risky and volatile credit while others have

[1] See Radalet and Sachs (1999), Caballero and Krishnamurthy (2001), Aghion, Bacchetta and Banerjee (2001), Chang and Velasco (2002), Arellano and Mendoza (2003), Mendoza and Smith (2006) models of crises motivated by collateral constraints.

safe loans and more stable access.

The purpose of this paper is to offer a fourth type of emerging-market financial-crisis model. We take seriously the idea that investment necessary for growth is risky. Business cycle research views the downside of that risk as a recession. We present a model in which financial market imperfections, likely to characterize emerging markets, can amplify the effects of a negative productivity shock to generate a financial crisis. The model for emerging market economies does not exhibit multiple equilibria, and in the absence of the negative shock, there is no crisis. Therefore, crises are attributable to fundamentals. A crisis is characterized by increasing interest rates and an endogenous sudden stop of capital. Depending on policy, the negative productivity shock and sudden stop of capital can be accompanied by widespread private default and/or a currency crisis. Hence, this model attributes emerging market financial crises to the interaction of negative productivity shocks with the imperfect financial markets which characterize emerging economies. Policy does affect the severity of the crisis, but there is no particular policy, like government bailouts, without which the crisis would not occur. The model therefore does not place blame for financial crises on government policy, but attributes them to the same productivity shocks which create business cycles.

Financial crises in emerging markets are complex affairs, and undoubtedly all four types of models are relevant. This paper fills a gap by considering the implications of standard productivity shocks, which are the cornerstone of business cycle analysis, for financial crises. The financial market imperfections we consider are present in advanced economies, but are less severe, as reflected by different parameter values. We show that these different parameter values divide countries into credit clubs. Countries without access to technology, with low wealth, or with weak domestic capital markets have no access to international credit markets. Technology and a minimum level of initial resources, together with reasonably strong domestic credit markets, give a country access, but that access is volatile and interest rates carry default risk premia. A country acquires stable access to international credit with risk-free loans once resources and the strength of domestic credit markets cross thresholds.

The paper is organized as follows. The next section provides a description of the emerging market economy. It includes a characterization of partial equilibrium in credit markets, conditional on choices for debt, and first order conditions describing optimal behavior by the domestic agent. Section 3 characterizes general equilibrium. Section 4 describes equilibrium debt crises and currency crises. Section 5 considers government policy, and Section 6 contains conclusions.

2 Model of a Small Emerging-Market Economy

2.1 Assumptions

The domestic economy is small and open. The world interest rate (r) is fixed, and foreign creditors are risk-neutral. The world price of the single good is fixed and normalized at unity. The single good rules out changes in a relative price or the real exchange rate as a cause for crises in the model.[2] Agents in the country have access to technology which provides them with a risky investment opportunity with expected returns substantially greater than the

[2] These changes undoubtedly occur and are emphasized in the collateral constraint models. We rule them out here to focus on aggregate productivity shocks.

world interest rate. There are three periods in the model, labeled 0, 1, and 2. Period 0 is a planning period in which agents choose whether or not to undertake the risky investment project. This decision affects the expected values of consumption in periods 1 and 2.

We assume that capital markets are subject to imperfections which are likely to exist in emerging markets. We make assumptions about fundamental credit market imperfections and use those to derive constraints and other imperfections which characterize capital markets in equilibrium. The fundamental imperfections are chosen to yield a particular set of derivative financial market characteristics in equilibrium. These include international borrowing to finance risky investment, the possibility of default in equilibrium, credit ceilings which are increasing both in the strength of domestic capital markets and in expectations of future output, and maturity mismatch on loans and investments.

To generate external financing, we assume that agents in the emerging market have the opportunity to undertake profitable investment, but that they do not have sufficient resources to finance a project without entering international financial markets. Resources include an initial endowment of Y together with small government transfers $\tau_0 \geq 0$. Investment projects are of fixed size, exceeding initial resources $I > Y + \tau_0$. Agents can choose to allocate their resources to the safe international bond (B_0) as an alternative to investment. Agents choose whether or not to invest with the objective of utility maximization. When they choose investment, they must use external financing.

To generate financing with debt contracts subject to ceilings and the possibility of default in equilibrium, we follow the costly state-verification literature and assume that agents cannot commit to repay and that information on the success or failure of the risky investment project is private and accessible to the creditor only after payment of a state-verification fee (Townsend 1979).[3] The optimal contract minimizes payment of the state-verification fee and has characteristics of a standard debt contract.[4] Agents pay an amount agreed ex ante in states for which this quantity is less than output and otherwise surrender output. When agreed repayments, interpreted as debt with interest, are less than output, agents prefer to repay, implying that there is no need to verify output. When output is less than the agreed amount, agents are unable to pay, verification occurs, and agents surrender output. We interpret failure to repay and surrender of output as bankruptcy with default. In equilibrium, there is an upper bound on debt which creditors are willing to extend, assuring that agents with high output have the incentive to repay.

The costly-state-verification model was originally introduced to model credit market imperfections in advanced economies. We modify the model in two ways to generate stronger imperfections in emerging markets. To model credit market strength and generate responsiveness of loan ceilings to the strength of credit markets, we assume that bankruptcy courts cannot award the entire output of a project in the event of a failure to repay. Instead, they award a fraction $\eta < 1$ of output, in contrast to $\eta = 1$ for advanced economies. We interpret a value for η close to one as a relatively strong credit market, likely to characterize an emerging market, while a very small η would characterize an underdeveloped country. We

[3] Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997) present general equilibrium models with asymmetric information and costly state verification in which shocks to firm wealth affect the cost of investing, thereby causing and propagating business cycles.

[4] See Romer (1996) for a presentation of the model and its solution.

show that the magnitude of the equilibrium credit ceiling is directly related to η, implying that stronger credit markets enjoy larger credit ceilings relative to output.

The second modification is made to generate maturity mismatch in loans and investment projects. We assume that agents must present evidence of investment plans before they can receive a loan, eliminating any moral hazard with respect to investment. However, there is moral hazard with respect to the choice of investment projects. Agents choose between a safe one-period investment project and a two-period project with higher expected returns. The safe one-period project yields a fixed quantity of output in period 1 $(Y_1^s > (1+r)(I-Y))$ and no output in period 2. The risky two-period project yields a small fixed quantity of output in period 1 (Y_1), and final output of HI for agents of type h, and LI for agents of type l in period 2. Agents do not know their type until period 2 when output of the risky project is revealed. The fraction of agents with high productivity (ρ) is stochastic with a lower bound of ρ^l. We assume that LI is small relative to HI and represents scrap value of the project. Each project requires the agent's full labor endowment, implying that he can choose only one. We assume that expected output from the risky two-period project is sufficiently greater than output from the safe one-period project that agents who expect to honor debt contracts prefer the risky project.[5]

[5] We need specific assumptions about magnitudes for later proofs. The second-period output of the investment project for $h-$agents is sufficiently high relative to Y_1 and LI that, in the worst state $\left(\rho^l\right)$.

$$\rho^l (HI - LI) > (1+r)(Y_1 - LI),$$

Additionally, we assume that H/L is sufficiently large that

$$HI > \rho^l f\left(\rho^l\right) LI.$$

We elaborate further on magnitude restrictions in the appendix after we solve for equilibrium values.

However, if an agent could finance the one-period project with a two-period loan, he could completely escape debt obligations since there would be no period-2 output to support repayment when the loan matures. When utility with period-1 income of Y_1^s exceeds utility with expected income from the risky project, the agent plans to finance the single-period project with a two-period loan and walk away. The moral hazard problem can be solved with single-period debt contracts. If agents use single-period loans to finance the safe one-period project and fail to repay, creditors can claim ηY_1^s, leaving agents with income of $Y_1^s(1-\eta)$.[6] Under the assumption that utility with income of $Y_1^s(1-\eta)$ is less than expected utility with income from the risky two-period project, the moral hazard problem is solved when creditors offer only single-period loans. Agents choose the risky investment project in equilibrium.

We generate sudden stops in this environment by assuming that in period 1, the market receives a signal on the number, but not the identity of type $h-$agents, given by ρ.[7] To simplify, we assume that the signal is perfect such that aggregate uncertainty is resolved in period 1. We show that in equilibrium, the endogenous credit ceiling for new loans depends on the number of productive agents. A low number of productive agents yields a low credit ceiling, which we interpret as a sudden stop of capital flows. Agents, facing a sudden stop of capital flows, choose whether or not to repay or default on initial loans.

To permit an exchange rate crisis, we introduce a government which is responsible for money (M) and monetary policy. The government sells money to the private sector and uses the proceeds to buy foreign exchange reserves. It also makes monetary transfers $(\tau_0 \geq 0)$.

[6] Since agents must provide credible evidence of investment plans for each single-period loan, the creditor who is not approached for a new loan by the defaulting client, will claim the bankruptcy settlement.
[7] Think of this as general information on the state of the economy.

Money, backed by foreign exchange reserves, can serve as a hedge asset to support consumption in the event of a sudden stop of capital flows. To allow a currency crisis to accompany a sudden stop of capital in the model, we make assumptions which assure that, in an equilibrium with risky investment, agents choose domestic money as the hedge asset. Following Obstfeld and Rogoff (1996), we assume that all international bonds would be confiscated in the event of bankruptcy, and that the holding cost of foreign currency is prohibitively high, such that agents do not choose to hold foreign currency.[8] Additionally, we assume that the government has a fixed-price-level target and maintains a fixed exchange rate system until its reserves are exhausted, allowing interpretation of depreciation as a failure of the fixed exchange rate. These assumptions, restricting possible hedge assets and fixing exchange rates, are not necessary to generate sudden stops of capital with the possibility of default, but they are necessary to introduce the possibility of a currency crisis accompanying a sudden stop.

The introduction of money requires some assumption about the denomination of the private debt contracts (D_t). If debt contracts were denominated in domestic currency, then the domestic government could completely inflate away their residents' debt, implying that international creditors could not expect to receive the market rate of return. Since the government has no way to commit not to inflate, especially in a bad state, in equilibrium debt will be denominated in foreign currency.

[8] The government can hold interest-bearing foreign bonds as foreign exchange reserves, and these are not confiscated in the event of private default. Therefore, it is optimal for agents to hold domestic money, backed by interest-bearing international bonds, instead of foreign currency, justifying the imposition of prohibitively high holding costs. These could be something like failure to provide implicit or explicit backing for domestically-held foreign-currency deposits or rules which prevent banks from accepting foreign-currency deposits from domestic residents.

There is no role for government spending in the model, so we assume that spending is zero. Additionally, since the model is about private debt, not government debt, we assume that the government cannot borrow internationally, implying that there is no sovereign debt. And, finally, since productivity across agents differs and the model is not about risk-sharing across agents, we assume that the government redistributes income using the tax system in period 2 (τ_2) to achieve perfect risk-sharing across agents.[9]

Agents maximize expected utility, which depends on consumption and real money balances in periods 1 and 2. Since agents are identical until the final period, they make the same decisions in periods 0 and 1. Assuming that agents have unit mass, expected utility can be expressed as

$$\int_{\rho^l}^{1}\left[\ln c_1+\theta\ln\frac{M_0}{P_1}+\beta(\rho\ln c_{2h}+(1-\rho)\ln c_{2l}+\theta\ln\frac{M_1}{P_0})\right]f\left(\rho\right)d\rho. \tag{1}$$

where $P_2 = P_0$ since the government can set period-2 taxes to achieve its fixed-price target in any state. In equation (1), β is the discount factor, assumed to equal the inverse of the world gross interest rate ($\beta = (1+r)^{-1}$), θ, restricted by $0 < \theta < \frac{r}{1+r}$, is a fixed parameter, $f(\rho)$ is the density function for the number of high-productivity agents, c denotes consumption, subscripts 1 and 2 denote periods, and subscripts h and l denote the high-productivity and low-productivity agent respectively.

[9] This requires that the government can learn the identity of agents without paying the state-verification fee.

It is convenient to represent a time line for the economy as follows

Period 0	Period 1	Period 2
receive transfers (τ_0)	learn ρ and debt ceiling	realize output, agents learn identity
choose investment project	receive transfers (τ_1)	creditors pay verification fees
choose $\frac{M_0}{P_0}$, implying D_0	choose to repay or default	l-agents default and pay ηLI
	choose M_1 and c_1	h-agents pay $min[\eta HI,\ (1+r_1)D_1]$
	currency crisis realized or not	consume, c_2, and pay taxes $\tau_2 < 0$

2.2 Partial Equilibrium in International Credit Markets

This section characterizes a partial equilibrium in an emerging market economy, in which two-period risky investment projects are financed by single-period international loans, provided by risk-neutral foreign creditors. We define an emerging economy as one which needs external financing for investment and for which η is less than unity, but not too small, as quantified below. Conditional on the agent's choice of values for D_0 and D_1, we solve for equilibrium values of the interest rates. The interest rate is the price at which risk-neutral international creditors would be willing to supply different quantities of loans. We show that when D_0 and D_1 exceed certain values, there is no equilibrium, implying upper bounds. In the general equilibrium, defined subsequently, the agent's choice for debt is endogenous and depends on the interests rates required for different quantities of debt.

A partial equilibrium in credit markets is defined for given values of D_0 and D_1, as interest rates in each period, $\{r_0,\ r_1\}$ and debt ceilings in each period $\left\{\bar{D}_0,\ \bar{D}_1\right\}$ such that risk-neutral international creditors willingly provide loans when they expect to receive the risk-free interest rate, and consumers choose between repayment or surrender of bankruptcy awards in order to maximize utility.

2.2.1 Period-1 Debt

Working backwards, consider the equilibrium values for the debt ceiling and interest rate in period 1. At the beginning of period 1, the market receives a signal on the number of productive agents (ρ), but agents do not learn their own identity. We interpret failure to repay contractual debt with interest as default. As in the costly-state-verification literature, agents with low productivity, l-agents, will default on their period-1 debt (D_1) in period 2 since their debt obligations exceed the bankruptcy settlement. Creditors offer new period-1 debt (D_1) up to a ceiling at which debt repayments equal the compensation $h-$agents must pay creditors in the event of default. This assures that $h-$agents have the incentive to repay in period 2, and implies a ceiling on period-1 debt given by

$$(1+r_1)\,D_1 \leq (1+r_1)\,\bar{D}_1 = \eta HI, \tag{2}$$

where $\bar{D}_1$ is the ceiling on period-1 debt.

The equilibrium interest rate on period-1 debt depends on the realization of ρ. Assuming that loans to the emerging economy's agents can be pooled to yield a risk-free asset, arbitrage requires that the period-1 interest rate equate the payments from loans to agents in the emerging market with the payments on the same loans in the risk-free international market. Given the debt ceiling, the ρ agents of type h always pay $1+r_1$ on D_1, and the $1-\rho$ agents of type l pay ηLI after the international creditor pays the state-verification fee of ϖ. Therefore, arbitrage requires

$$(1+r)\,D_1 = \rho\,(1+r_1)\,D_1 + (1-\rho)\,(\eta LI - \varpi)\,.$$

Simplifying by assuming that $\eta LI = \varpi$, the period 1 gross interest rate is given by

$$1 + r_1 = \frac{1+r}{\rho}. \tag{3}$$

A bad signal about the number of productive agents, represented by a low value for ρ, raises interest rates because only agents of type h will repay. Therefore, interest rates are rising as the fraction of $h-$agents, given by ρ, falls.

Equations (2) and (3) can be solved for the period-1 debt ceiling $\left(\bar{D}_1\right)$ to show that it is increasing in both in the number of productive agents (ρ) and in the fraction of output that creditors can claim in the event of default (η),

$$\bar{D}_1 = \frac{\rho\eta HI}{1+r}. \tag{4}$$

A low realization for ρ implies a low debt ceiling and has the interpretation of capital flight. The assumption that η is relatively large in emerging markets is interpreted as requiring that η be large enough that the debt ceiling is increasing in ρ faster than desired debt. This implies that the debt ceiling is more likely to bind for a low realization of ρ.

Now, consider the agent's default decision in period 1, on debt taken out in period 0, D_0, after information on ρ is revealed. Let D_1 represent the agent's choice for period-1 debt. When D_1 is less than the debt ceiling $\left(\bar{D}_1\right)$, default is not an issue. This is because for $D_1 \geq (1+r_0)\,D_0$, the agent can repay existing loans with new loans, and for

$D_1 < (1 + r_0) D_0$, the agent reduces his debt by repaying it. When debt is constrained by the ceiling and $\bar{D}_1 \geq (1 + r_0) D_0$, the agent can still repay with new loans, implying that he does not default.[10] However, when $\bar{D}_1 < (1 + r_0) D_0$, default becomes a possibility. The agent chooses between default and repayment to maximize utility, conditional on bankruptcy awards.

These awards depend on the creditor's reaction to the agent's failure to repay. The creditor has two choices. He could claim bankruptcy awards in period 1 of ηY_1. Alternatively, he could roll over the loan, effectively offering the agent period-1 debt equal to $(1 + r_0) D_0 > \bar{D}_1$. This would give the agent a period-2 choice of repaying $(1 + r_1)(1 + r_0) D_0$, or surrendering ηHI, if he is an $h-$agent, or ηLI, if he is an $l-$agent. Since bankruptcy awards would be less than debt with interest (since debt exceeds the ceiling), both types of agents would declare bankruptcy in period 2. Since awards are conditional on output, the international creditor would verify output and pay state verification fees. Given the assumption that $\varpi = \eta LI$, the present value of the net bankruptcy awards to the creditor from rolling over debt is given by $\frac{\rho\eta(HI-LI)}{1+r}$, and the net value of immediate bankruptcy awards is $\eta (Y_1 - LI)$. Given the assumption above that HI is large relative to Y_1, the creditor prefers to roll over the loan, rescheduling debt in response to the default.[11]

Lemma 1 *Given initial debt with interest,* $(1 + r_0) D_0$, *there is a critical value of* $\rho = \rho^d$, *below which agents choose default and above which agents choose repayment.*

[10]Even if he did not repay, the creditor would reduce net new loans so that they did not exceed the debt ceiling, and there would be no other consequences.

[11]Recall that if there were any agents who chose a single-period project, these agents would have no investment project, thereby revealing their identity. The creditor would treat these agents differently, claiming bankruptcy awards of ηY_1^s instead of rolling over the loan. Given that agents know the creditor will do this, they prefer the two-period investment project, and no agents will choose the single-period project in equilibrium.

Proof. Let the realization for ρ be large such that $\frac{\rho\eta HI}{1+r} > (1+r_0)\, D_0$. Agents use new debt to repay old, and default has no meaning. Let the realization for ρ be small such that $\frac{\rho\eta HI}{1+r} < (1+r_0)\, D_0$. Default in period 1 does not affect the expected value of debt repayments in period 2 because both agents will default and surrender η of output. Default increases current resources since debt repayments exceed the current debt ceiling. With constrained debt, current resources are more valuable than future resources for consumption smoothing. Agents prefer default. ■

Therefore, agents default when the upper bound on period-1 debt, given by $\frac{\rho\eta HI}{1+r}$ from equation (4), is less than repayment of principle and interest on initial debt. In equilibrium, for a given value of $(1+r_0)\, D_0$, the critical value of ρ below which agents default in period 1, ρ^d, is given by

$$\rho^d = \max\left[\frac{(1+r)\,(1+r_0)\,D_0}{\eta HI}, \rho^l\right]. \tag{5}$$

When ρ^d takes on the value of its lower support, given by ρ^l, there is no value of ρ for which agents would default, and loans are perfectly safe.

These results imply that there is a critical value for initial debt $\left(D_0^R\right)$, above which debt is risky and below which debt is safe. Using equation (5) and setting $r_0 = r$ at the critical value yields

$$D_0^R = \frac{\rho^l \eta HI}{(1+r)^2}. \tag{6}$$

When $D_0 \leq D_0^R$, substituting into equation (5) yields $\rho^d = \rho^l$, implying that debt is perfectly safe since no value for ρ could elicit default.

2.2.2 Initial-Period Debt

Now, consider the equilibrium interest rate on loans made in period 0 (D_0), when there is uncertainty regarding ρ. If agents choose default in period 1, creditors claim η of second period output at the cost of the verification fee of ϖ. Given risk-neutral creditors, the value of expected debt repayments on the risky loan must equal the value of debt repayments on a safe international loan. With the simplifying assumption that $\eta LI = \varpi$, the period-0 interest rate must satisfy

$$(1+r)\,D_0 = (1+r_0)\,D_0 \int_{\rho^d}^{1} f(\rho)\,d\rho + \int_{\rho^l}^{\rho^d} \left(\frac{\eta\,(HI-LI)}{1+r}\right) \rho f(\rho)\,d\rho,$$

where the first integral represents the probability that agents repay in period 1 and the second represents the expected present value of net repayments in period 2, arising from default and rescheduling in period 1. Solving for contractual debt repayments $[(1+r_0)\,D_0]$ as a function of D_0 and ρ^d yields

$$(1+r_0)\,D_0 = \frac{(1+r)\,D_0 - \int_{\rho^l}^{\rho^d} \left(\frac{\eta(HI-LI)}{1+r}\right) \rho f(\rho)\,d\rho}{\int_{\rho^d}^{1} f(\rho)\,d\rho}. \tag{7}$$

Equations (5) and (7) constitute a pair of non-linear equations which can be used to determine equilibrium values for r_0 and ρ^d for a given value of initial debt, D_0. Linearization would distort a central feature of the equilibrium, which we want to emphasize, and we therefore characterize the solution graphically.[12]

[12]The feature is the upper bound on debt.

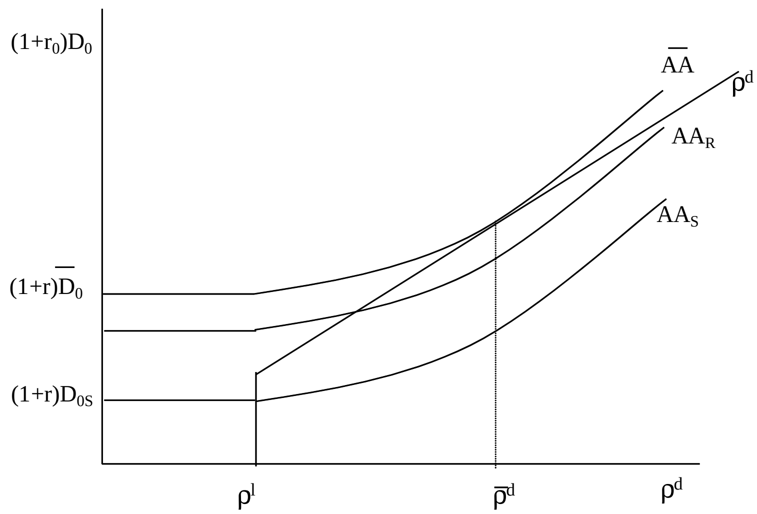

Figure 1: Financial Markets

In Figure 1, the curve labeled ρ^d plots the equilibrium relationship between the value of ρ which elicits default, given by ρ^d, and contractual debt obligations, given by $(1 + r_0)\, D_0$, from equation (5). Since ρ^l is the lower support of the distribution, there is a range of values for $(1 + r_0)\, D_0$ for which no value of ρ would elicit default, implying that for low contractual debt obligations, ρ^d is vertical at ρ^l. As $(1 + r_0)\, D_0$ increases, equation (5) implies a positive linear relationship between $(1 + r_0)\, D_0$ and ρ^d.

The curves labeled AA plot the arbitrage relationship in equation (7) between ρ^d and $(1 + r_0)\, D_0$, for different values of period-0 debt (D_0). For values of $\rho^d \leq \rho^l$, there is no risk of default and the interest rate equals the world rate, implying that the AA curves have intercepts at $(1 + r)\, D_0$ and that they are horizontal for $\rho^d \leq \rho^l$. Higher initial debt is represented by curves with higher intercepts. For $\rho^d > \rho^l$, equation (7) implies that $(1 + r_0)\, D_0$ rises at an increasing rate as ρ^d rises, requiring that the AA curves have an increasing slope.

The first intersection of a particular AA curve with the ρ^d curve is a stable equilibrium and gives the equilibrium values for ρ^d and r_0 for a given value of $D_{0.}$ The curve labeled AA_S is drawn for a low value of initial debt (D_{0S}). It intersects the ρ^d curve along the vertical portion, implying that this level of debt is perfectly safe, hence the subscript S for safe. An increase in D_0 shifts the AA curve upwards. If the increase is large enough that the AA curve no longer intersects the ρ^d curve along the vertical portion, then this level of debt is risky. Both r_0 and ρ^d rise until the values of $(1+r_0)\,D_0$ and ρ^d are given by the intersection of the AA and ρ^d curves for the new level of D_0.

Note that there is a value for initial debt, given by $\bar{D}_0$, such that for $D_0 > \bar{D}_0$, there is no equilibrium. This establishes the upper bound on D_0 as the largest value for D_0 such that a solution to the equations (5) and (7) exists. Equivalently, $\bar{D}_0$ is the value of D_0, which yields a tangency between the two curves. The value of ρ^d for $D_0 = \bar{D}_0$ is labeled $\bar{\rho}^d$, and represents the largest feasible value for ρ^d.

An increase in the proportion of output awarded in bankruptcy, given by η, flattens the AA curve and increases the slope of the ρ^d curve, implying that the tangency occurs for a higher value of debt. Stronger credit markets are associated with higher debt ceilings.

Totally differentiating equations (5) and (7) with respect to ρ^d, r_0, and D_0, for $\rho^d > \rho^l$, yields the equilibrium relationship between ρ^d and D_0, allowing the interest rate to adjust, and the equilibrium relationship between r_0 and D_0, allowing ρ^d to adjust, respectively, as

$$\frac{d\rho^d}{dD_0} = \frac{(1+r)^2}{\eta HI \int_{\rho^d}^{1} f(\rho)\,d\rho - \rho^d \eta LI f(\rho^d)} > 0 \qquad D_0 < \bar{D}_0, \tag{8}$$

$$\frac{dr_0}{dD_0} = \frac{\int_{\rho^l}^{\rho^d}\left(\frac{\eta(HI-LI)}{1+r}\right)\rho f(\rho)\,d\rho + (1+r_0)\,\rho^d f\left(\rho^d\right)\eta LI}{D_0\left[\eta HI\int_{\rho^d}^{1} f(\rho)\,d\rho - \rho^d\eta LIf\left(\rho^d\right)\right]} > 0 \qquad D_0 < \bar{D}_0. \tag{9}$$

The signs reflect the choice of the stable equilibrium at the lower value of ρ^d, together with assumption that H is large relative to L so that the denominator is positive at $\rho^d = \rho^l$. As ρ^d rises (in response to the increase in D_0), the value of the denominator shrinks, eventually reaching zero as D_0 reaches $\bar{D}_0$, with ρ^d reaching $\bar{\rho}^d$. Equilibrium values for ρ^d and r_0 are rising in D_0 at increasing rates, with the rate of increase reaching infinity at upper bounds.

To summarize, the resulting equilibrium in credit markets has characteristics of credit markets in emerging economies. Creditors offer single-period debt contracts to finance longer-term investment, yielding maturity mismatch. Equilibrium interest rates are increasing in the magnitude of the loan. Creditors impose endogenous credit ceilings, conditional on awards they would receive in bankruptcy court in the event of default. The period-1 credit market can be characterized by binding debt ceilings which fluctuate with news, yielding sudden stops in the event of bad news. In contrast to the costly state-verification models of developed economies (Bernanke and Gertler 1989 and Carlstrom and Fuerst 1997), in which shocks primarily affect the demand for loans and investment by affecting costs, shocks affect the availability of credit.

2.3 Budget Constraints

2.3.1 Period 0

In period 0, the government sells nominal money to agents and issues monetary transfers (τ_0). Goods and foreign exchange, received in exchange for money net of transfers, are used to buy the international bond for use as foreign exchange reserves (F_0). The government's period zero budget constraint is therefore given by

$$F_0 = \frac{M_0}{P_0} - \tau_0 > 0. \tag{10}$$

The assumptions that the government maintains fixed exchange rates in good states and that it cannot borrow on foreign markets requires that initial transfers are small enough to assure $F_0 > 0$. Note that $\tau_0 = 0$ implies that money is completely backed by foreign exchange reserves, as with a currency board.

In period 0 agents receive an endowment (Y) and exogenous government transfers (τ_0). They use these together with new debt (D_0) to finance the investment project (I) and to acquire real money balances $\left(\frac{M_0}{P_0}\right)$ and the safe international bond (B_0).[13]

$$Y + \tau_0 + D_0 = I + \frac{M_0}{P_0} + B_0. \tag{11}$$

Given that international bonds would be confiscated in the event of default, that investment is relatively more profitable than the safe international bond, and that the agent must borrow at a risk-adjusted interest rate if he chooses to invest, an equilibrium with investment implies that the agent chooses $B_0 = 0$.

Consider the effect of the upper bound on debt for an agent's access to international credit. Equation (11) with $B_0 = 0$ together with the upper bound on debt implies that in an equilibrium with investment

$$D_0 = I - Y + \frac{M_0}{P_0} - \tau_0 \leq \bar{D}_0. \tag{12}$$

[13]The assumption that holding costs of foreign currency are prohibitively high implies that agents do not hold foreign currency.

Proposition 1 *A country with $I - Y \geq \bar{D}_0$ does not have access to international financial markets.*
Proof. *Equations (11) and (10), together with assumptions, yielding $F_0 > 0$, imply that a country with $I - Y \geq \bar{D}_0$ would require $D_0 > \bar{D}_0$.* ■

Therefore, countries with too few resources relative to the size of investment projects find that they cannot access credit markets. Very poor countries cannot borrow enough to engage in risky investment projects because the risk of default would be too high. Additionally, since $\bar{D}_0$ is increasing in η, a stronger credit market can allow access for a given value of $I - Y$.

2.3.2 Period 1

In period 1, the government receives interest (r) on foreign exchange reserves and distributes the interest to agents as transfer payments $(\tau_1 = rF_0)$. Additionally, the government buys domestic currency using foreign exchange reserves at the pegged exchange rate up until the point at which it exhausts reserves. Therefore, the period-1 government budget constraint becomes

$$F_1 = \max\left\{(1+r)\,F_0 - \tau_1 + \frac{M_1}{P_0} - \frac{M_0}{P_0} = \frac{M_1}{P_0} - \tau_0,\ 0\right\}. \tag{13}$$

The max is necessary because foreign exchange reserves are constrained to be non-negative. That is, the government's reserves evolve according to the first term in equation (13) as long as $F_1 \geq 0$. The second equality comes from using the government's period-0 budget constraint to substitute for $\frac{M_0}{P_0}$. Once reserves are exhausted, the government is no longer able to buy money in exchange for foreign exchange reserves, limiting the possible reduction in the quantity of money. This implies that $\frac{M_1}{P_0}$ has a lower bound at τ_0, where $F_1 = 0$. If foreign exchange reserves are exhausted in period 1, then the price level and the exchange rate become endogenous.

Now, consider the period-1 budget constraint for the agent. In period 1, a value for ρ is realized, but agents do not learn their own identity. When $\frac{\rho\eta HI}{1+r} \geq (1+r_0) D_0$, agents can increase debt and use these resources, together with period-1 output (Y_1), government transfers $(\tau_1 = rF_0)$, and real money acquired in the planning period $\left(\frac{M_0}{P_1}\right)$, to consume (c_1) and to hold real money balances $\left(\frac{M_1}{P_1}\right)$. When $\frac{\rho\eta HI}{1+r} < (1+r_0) D_0$, Lemma (1) states that the agent defaults. In response, the international creditor rolls over the loan so that $D_1 = (1+r_0) D_0$, implying no change in the agent's outstanding debt.

Since all agents are alike in period 1, they cannot increase aggregate consumption above current output without exchanging domestic money for foreign exchange and purchasing foreign goods. Each agent is allowed to exchange money balances for foreign exchange up until the point at which authorities exhaust their foreign exchange reserves. When there are insufficient reserves, we assume that agents each receive an equal per capita share of foreign exchange reserves. The agent acts as though any additional transactions, exceeding the agent's share of foreign exchange reserves, must be carried out at the new equilibrium price level (P_1).[14]

Letting ζ be an indicator function, which takes on the value of unity in the absence of

[14]Agents face two prices in period 1 when the fixed exchange rate fails. To the extent that agents can exchange domestic money for foreign exchange at the fixed exchange rate of P_0, they can use the proceeds of the exchange to buy goods for $P_0 \cdot P^*$,where $P^* = 1$ is the foreign-currency price of goods. To the extent that they cannot get as much foreign currency as they want goods, agents act as though other transactions must be made at the new equilibrium price of P_1. However, since all agents are alike, there are no agents willing to trade goods for money. We show below that the higher price is necessary in equilibrium to convince agents to postpone consumption.

default and zero otherwise, the agent's period-1 budget constraint is given by[15]

$$c_1 = Y_1 + \zeta\left[D_1 - (1 + r_0)\, D_0\right] + (1 + r)\, F_0 + \left(\frac{M_0}{P_0} - F_0\right)\frac{P_0}{P_1} - \frac{M_1}{P_1}. \tag{14}$$

2.3.3 Period 2

The government does not carry reserves forward in the final period because the reserves would have no value. Therefore, the government uses any remaining foreign exchange reserves together with period-2 taxes $(-\tau_2)$ to buy the remaining money supply at $P_2 = P_0$. The period-2 budget constraint is given by

$$F_2 = 0 = (1 + r)\, F_1 - \frac{M_1}{P_0} - \tau_2. \tag{15}$$

Using equations (13) and (15), period-2 transfers can be expressed as

$$\tau_2 = r\frac{M_1}{P_0} - (1 + r)\, \tau_0. \tag{16}$$

In period 2, the $h-$agent receives output of HI. He pays the minimum of ηHI and $(1 + r_1)\, D_1$ to international creditors. If he defaulted in period 1, effectively forcing international creditors to roll over period-0 debt, then bankruptcy awards are smaller than debt obligations. He uses resources, net of debt repayments or bankruptcy awards, together with money to consume and to pay taxes, based on his income (τ_{2h}). His period-2 budget constraint is given by

$$c_{2h} = \zeta\left[HI - (1 + r_1)\, D_1\right] + (1 - \zeta)(1 - \eta)\, HI + \frac{M_1}{P_0} + \tau_{2h}. \tag{17}$$

[15]The agent could choose to buy international bonds, but he will not do so in equilibrium since this would require additional borrowing at a risk-adjusted interest rate and since international bonds have no hedge value in period 1. He will not hold foreign currency since it yields no utility and has high holding costs.

The $l-$agent defaults in period 2 because his debt exceeds liabilities assessed by the bankruptcy court. Therefore, he receives output LI and pays ηLI to international creditors. He uses these net resources together with money to consume and to pay taxes (τ_{2l}), based on his income. His budget constraint is given by

$$c_{2l} = (1-\eta) LI + \frac{M_1}{P_0} + \tau_{2l}. \tag{18}$$

Agents understand that the government chooses income-specific tax and transfer rates to equalize income net of debt repayments across agents subject to a constraint that aggregate transfers satisfy

$$\rho\tau_{2h} + (1-\rho)\tau_{2l} = \tau_2.$$

This implies that period-2 budget constraints for agents are equivalent and are given by

$$c_2 = \rho\left\{\zeta\left[HI - (1+r_1) D_1\right] + (1-\zeta)(1-\eta) HI\right\} + (1-\rho)(1-\eta) LI + \frac{M_1}{P_0} + \tau_2. \tag{19}$$

2.4 First Order Conditions

In making decisions, agents take taxes, foreign exchange reserves, interest rates, and prices as given. The problem is solved backwards, beginning with choices made in period 1.

In period 1, uncertainty about the number of $h-$agents is resolved with a realization of ρ, but agents do not learn their own identity. They choose period-1 debt (D_1), period-1 consumption (c_1), real money $\left(\frac{M_1}{P_1}\right)$, and whether or not to default, $\zeta \in \{0,1\}$, to maximize expected utility, given the current value of initial money $\left(\frac{M_0}{P_1}\right)$, subject to budget constraints given by (14) and (19) and the inequality constraint, given by equation (2).

Consider the agent's choice for first-period debt in the case in which he chooses to repay period-0 debt in period 1 $(\zeta = 1)$, giving him the right to increase debt at the period-1 interest rate. The Euler equation has an inequality depending on whether or not the debt ceiling in period 1 is binding

$$\frac{1}{c_1} \geq \frac{\beta\rho\,(1+r_1)}{c_2} = \frac{\beta\,(1+r)}{c_2}, \tag{20}$$

where the equality uses equation (3) to substitute for $(1+r_1)$. Given the assumption that $\beta = (1+r)^{-1}$, equation (20) implies that, when the debt ceiling does not bind, agents choose equal consumption across periods. Otherwise, second-period consumption exceeds first period consumption with debt given by the ceiling in equation (4).

Now, consider the agent's choice for period-1 real money balances $\left(\frac{M_1}{P_0}\right)$. Period-1 money is chosen to maximize utility, given by equation (1) with known ρ, subject to the budget constraints, equations (14) and (19). Imposing $\beta\,(1+r) = 1$, the first order condition can be expressed as

$$\frac{M_1}{P_0}\left[\frac{(1+r)\,P_0}{P_1 c_1} - \frac{1}{c_2}\right] = \theta. \tag{21}$$

When the Euler equation (20) holds with equality, the money demand function reduces to its familiar form in which expenditures on money are proportional to consumption.

$$\theta c_1 = \frac{M_1}{P_0}\left[(1+r)\,\frac{P_0}{P_1} - 1\right],$$

where P_0 is the price level in period 2, giving $(1+r)\,\frac{P_0}{P_1}$ the interpretation of the gross nominal interest rate. When agents cannot smooth consumption using debt, so that equation (20) holds with inequality $(c_2 > c_1)$, they can still obtain some consumption-smoothing using real money balances. When $\frac{P_0}{P_1} = 1$, agents could reduce $\frac{M_1}{P_0}$ by a large enough amount to raise c_1 and reduce c_2 until they are equal, but they will not choose to do this. Money directly yields utility, and the utility value of complete consumption smoothing is not large enough for agents to sacrifice the utility from real money balances required for complete consumption smoothing. Note that real money balances are lower the larger the difference in consumption across periods because agents do sacrifice some utility from money to gain utility from consumption-smoothing.

Now, consider the choices made in the planning period. In period 0, the agent chooses initial money balances $\left(\frac{M_0}{P_0}\right)$ implying a desired level of initial debt, D_0, subject to the upper bound $\bar{D}_0$. In choosing initial money, he takes interest rates, prices, and government transfers from foreign exchange reserves as given. He also considers how initial money will affect future choices which determine expected utility from consumption and money balances over the next two periods. The first-order condition on $\frac{M_0}{P_0}$ is given by[16]

$$\int_{\rho^l}^{1} \left\{ \frac{1}{c_1} \left[-\zeta\left(1 + r_0\right) + \frac{P_0}{P_1} \right] + \theta \frac{P_0}{P_1} \frac{P_0}{M_0} \right\} f\left(\rho\right) d\rho \geq 0. \tag{22}$$

$$\left\{ \int_{\rho^l}^{1} \left\{ \frac{1}{c_1} \left[-\zeta\left(1 + r_0\right) + \frac{P_0}{P_1} \right] + \theta \frac{P_0}{P_1} \frac{P_0}{M_0} \right\} f\left(\rho\right) d\rho \right\} \left\{ \frac{\bar{M}_0}{P_0} - \frac{M_0}{P_0} \right\} = 0, \tag{23}$$

where equation (11) determines

$$\frac{\bar{M}_0}{P_0} = \bar{m}_0 = \bar{D}_0 - I + Y + \tau_0. \tag{24}$$

The net marginal cost of holding money is the negative of the first term in equation (22). It represents the consumption value of the cost of the increase in debt necessary to hold the

[16]Note that the coefficient on the partial derivatives of $\frac{M_1}{P_0}$ and D_1 with respect to $\frac{M_0}{P_0}$ are zero by first order conditions, given by equations (21) and (20) respectively.

additional money, integrated over all values of ρ. For states in which debt is repaid $(\zeta = 1)$, the net marginal cost is given by $\frac{1}{c_1}\left(r_0 + \frac{P_1 - P_0}{P_1}\right)$, which is approximately the consumption value of the nominal interest rate. For states in which debt is not repaid $(\zeta = 0)$, the net marginal cost is $-\frac{P_0}{P_1 c_1}$, indicating that holding additional money yields benefits only since money is used to increase consumption at the period-1 price level and debt issued to acquire money is not repaid. The net marginal cost is the integral of these marginal costs over all states. The marginal benefit of additional money is the utility value of money, given by the last term. The inequality reflects the presence of the upper bound on debt implying an upper bound on money.

When parameters take on values for which there is no probability of either default or currency crisis in equilibrium, $\rho^d = \rho^l$, and $\zeta = 1$ and $P_0 = P_1$ in all states, simplifying the first order condition to the standard one given by

$$r \int_{\rho^l}^{1} \frac{1}{c_1} f(\rho)\, d\rho = \theta \frac{P_0}{M_0}. \tag{25}$$

3 General Equilibrium

3.1 Definition

We consider an equilibrium for parameter values such that agents choose the risky two-period investment project. Given government policy, including values for τ_0 and η, the distribution of ρ, and a realization for ρ in period 1, equilibrium is defined as the set of values for consumption and real money balances in each period, foreign exchange reserves in each period, relative prices, interest rates, debt ceilings, period-2 taxes, and a value for

ρ^d, $\left\{c_1,\ c_{2,}\ \frac{M_0}{P_0},\ \frac{M_1}{P_0},\ \frac{P_0}{P_1}, r_0,\ r_{1,}\ \bar{D}_0,\ \bar{D}_1,\ F_0,\ F_1,\ \tau_2,\ \rho^d\right\}$, for which arbitrage conditions on interest rates (equations 3 and 7) and the period-1 debt ceiling (equation 4) are satisfied, government budget constraints hold with positive period-0 foreign exchange reserves and non-negative period-1 reserves (equations 10, 13, and 15), agent budget constraints (equations 14 and 19) and first order conditions (equations 20, 21, 22, and 23)) hold, agents choose default optimally (equation 5), and expectations are rational.

3.2 Equilibrium Debt and Interest Rates

Equations (5) and (7) allow characterization of $(1 + r_0)\, D_0$ and ρ^d as a function of the agent's choice for D_0. These equations also imply an upper bound on initial debt. In this section, we characterize the general equilibrium by solving for the agent's choice for initial debt, D_0. Consider the demand for initial debt in a country for which $\bar{D}_0 > I - Y - \tau_0 > 0$. Given that we are focusing on an equilibrium in which agents choose the risky investment project, they must borrow at least $I - Y - \tau_0$. Given that money yields utility, they will choose to borrow more to hold real money balances. Hence we focus on the demand for initial real money as the determinant of D_0. This is given by the solution to equations (22) and (23).

The objective is to characterize money demand, and hence D_0, as a function of the interest rate (r_0) and the critical value of ρ eliciting default $\left(\rho^d\right)$. Using equation (22), Figure 2 graphs the marginal benefits of money, $\int_{\rho^l}^1 \left\{\theta \frac{P_0}{P_1} \frac{P_0}{M_0}\right\} f(\rho)\, d\rho$, and the net marginal costs, $\int_{\rho^l}^1 \left\{\frac{1}{c_1}\left[\zeta(1 + r_0) - \frac{P_0}{P_1}\right]\right\} f(\rho)\, d\rho$, as a function of initial real money balances $\left(m_0 = \frac{M_0}{P_0}\right)$, allowing values for ρ^d and r_0 to change according to equations (8) and (9), with $\frac{dm_0}{dD_0} = 1$.

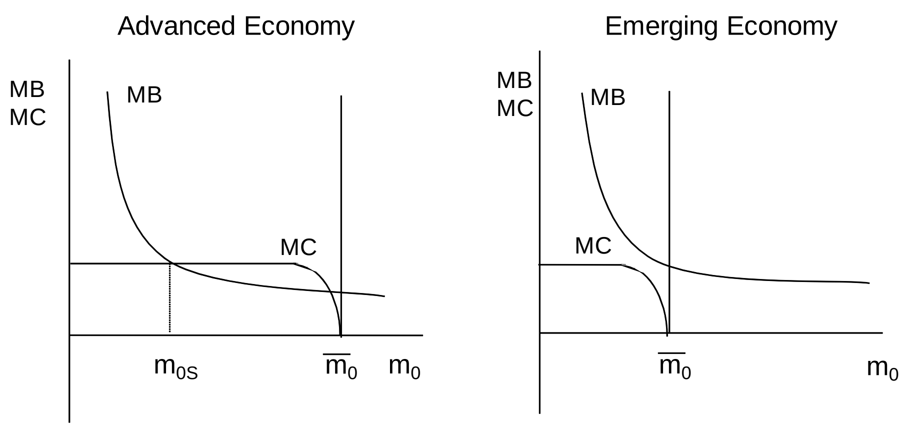

Figure 2: Equilibrium Initial Money

Marginal benefits of money, defined as the additional utility of holding additional money and labeled MB, are independent of r_0 and ρ^d. Given assumptions on the utility function, marginal benefits are falling at a decreasing rate as real money increases. Marginal benefits are drawn as identical in both panels of Figure 2.

The marginal cost of money is the opportunity cost of holding money. Simplifying by assuming that $\tau_0 = 0$, so that prices are constant in all states, net marginal costs can be expressed as[17]

$$MC = \int_{\rho^d}^{1} \frac{r_0}{c_1} f(\rho)\, d\rho - \int_{\rho^l}^{\rho^d} \frac{1}{c_1} f(\rho)\, d\rho, \tag{26}$$

with ρ^d given by equation (5). For low levels of m_0, D_0 is low and the results above imply that $r_0 = r$ and $\rho^d = \rho^l$. Small increases in money keep D_0 low enough that r_0 and ρ^d do not change from their risk-free values. Agents incur the interest rate as the opportunity cost of holding additional money, but they receive this back in equilibrium as lump-sum transfers of

[17]We show below that this is sufficient to fix prices in all states. The implications allowing $\tau_0 > 0$, are not very different as long as the government maintains a fixed exchange rate in good states as assumed. This prevents relative prices from moving too far from unity even in bad states.

interest earnings on foreign exchange reserves, keeping the value of equilibrium consumption constant. Therefore, for low values of m_0, the second term in equation (26) is zero, and marginal cost is positive and constant as m_0 increases.

As m_0 continues to increase, D_0 reaches a critical value, given by D_0^R in equation (6), at which r_0 and ρ^d increase. When m_0 takes on a value such that $D_0 = D_0^R$, marginal costs fall because in states with $\rho < \rho^d$, positive values for marginal costs $\left(\frac{r_0}{c_1}\right)$ are replaced with negative values $\left(-\frac{1}{c_1}\right)$. Since D_0^R is increasing in the strength of credit markets (η), marginal costs fall at a lower value of debt (and money) for the emerging economy than for the advanced economy.

For values of initial money such that $D_0^R < D_0 < \bar{D}_0$, increases in m_0 cause both r_0 and ρ^d to rise at increasing rates. The increase in r_0 and the adjustment of c_1 to increasing m_0 increase the marginal cost of money in states for which $\rho > \rho^d$. The increase in ρ^d replaces states in which agents repay and incur positive marginal cost of money with states in which agents default and incur negative marginal costs, reducing marginal costs. We argue in the appendix that in this range, the marginal cost of money is falling in m_0 for reasonable parameter values.

The rate of change of marginal costs reaches negative infinity at the value for m_0 for which $\frac{d\rho^d}{dD_0} \to \infty$, equivalently at the upper bound on m_0. In Figure 2, the upper bound on money, labeled $\bar{m}_0$, is drawn as occurring at a lower value for m_0 in the panel labeled "Emerging Economy" than in the panel labeled "Advanced Economy". The upper bound on money for an advanced economy is higher because that economy has higher income, and therefore less need to borrow for investment, and a stronger credit market, reflected by a larger η.

To summarize, MC is initially flat in m_0. At the critical value of m_0 at which debt becomes risky, marginal costs begin falling, and the rate of change of marginal costs reaches negative infinity at $m_0 = \bar{m}_0$.

Equilibrium real money must satisfy equations (22) and (23). This requires either that net marginal costs equal marginal benefits at a value of money less than the maximum or that marginal benefits be greater than or equal to marginal costs at the maximum. Since the rate of change of net marginal costs approaches negative infinity at the upper bound, there is always an equilibrium at the upper bound. This equilibrium has maximum risk.

If there is a safe equilibrium, then net marginal cost computed at the risk-free interest rate, must intersect marginal benefit before m_0 gets large enough to cause ρ^d to rise above ρ^l. A safe equilibrium is given by D_{0S} in Figure 1 and m_{0S} in the first panel of Figure 2. For the emerging market, only the risky equilibrium at $\bar{m}_0$ exists. Note that both the safe equilibrium and the risky one are locally stable since agents reduce (increase) real money when net marginal costs exceed (are less than) marginal benefits.

Therefore, advanced economies exhibit multiple equilibria, where one is safe and the other has maximum risk, while emerging markets have only the equilibrium with maximum risk. If we assume that in normal times, agents coordinate on the safe equilibrium for advanced economies, then the following proposition holds.

Allowing τ_0 to be small, but positive, does not change the thrust of this result. We show below that when $\tau_0 > 0$, an increase in m_0 has no effect on $\frac{P_0}{P_1}$ unless foreign exchange reserves in period 1 are exhausted in some states. For small m_0, F_0 is small implying that foreign exchange reserves could be exhausted in some states. As m_0 increases, F_0 increases reducing the number of states in which foreign exchange reserves will be exhausted and reducing the magnitude of the fall in $\frac{P_0}{P_1}$ for those states in which foreign exchange reserves are exhausted. From equation (22), this increases the marginal benefits of money and reduces net marginal costs. The assumption that τ_0 is small is taken to imply that the slopes do not change much so that the slope of MB is less than the slope of MC for small values of m_0. When $\tau_0 > 0$, we will show below that the safe equilibrium, when it exists, is without default risk, but not without exchange rate risk.

Proposition 2 *Assume that agents coordinate on the safe equilibrium when it exists. Given values for η, H, L, and I, there is a critical value of $Y + \tau_0$, above which equilibrium debt is low enough that countries have no risk of widespread default and below which equilibrium debt is driven to its upper bound $\left(\bar{D}_0\right)$ such that risk takes on its maximum value. This divides countries into two groups, a safe group and a risky group. A country with a stronger credit market, represented by a larger value for η, has a lower critical value.*

Proof. Using equations (12) and (6), define a critical value for money, below which debt is perfectly safe by

$$m_0^R = D_0^R - (I - Y - \tau_0) = \frac{\rho^l \eta H I}{(1+r)^2} - I + Y + \tau_0.$$

A safe equilibrium exists if MB intersects MC at a value of $m_0 \leq m_0^R$. Otherwise, debt is driven to its upper bound with maximum risk $\left(\bar{D}_0\right)$. The critical value of money is increasing in both $Y + \tau_0$ and η. Given a value for η, this implies a critical value for $Y + \tau_0$ such that for values above the critical value, debt is perfectly safe in equilibrium. ■

Note that this proposition implies that there are no countries which are a "little" risky. A country which would have a small probability of default at the risk-free interest rate has the incentive to borrow to hold money as a precaution against a default state. The increase in debt increases both the probability of default and the interest rate, giving him incentives to hold even more money. This in turn increases the interest rate and the probability of default until values are driven to their upper bounds. Additionally, stronger credit markets, represented by a higher value for η, have a higher critical value for $I - Y - \tau_0$, implying that differences in credit markets contribute to determination of whether a country belongs in the safe club or the risky club.

We should note that we cannot rule out the possibility of the risky equilibrium for advanced economies. There could be turbulent times where agents coordinate on the risky equilibrium even for the advanced economies. In these times, all economies would join the risky club.

4 Crises

4.1 Debt Crisis

Under the assumption that markets coordinate on the safe equilibrium when it exists, a country has risk of a debt crisis if values of $Y + \tau_0$ and η are low relative to an advanced country, so that the safe equilibrium does not exist, but high relative to an underdeveloped country so that creditors are willing to lend.

Proposition 3 *In period 1, a value for $\rho < \bar{\rho}^d$ triggers a debt crisis in which all agents choose not to repay debt.*

Proof. Proposition 2 implies that debt is driven to its upper bound, driving ρ^d to $\bar{\rho}^d$. When the stochastic realization of $\rho < \rho^d = \bar{\rho}^d$, agents default by Lemma 1. ■

In equilibrium, international creditors impose a ceiling on period-1 debt, given by $\frac{\rho\eta HI}{1+r}$, to assure that they receive the expected risk-free rate of return. A small realization for ρ implies a low credit ceiling and has the interpretation of capital flight. The sudden stop in capital flows, created by a small number of $h-$agents, implies that agents could be better off defaulting on their debt. Agents choose to default on their debt whenever the realization for $\rho < \rho^d = \bar{\rho}^d$, such that debt repayments would exceed new loans available. Agents do face consequences for their default decision in period 2, when they must surrender bankruptcy awards based on realized output.

Therefore, a sudden stop in capital flows, triggered by a negative shock to expected future output, can create a debt crisis with widespread default. Consider the implications of the same type of sudden stop for an exchange rate crisis.

4.2 Exchange Rate Crisis

A sudden stop in capital flows can create an exchange rate crisis with or without default. Agents anticipate that period-2 output will be substantially higher than period-1 output since they will reap the rewards of the risky investment project in period 2. Therefore, to smooth consumption, they want to borrow in period 1 against period-2 income. Additionally, since they know that negative news about future output would tighten the constraint on the amount they can borrow, they hold money as a hedge asset which can finance period-1 consumption. When a sudden stop occurs, agents seek to smooth consumption by exchanging money for foreign exchange with which they can buy goods. We have assumed that the government has committed to hold enough foreign exchange reserves to fix the exchange rate in good states, so we consider the possibility of an exchange rate crisis only when agents are constrained in period-1 debt.

An exchange rate crisis occurs in period 1 when agents want more consumption, at the price level implied by the initial exchange rate and the constrained debt level, than is available in equilibrium. Exchange rate overshooting, in which the exchange rate in period 1 exceeds its value in period 2, convinces them to postpone consumption. Equilibrium in a currency crisis requires an increase in an intertemporal price, not a relative price, requiring the exchange rate to overshoot.

To demonstrate how this occurs within the model, it is necessary to consider the determinants of period-1 real money balances when D_1 is constrained by $\frac{\rho\eta HI}{1+r}$.

Lemma 2 *Optimal real money balances in period 1 for $P_1 = P_0$ are convex in ρ with a minimum at ρ^d.*

The proof, contained in the appendix, is based on the first order condition on period-1 real money balances, equation (21), with $P_1 = P_0$, given by

$$\frac{M_1}{P_0}\left[\frac{1+r}{c_1} - \frac{1}{c_2}\right] = \theta. \tag{27}$$

For $\rho > \rho^d$, money demand is increasing in ρ because an increase in ρ raises equilibrium consumption in both periods. For $\rho < \rho^d$, money demand is falling in ρ because consumption in period 1 no longer responds directly to ρ since agents are in default. Therefore, for high values of ρ money demand falls as ρ falls, reaching a minimum at ρ^d and then rises as ρ continues to fall.

Proposition 4 *If optimal real money balances in period 1* $\left(\frac{M_1}{P_0}\right)$ *at* ρ^d *are less than* τ_0*, then a currency crisis occurs over a range of values for* ρ *given by* $\rho^x < \rho^d < \rho^X$*, where* ρ^x *and* ρ^X *are values of* ρ *at which optimal real money equals* τ_0*. If optimal real money balances at* ρ^d *are greater than or equal to* τ_0*, then a currency crisis cannot occur.*

Proof. Using equation (13), foreign exchange reserves in period 1 depend on the quantity of period-1 real money balances according to

$$F_1 = \frac{M_1}{P_0} - \tau_0 \geq 0. \tag{28}$$

Foreign exchange reserves cannot be negative. Therefore, τ_0 acts as a lower bound on real money balances. When equation (21) implies a value of $\frac{M_1}{P_0} \geq \tau_0$, there is no exchange rate crisis. Otherwise, $\frac{M_1}{P_0}$ must take on the value of its lower bound, and $\frac{P_1}{P_0}$ solves equation (21) with $\frac{M_1}{P_0} = \tau_0$, given by

$$\theta = \tau_0 \left[\left(\frac{1+r}{c_1}\right)\left(\frac{P_0}{P_1}\right) - \frac{1}{c_2} \right]. \tag{29}$$

Therefore, if an exchange rate crisis is possible, then as ρ falls from one, an exchange rate crisis must occur for values of $\rho \geq \rho^d$. As ρ continues to fall from ρ^d, real money demand is rising, implying that a currency crisis occurs for values of $\rho < \rho^d$ until ρ has fallen enough to restore money demand to τ_0. ■

Corollary 1 *When a currency crisis is possible, depreciation is concave in* ρ *with a maximum at* ρ^d*.*

The proof is contained in the appendix. Intuitively, once period-1 foreign exchange reserves (F_1) reach zero, period-1 nominal money balances can no longer fall, and the ability to transfer consumption across time has been eliminated. Instead, relative prices must adjust to satisfy the first order condition on period-1 real money balances given by equation (21).

Depreciation increases the relative price of current compared to future goods, convincing agents to postpone consumption. Since real money demand reaches a minimum at $\rho = \rho^d$, prices must reach a maximum.

5 Government Policy

The government affects both the probability of an exchange rate crisis and the magnitude of depreciation in a crisis with its choice of a value for initial transfers, τ_0.

Proposition 5 *In an economy with a non-zero probability of default at* $\tau_0 = 0$, *there is a critical value of initial transfers given by* τ_0^x, *below which an exchange rate crisis is not possible and above which an exchange rate crisis is possible. For* $\tau_0 \geq \tau_0^x$, *both the probability of an exchange rate crisis and the magnitude of depreciation in an exchange rate crisis are increasing in* τ_0.

The proof is in the appendix. It relies on the fact that an increase in initial transfers reduces initial foreign exchange reserves, increasing the probability that there will not be enough reserves to allow consumers to smooth consumption.

Proposition 5 implies that an increase in τ_0 can raise the probability of a currency crisis while Proposition 2 implies that an increase in τ_0 can eliminate default risk if Y is near the critical value for graduating into the safe credit club.[18] Therefore, there can be a trade-off in the risk of a debt crisis and the risk of a currency crisis.

Corollary 2 *A currency board sets* $\tau_0 = 0$, *implying that a productivity shock, which triggers a sudden stop of capital flows, cannot create a currency crisis.*

Proof. From equation (28), $\tau_0 = 0$ implies that foreign exchange is fully backed by money at the designated exchange rate, the definition of a currency board. Since money demand is

[18]When the economy is far from the safe equilibrium, then the increase in τ_0, necessary to yield the safe equilibrium, would not be feasible, given the government's assumed commitment to maintain fixed exchange rates in good states.

always positive, foreign exchange reserves are always positive, and a currency crisis cannot occur. ■

The government's decision to maintain a fixed exchange rate system allows foreign exchange reserves to serve as a hedge asset in the event of default.

Proposition 6 *Reserves provide insurance for the country, increasing present-value resources for states in which aggregate income is low and the country is in default, at the cost of fewer resources in states in which income is high and the country chooses to repay.*

Proof. From Proposition 1, when ρ is high (low) agents repay (default) in period 1. Period-2 gross income is given by $\rho HI + (1-\rho) LI$, which is increasing in ρ. Therefore, agents default in low-income states and repay in high-income states. Equation (30) in the appendix together with equations (10) and (12) show that the agent's intertemporal budget constraint is given by

$$c_1 + \frac{c_2}{1+r} = Y_1 + (1+r) F_0 + \frac{\rho HI + (1-\rho)(1-\eta) LI}{1+r} - \left[\zeta (1+r_0)(F_0 + I - Y) + (1-\zeta) \frac{\rho \eta HI}{1+r} \right],$$

where $\zeta \in \{0, 1\}$ is chosen to maximize the term in brackets. Foreign exchange reserves affect the present-value of the country's resources, conditional on realizations for ρ. With high ρ and debt repayment ($\zeta = 1$), foreign exchange reserves represent a net cost, since $r_0 > r$, whereas with low ρ and default ($\zeta = 0$), they represent a net gain of resources. ■

Note that net resources are reduced due to credit market imperfections, which require creditors to pay the costly-state-verification fee, equal to ηLI, before they can obtain compensation after an agent defaults.

6 Conclusion

Investment necessary for growth and development in an emerging market is risky and takes time. When an emerging market country has little wealth relative to the size of its investment project, it must access international financial markets if it chooses to invest. A sudden stop in capital flows, caused by a negative productivity shock in the presence of capital market imperfections can magnify a solvency crisis, in which there is a relatively large number of less-productive agents who default, into a widespread liquidity crisis, in which all agents default. The negative productivity shock and associated sudden stop can also trigger a currency crisis.

This model fills a gap in the literature on emerging market financial crises by explicitly considering the possibility that a financial crisis can be generated by the same fundamentals responsible for business cycles, productivity shocks. A relatively low number of productive projects is considered a negative productivity shock. When agents must borrow to engage in risky investment, and when information on output is subject to costly state verification, creditors impose ceilings on loans to assure that agents with high productivity prefer repayment to default with the surrender of a bankruptcy penalty. However, agents with low productivity always default, and a low number of productive agents implies both low aggregate output and a larger than average number of defaults. Additionally, when there is also mismatch between the maturities for loans and investment, credit ceiling on new loans will be determined after creditors receive a signal about the number of productive agents. When this signal indicates a low number, interest rates are high and loan ceilings are low, implying a sudden stop of capital flows. A large enough sudden stop of capital creates default, because agents lose less with default than with repayment. Additionally, the sudden stop of capital forces agents to use money to support consumption. They swap money for foreign exchange and use the foreign exchange to buy goods. If the monetary authority has sufficient reserves to satisfy demand at the current price level, then there is no currency crisis. If not, the exchange rate depreciates, overshooting to raise the relative price of current goods compared to future goods, thereby convincing agents to postpone consumption.

The model also demonstrates that the world can be divided into three credit clubs, those with no access, those with stable access at low interest rates, and those with volatile access at interest rates reflecting risk. Countries with very little wealth or very weak capital markets have no access to international credit. Countries with high wealth and strong credit markets enjoy high credit ceilings relative to their needs to borrow. For these countries, desired debt at the risk-free interest rate is so low that agents always prefer repayment to sacrificing the large fraction of output implied by the strong credit market. These are the advanced economies who have stable access to international credit at low interest rates. Countries in between, those with some wealth and a reasonably strong credit market, are driven to the risky credit club. This occurs because debt desired at the risk-free interest rate is high enough that agents prefer default to repayment in some states. Money demand is increasing in the number of states requiring default since additional money allows a one-for-one increase in consumption in bad states at the cost of reduction in consumption in good states equal to the interest rate. Therefore, once debt is high to elicit default in some states, marginal benefits of money exceed marginal costs driving agents to acquire the maximum amount of money and debt. This drives these countries to the risky club with volatile credit access.

The model highlights the role of reserves as insurance. Although government policy is not responsible for the productivity shock generating the crisis, its reserve policy affects the economy's response to the shock. Reserves have the role of insurance, transferring resources to the country in times of default when output is low, at the cost of a default-risk premium paid on loans when output is high. In default, the capital account shuts down, and agents use reserves in their Bretton Woods era function to finance a temporary current account imbalance, thereby sustaining consumption. When reserves are insufficient to satisfy consumption-demand with fixed exchange rates, exchange rate overshooting convinces agents to postpone consumption. Therefore, a large enough quantity of reserves can prevent a sudden stop from becoming an exchange rate crisis. Additionally, if the country is near the threshold for graduating into the safe country club, then a small reduction in reserves, created by increasing initial transfers (τ_0) allows graduation, by reducing the demand for initial loans, possibly at the cost of increasing the probability of a currency crisis.

The model can be used to explain financial crises with origins in the private sector. The 1997 Asian financial crises are often viewed as private debt crises, which also produced currency crises. Corsetti, Pesenti and Roubini (1999) present evidence to show that just prior to the 1997 financial crises in the Southeast Asian countries, bankruptcies and non-performing loans were high, both evidence of some degree of default on the part of private agents. The crises brought more default with closures of banks and finance companies.

Large estimates of the fiscal costs of restoring solvency to the financial sector after these crises, presented by Burnside, Eichenbaum and Rebello (2001), is further evidence of private default and debt rescheduling. Additionally, these countries experienced capital flight and most devalued their currency. None of them experienced sovereign default. This model formally endogenizes the sudden stop of international credit which is often viewed as the cause of the debt and currency crises. News about bankruptcies could be interpreted as reducing expectations for future output, triggering the sudden stop in capital flows. The model also explains the associated currency crises in countries which did not have a currency board, and the absence of a currency crisis in Hong Kong, a country with a currency board.

7 Appendix

7.1 Equilibrium Values

Some proofs and other results require solution for equilibrium values of consumption as a function of the initial period choice for money $\left(\frac{M_0}{P_0}\right)$ and equivalently for debt (D_0).

Combining agent budget constraints, given by equations (14) and (19), using equation (3) for the equilibrium period-1 interest rate and government flow budget constraints, given by equations (10), (13), and (15), yields the agent's intertemporal budget constraint

$$c_1 + \frac{c_2}{1+r} = Y_1 - \zeta(1+r_0)D_0 + (1+r)F_0 + \frac{[\zeta + (1-\zeta)(1-\eta)]\,\rho HI + (1-\rho)(1-\eta)\,LI}{1+r}, \tag{30}$$

where $\zeta \in \{0,1\}$.

When agents are unconstrained in the amounts they can borrow in period 1, $\left(D_1 < \bar{D}_1\right)$, they choose to repay, such that $\zeta = 1$. The first order equation on consumption implies that agents achieve complete consumption-smoothing. Substituting $F_0 = \frac{M_0}{P_0} - \tau_0 = D_0 - I + Y$ yields the value for equilibrium consumption as

$$c_1 = c_2 = \frac{\rho HI + (1-\rho)(1-\eta)\,LI + (1+r)\left[Y_1 - (r_0 - r)\,D_0 + (1+r)(Y-I)\right]}{2+r}. \tag{31}$$

The equilibrium value of period-1 debt is given by

$$D_1 = \frac{[\rho HI + (1-\rho)(1-\eta)\,LI]\,(r+\theta) - [Y_1 + (1+r)\,F_0]\,[r - \theta\,(1+r)]}{r\,(2+r)}, \tag{32}$$

and parameter are assumed to take values such that $D_1 > 0$..

When agents are constrained in the amount they can borrow in period 1 $\left(D_1 = \bar{D}_1 = \frac{\eta\rho HI}{1+r}\right)$, equilibrium consumption is given by

$$c_1 = \zeta\left(\frac{\eta\rho HI}{1+r} - (1+r_0)D_0\right) + Y_1 + (1+r)(F_0) - \left(\frac{M_1}{P_0} - \tau_0\right) \tag{33}$$

$$c_2 = (1-\eta)\left[\rho HI + (1-\rho) LI\right] + (1+r)\left(\frac{M_1}{P_0} - \tau_0\right) \tag{34}$$

where $\zeta \in \{0,1\}$ is chosen to maximize utility and therefore c_1. The value for $\frac{M_1}{P_0}$ satisfies equation (21), subject to $\frac{M_1}{P_0} \geq \tau_0$. Note that the magnitude of exchange rate depreciation itself does not affect consumption in equilibrium because there are no agents with whom to trade goods for money in the event that foreign exchange reserves are exhausted. Exchange rate depreciation must be large enough to convince agents to postpone consumption.

Equations (10) and (12) imply $\frac{dF_0}{dm_0} = \frac{dD_0}{dm_0} = 1$. Therefore, when period-1 debt is constrained, $\frac{dc_1}{dm_0} = 1 + r - \zeta(1+r_0) - \frac{1}{P_0}\frac{dM_1}{dm_0} \approx 1 + r - \zeta(1+r_0)$, since $\frac{dM_1}{dm_0}$ is very small when c_1 is constrained. When agents fail to repay, $\zeta = 0$, and the effect of an increase in initial money balances on c_1 is positive and much larger because agents can use the resulting increase in foreign exchange reserves with interest to finance consumption. When period-1 debt is not constrained $\frac{dc_1}{dm_0} = \frac{(1+r)(r-r_0)}{2+r}$.

7.2 Marginal Cost Falling in m_0

The derivative of the net marginal cost of money (equation 26) with respect to m_0, for $D_0 \geq D_0^R$, allowing r_0 and ρ^d to change endogenously from equations (8) and (9) is given by

$$\frac{\partial MC}{\partial m_0} = \int_{\rho^d}^{1} \frac{c_1\frac{dr_0}{dD_0} - \frac{\partial c_1}{\partial m_0} r_0}{c_1^2} f(\rho)\, d\rho + \int_{\rho^l}^{\rho^d} \frac{\frac{\partial c_1}{\partial m_0}}{c_1^2} f(\rho)\, d\rho - \frac{f(\rho^d)(1+r_0)}{\tilde{c}_1}\frac{d\rho^d}{dD_0},$$

where $\tilde{c}_1$ equals c_1 evaluated at $\rho = \rho^d$.

Substituting values above for $\frac{\partial c_1}{\partial m_0}$ in different states yields an upper bound on the derivative of the marginal cost of money with respect to money

$$\frac{\partial MC}{\partial m_0} < \int_{\rho^d}^{1} \frac{c_1 \frac{dr_0}{dD_0} + (r_0 - r)\, r_0}{c_1^2} f(\rho)\, d\rho + \int_{\rho^l}^{\rho^d} \frac{1+r}{c_1^2} f(\rho)\, d\rho - \frac{f\left(\rho^d\right)(1+r_0)}{\tilde{c}_1} \frac{d\rho^d}{dD_0}.$$

Substituting equation (9) for $\frac{dr_0}{dD_0}$, and dropping the term $(r_0 - r)\, r_0$ as being second-order small yields

$$\frac{\partial MC}{\partial m_0} < \left(\frac{\eta HI}{(1+r)\, D_0}\right) \left\{ \frac{d\rho^d}{dD_0} \left[\int_{\rho^d}^{1} \frac{1}{c_1} f(\rho)\, d\rho - \frac{\rho^d f\left(\rho^d\right)}{\tilde{c}_1} \right] - \int_{\rho^d}^{1} \frac{1}{c_1} f(\rho)\, d\rho \right\} + \int_{\rho^l}^{\rho^d} \frac{1+r}{c_1^2} f(\rho)\, d\rho.$$

The term in square brackets is negative because $\int_{\rho^d}^{1} \frac{1}{c_1} f(\rho)\, d\rho < \frac{1}{\tilde{c}_1}$ and $1 < \rho^d f\left(\rho^d\right)$, when ρ^d has a reasonable lower support. Specifically, if ρ is uniform, a lower support exceeding one-half is sufficient. Even in the worst state, it is reasonable to assume that more than fifty percent of firms are successful. This implies that the term in curly brackets is negative always. The final term is zero when $D_0 = D_0^R$, such that $\frac{d\rho^d}{dD_0} > 0$, but $r_0 = r$ and $\rho^d = \rho^l$. Therefore, at $D_0 = D_0^R$, marginal cost falls as m_0 increases. As money continues to increase, the final term becomes positive. However, $\frac{d\rho^d}{dD_0}$ is rising at an increasing rate in D_0 such that $\frac{d\rho^d}{dD_0} \to \infty$ as $D_0 \to \bar{D}_0$. Therefore, MC is falling at values of debt for which $D_0 = D_0^R$ and for which $D_0 = \bar{D}_0$.

We cannot prove that MC is falling in m_0 between these two points. Define $A = \left(\frac{\eta HI}{(1+r)D_0}\right) \left\{ \frac{d\rho^d}{dD_0} \left[\int_{\rho^d}^{1} \frac{1}{c_1} f(\rho)\, d\rho - \frac{\rho^d f\left(\rho^d\right)}{\tilde{c}_1} \right] \right\}$, $B = \left(\frac{\eta HI}{(1+r)D_0}\right) \left\{ - \int_{\rho^d}^{1} \frac{1}{c_1} f(\rho)\, d\rho \right\}$, and $C = \int_{\rho^l}^{\rho^d} \frac{1+r}{c_1^2} f(\rho)\, d\rho$, where A and B are negative and C is positive. For marginal cost to be rising at any point would require $C > -(A+B)$ For D_0 near D_0^R, C is small relative to $-B$, since the integral for C includes relatively few states. As D_0 increases, C becomes larger, but $-A$ becomes larger at a rapidly increasing rate because it must reach infinity once D_0 reaches $\bar{D}_0$. Experimentation with numerical values assuming a uniform distribution always yields marginal costs which fall at an increasing rate.

7.3 Assumptions on Magnitudes

In equilibrium, the expected utility from the risky project can be computed by taking the time-zero expectation of the intertemporal budget constraint, equation (30), using equation (7), to yield

$$E_0\left[c_1 + \frac{c_2}{1+r}\right] = \int_{\rho^l}^{1} \frac{\rho HI + (1-\rho)(1-\eta) LI}{1+r} f(\rho)\, d\rho + Y_1 - (1+r)(I-Y) - \int_{\rho^l}^{\bar{\rho}^d} \left(\frac{\eta LI}{1+r}\right) \rho f(\rho)\, d\rho.$$

For agents to choose the risky investment project, the expected present-value of consumption, given above, must be enough larger than resources from the single-period project with default $[(1-\eta) Y_1^s]$ and from the single-period project with repayment $[Y_1^s - (1+r)(I-Y)]$ to compensate agents for risk.

The assumption that the endogenous debt ceiling, given by $\frac{\rho\eta HI}{1+r}$, increases more rapidly in ρ than the desired level of debt, given by equation (32), requires that the derivative of equation (32) with respect to ρ, given by $\frac{[HI-(1-\eta)LI](r+\theta)}{r(2+r)}$, exceed the derivative of the debt ceiling with respect to ρ, given by $\frac{\eta HI}{1+r}$. Solving the inequality for η yields

$$\eta > \frac{(HI - LI)(1+r)(r+\theta)}{(HI - LI)(1+r)(r+\theta) + HI(r - \theta(1+r))}. \tag{35}$$

For the value of η satisfying equation (35) to be less than one, $\theta < \frac{r}{1+r}$, as assumed.

We confirm that $l-$agents optimally choose default in period 2. When period-1 debt is unconstrained, equations (32) together with the inequality in equation (35), imply that

$(1+r_1)\,D_1 > \eta LI$. When period-1 debt is constrained, the cost of the bankruptcy settlement, ηLI, is less than interest and debt at the debt ceiling for a total of ηHI.

7.4 Proof of Lemma 2

Proof. Let $D_1 = \bar{D}_1 = \frac{\rho\eta HI}{1+r}$. Differentiation of equation (27) with respect to ρ, using equations (33) and (34) yields

$$\frac{d\frac{M_1}{P_0}}{d\rho} = \left(\frac{M_1}{P_0}\right)^2 \left[\frac{\zeta\eta HIc_2^2 - (1-\eta)\,(HI - LI)\,c_1^2}{\theta c_1^2 c_2^2 + \left(\frac{M_1}{P_0}\right)^2 (1+r)\,(c_1^2 + c_2^2)}\right]$$

When $\zeta = 1$, the Euler equation with $c_2 > c_1$ together with equation (35), which requires

$$\eta > \frac{(HI - LI)\,(1+r)\,(r+\theta)}{(HI - LI)\,(1+r)\,(r+\theta) + HI(r - \theta\,(1+r))} > \frac{(HI - LI)}{(2HI - LI)},$$

imply a positive derivative. When $\zeta = 0$, the derivative is negative. Therefore, real money is increasing in ρ for $\rho > \rho^d$ and decreasing in ρ for $\rho < \rho^d$, reaching a minimum at $\rho = \rho^d$.

■

7.5 Proof of Corollary 1

Proof. Setting $\frac{M_1}{P_0} = \tau_0$ in equation (21) yields equation (29). Totally differentiating with respect to $\frac{P_1}{P_0}$ and ρ yields

$$\frac{d\left(\frac{P_1}{P_0}\right)}{d\rho} = -\frac{P_1}{P_0\,(1+r)}\left[\frac{\zeta\eta HIc_2^2 - (1-\eta)\,(HI - LI)\,c_1^2}{c_1 c_2^2}\right].$$

For $\rho > \rho^d$, $\zeta = 1$, and the derivative is negative. For $\rho < \rho^d$, $\zeta = 0$, and the derivative is positive. Therefore, $\frac{P_1}{P_0}$ is decreasing in ρ for $\rho > \rho^d$ and increasing in ρ for $\rho < \rho^d$, reaching a maximum at $\rho = \rho^d$. ■

7.6 Proof of Proposition 5

Proof. When there is a non-zero probability of default, D_0 takes on its upper bound value. This fixes the value of initial foreign exchange reserves, F_0. For $\tau_0 = 0$, an exchange rate crisis is not possible because money always exceeds zero since it enters the utility function directly. Equation (13) implies that with $\tau_0 = 0$, $F_1 = \frac{M_1}{P_0} > 0$. With F_0 fixed, an increase in τ_0 creates an equal increase in $\frac{M_0}{P_0}$ from equation (10). Differentiating equation (27), using equations (33) and (34), shows that an increase in τ_0 reduces period-1 reserves

$$\frac{dF_1}{d\tau_0} = \frac{d\frac{M_1}{P_0}}{d\tau_0} - 1 = \frac{-\theta\left(\frac{M_1}{P_0}\right)c_1^2c_2^2}{\left(\frac{M_1}{P_0}\right)^2(1+r)\left(c_1^2+c_2^2\right)+\theta\left(\frac{M_1}{P_0}\right)c_1^2c_2^2} < 0.$$

With foreign exchange reserves in period 1 falling as τ_0 increases, there will be a value of τ_0, given by τ_0^x, at which $F_1\left(\rho^d\right) = 0$. Further increases in τ_0 imply further decreases in money demand and imply that money demand falls to τ_0 at larger values of ρ. Therefore, the largest value of ρ which would elicit a currency crisis, ρ^x, is increasing in τ_0. By Corollary 1, the magnitude of depreciation is increasing in τ_0 for a given realization of ρ. The government affects both the probability of an exchange rate crisis and the magnitude of depreciation in a crisis with its choice of a value for initial transfers, τ_0. ■

References

[1] Aizenman, Joshua and Nancy Marion, "International Reserve Holdings with Sovereign Risk and Costly Tax Collection," *Economic Journal* 114 (July, 2004) 569-91.

[2] Arellano, Cristina, and Enrique G. Mendoza, "Credit Frictions and 'Sudden Stops' in Small Open Economies: An Equilibrium Business Cycle Framework for Emerging Markets Crises," in *Dynamic Macroeconomic* Analysis: Theory and Policy in General Equilibrium. ed. S. Altug, J. Chadha, and C. Nolan. Cambridge University Press (2003), Ch. 7

[3] Aghion, Philippe, Philippe Bacchetta and Abhijit Banerjee, "Currency Crises and Monetary Policy in an Economy with Credit Constraints, *European Economic Review* 45 (2001), 1121-1150.

[4] Atkeson, Andrew, "International Lending with Moral Hazard and Risk of Repudiation." *Econometrica* 59(4) (July,1991), 1069-1089.

[5] Bernanke, B.D. and M. Gertler, "Agency Costs, Net Worth, and Business Fluctuations." *American Economic Review* 79(1) (March 1989), 14-31.

[6] Bulow, Jeremy and Kenneth Rogoff, "A Constant Recontracting Model of Sovereign Debt." *Journal of Political Economy* 97 (February 1989), 155-178.

[7] Burnside, Craig, Martin Eichenbaum and Sergio Rebelo. "Prospective Deficits and the Asian Currency Crisis," *Journal of Political Economy* 109 (December 2001), 1155-1197.

[8] Caballero, Ricardo J. and Arvind Krishnamurthy, "International and Domestic Collateral Constraints in a Model of Emerging Market Crises," *Journal of Monetary Economics* 48 (December, 2001), 513-548.

[9] Calvo, Guillermo A., "Capital Flows and Capital-Market Crises: The Simple Economics of Sudden Stops," *Journal of Applied Economics* 1 (November 1998), 35-54.

[10] Calvo, Guillermo A. and Carmen M. Reinhart, "When Capital Flows Come to a Sudden Stop: Consequences and Policy Options," in P.Kenen, M. Mussa, and A. Swoboda, *Reforming the International Monetary and Financial System*, (Washington, DC: International Monetary Fund, forthcoming) 2000a.

[11] Calvo, Guillermo A. and Carmen M. Reinhart, "Fear of Floating," *Quarterly Journal of Economics,* 117 *(*May, 2002), 379-408.

[12] Cespedes, Luis, Roberto Chang and Andres Velasco, "Balance Sheets and Exchange Rate Policy,." *American Economic Review* 94(4) (September 2004), 1183-93.

[13] Carlstrom, C.T. and T.S. Fuerst, "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis." *American Economic Review, 87(5), (December 1997), 893-910.*

[14] Chang, Roberto, "Financial Crises and Political Crises." *Journal of Monetary Economics* 54 (2007), 2409-2420.

[15] Chang, Roberto and Andres Velasco, "A Model of Financial Crises in Emerging Markets," *Quarterly Journal of Economics* 116 (May, 2001), 489-518.

[16] Corsetti, Giancarlo, Paolo Pesenti, and Nouriel Roubini, "What Caused the Asian Currency and Financial Crisis? Part I: A Macroeconomic Overview," *Japan and the World Economy* 11(3) (1999), 305-73.

[17] Corsetti, Giancarlo, Paolo Pesenti, and Nouriel Roubini, "Paper Tigers?: A Model of the Asian Crisis," *European Economic Review* 43(7), 1999, 1211-1236.

[18] Flood, R. P. and P. M. Garber, "Collapsing Exchange Rate Regimes: Some Linear Examples," *Journal of International Economics* 17 (August 1984), 1-13.

[19] Jeanne, Olivier and Romain Ranciere, "The Optimal Level of International Reserves for Emerging Market Countries: Formulas and Applications." memo, IMF (April 2005).

[20] Holmstrom, Bengt and Jean Tirole, "Financial Intermediation, Loanable Funds, and the Real Sector," *Quarterly Journal of Economics* 112(3) (August 1997), 663-692.

[21] Kaminsky, Graciella and Carmen Reinhart, "The Twin Crises: The Causes of Banking and Balance-of-Payments Problems," *American Economic Review* 89 (1999), 473-500.

[22] Kehoe, Timothy J. and David K. Levine, "Debt-Constrained Asset Markets,: *Review of Economic Studies* 60 (1993), 865-888.

[23] Krugman, P., "A Model of Balance of Payments Crises," *Journal of Money, Credit, and Banking* 11 (August 1979), 311-325.

[24] Heller, Heinz R., "Optimal International Reserves," *Economic Journal* 76 (1966) 296-311.

[25] Mendoza, Enrique and Katherine Smith, "Quantitative Implications of a Debt-Deflation Theory of Sudden Stops and Asset Prices." *Journal of International Economics* (September 2006).

[26] Obstfeld, Maurice and Kenneth Rogoff, *Foundations of International Macroeconomics,* Chapter 6. *(*MIT Press: Cambridge, MA) 1996.

[27] Radelet, S. and J. Sachs, "The Onset of the East Asian Financial Crisis," *Brookings Papers on Economic Activity* (1:1998), 1-90.

[28] Romer, David, *Advanced Macroeconomics,* Chapter 8 (McGraw-Hill Companies, Inc: New York) 1996.

[29] Ranciere, Romain, Aaron Torness, and Frank Westermann, "Systemic Crises and Growth." *The Quarterly Journal of Economics* (February 2008), 359-406.

[30] Tornell, Aaron, Frank Westermann, and Lorenza Martinez. "The Positive Link between Financial Liberalization," Growth and Crises, NBER working paper no. w10293 (February 2004).

[31] Wijnholds, J. Ono de Beaufort and Arend Kapteyn, "Reserve Adequacy in Emerging Market Economies," IMF Working Paper WP/01/143 (2001).