Implications of Productive Government Spending for Fiscal Policy

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Abstract

The standard assumption in macroeconomics that government spending is unproductive can have substantive implications for tax and spending policy. Productive government spending introduces a positive feedback between the tax rate, the productive capacity of the economy, and tax revenue. We allow marginal tax revenue to be optimally allocated between productive subsidies to human capital and utility-enhancing government consumption and calculate Laffer Curves for the US. Productive government spending yields substantially higher peaks and steeper slopes at low tax rates for the labor-tax Laffer curve. The use of tax revenue is an important determinant of the actual revenue that a tax rate increase generates.

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1 Introduction

A major role of government is to provide public goods, some of which enhance the productivity of the economy. Examples include the Eisenhower interstate highway system in the US, the extensive rail system in Europe, public education, government-funded research, among other projects. Yet, a standard simplifying assumption in macroeconomics is that government spending is unproductive. An even more extreme but common assumption is that government spending is entirely purposeless with purchases thrown into the ocean or tax revenue redistributed back to the same representative agent who paid it. These standard assumptions eliminate any positive direct effects of government spending on the economy. Those direct effects, however, must be the purpose of the spending and the reason for which the spending is undertaken.

This paper focuses on the importance of including the purpose of government spending when trying to understand the effects of an increase in distortionary tax rates on output, total tax revenue, and welfare. When a distortionary tax increase finances productive government spending, it increases the productivity of the economy. This increased productivity offsets some of the distortion from the increased tax rate, mitigating the output and welfare loss from the tax rate increase. When the spending is not productive, these effects are absent. Baxter and King (1993) wrote an early paper in which they demonstrated substantial differences in fiscal multipliers under distortionary taxation for the cases of productive and non-productive government spending. We follow their lead and compare Laffer curves and welfare with and without productive government spending. With productive spending, the slope of the Laffer curve at low tax rates and its peak are higher. The net welfare impact of a distortionary tax increase also depends on the type of spending financed.

We model productive government spending as subsidies to education, essentially subsidies to private investment in human capital. Following literature initiated by Lucas (1988)’s endogenous growth model, we provide a role for government subsidies to education by assuming that human capital has an externality in production, inducing the private market to provide too little of it. However, in the presence of distortionary labor taxation, the government would optimally choose to provide subsidies even in the absence
of the externality in order to offset some of the distortion created by the labor tax.

We provide a calibrated model to show that when the government allocates marginal tax revenue optimally between utility-enhancing spending on a public good and education subsidies, compared with allocating all marginal revenue to the public consumption good, the difference in the shape of the Laffer curve is economically significant. Peaks are higher and the slope at low tax rates is steeper. The peak of the labor-tax Laffer curve with optimal allocation provides an additional 71% in revenues compared with an additional 49% with full allocation to government consumption. The capital tax Laffer curve is much flatter and corresponding numbers are 12% and 8%. These numbers imply that optimal allocation of tax revenues yields additional revenue at the peak between 45% and 50% higher than possible with full allocation to the public consumption good. Our focus on a single type of productive government spending ignores other types of productive spending. This implies that our results yield a lower bound on the government’s ability to increase the slope and peak of the Laffer curve by allocating marginal tax revenue toward productive use.

Our assumption that marginal tax revenues are allocated toward welfare-enhancing uses allows us to meaningfully compute optimal labor and capital tax rates, conditional on exogenous government transfers. We find that the optimal capital tax should be zero, as in the optimal tax literature, and that the optimal labor tax is higher than its current rate to finance the productive and utility-enhancing government spending.

These issues are particularly relevant in current budgetary environments, where countries are facing difficult choices over spending cuts and tax increases needed to achieve long-run fiscal sustainability. Education expenditures have been widely targeted for cuts. Our analysis demonstrates that cuts to productive government spending are considerably less effective in achieving fiscal sustainability than cuts to utility-enhancing spending since the former will reduce the long-run productive capacity of the economy. This does not mean that all spending cuts should be to utility-enhancing spending because choices should be guided by welfare, not by maximizing tax revenues. However, in comparing costs and benefits of alternative spending cuts, their differing effects on marginal tax revenues should be included in both the budget-balancing and welfare calculations.

which compute steady-state Laffer curves by calibrating the steady state of an exogenous neoclassical growth model with capital and labor as inputs. We calibrate the Laffer curve to the steady state of a growth model, but make four significant departures.

The first departure is the focus of our paper. TU consider how far various economies are from the peaks of their Laffer curves, and find that some countries are close. In contrast, we want to understand how the possibility of productive government spending alters the ability to raise tax revenues with an increase in tax rates, and therefore the shape of the Laffer curve. Implicitly, a finding that the purpose of government spending has little effect on the shape of the Laffer curve would be a robustness finding for Laffer curves calibrated in TU.

The second departure is our focus on allowing some government spending to be productive, compared with the TU assumption that marginal tax revenue is allocated to transfers. Explicitly, we restrict uses of marginal tax revenue to those which are potentially welfare improving, one of which is productive government spending. Given the large fraction of government spending on education, and work initiated by Lucas (1988) on the productivity of human capital, we model productive government spending as subsidies to education expenditures.

The third departure is that we calibrate to a growth model with a third factor of production, human capital, as in Mankiw, Romer, and Weil (1992)(MRW). We generalize the MRW production function for human capital, which contains the single input of student time, to include three inputs, human capital, expenditures on education, and student time as in MRW.

Fourth, since additional spending financed by distortionary taxes has the potential to increase welfare in our model, we can make meaningful welfare calculations. Both TU and our model assume that some government expenditures are fixed. TU does not focus on welfare and assumes that marginal tax revenue finances transfer payments to the same representative agent who paid the distortionary taxes, always a net welfare loss. In contrast, we assume that marginal tax revenue is optimally allocated between productive government spending and government consumption, which is an imperfect substitute for private consumption in utility. At our constrained optimum, the capital tax is zero, and the labor tax is higher than the current rate.
Prescott (2002) has shown that allocation of marginal tax revenue to redistribution, compared with purposeless spending, does affect the shape of the Laffer curve, reducing both its slope at low tax rates and its peak. We demonstrate that allowing optimal allocation of marginal tax revenue to productive government spending compared with full allocation to utility-enhancing government consumption raises the slope of the Laffer curve at low tax rates and its peak. Therefore, our paper contributes to the literature demonstrating the importance of the purpose of government spending for the effect of an increase in the tax rate on steady-state output and tax revenues. The shape of the Laffer curve is not robust to alternative uses of marginal tax revenue.

The paper is organized as follows. We present the model in section 2. Section 3 discusses the calibration and parameterizations. Section 4 presents the results. Section 5 contains conclusions.

2 The Model

2.1 General Assumptions

In this section we specify and solve for the balanced growth path in an exogenous growth model and compute Laffer curves along the balanced growth path, following Trabandt and Uhlig (2011). We do not consider the transition to the balanced growth path or the implied full welfare effects of a policy change. Therefore, our policy implications are long-run implications only.\(^1\)

2.1.1 Firm

Following both Mankiw, Romer, and Weil (1992) and Trabandt and Uhlig (2011), we use an exogenous growth model, and following Mankiw, Romer, and Weil (1992), we add human capital to the production function.\(^2\) The representative firm produces output using a constant-returns-to-scale production function with inputs of physical capital \((K_t)\), human capital \((H_t)\), and labor \((n_{w,t}L_t)\), where \(n_{w,t}\) is hours per worker and \(L_t\) is the

\(^1\)It is not obvious how to set up a short-run transition to the optimal policy. Why is the government currently not following optimal policy? Is the change in policy expected or unexpected? These questions are beyond the scope of this paper.

\(^2\)Trabandt and Uhlig (2011) also consider an endogenous growth model.
number of workers. Technology, denoted by $z_t$, grows at exogenous rate $\xi$, such that $z_t = \xi^t$. We augment the constant-returns-to-scale Mankiw-Romer-Weil production function with an externality in the aggregate level of per-worker human capital $(h_{a,t})$. Since all representative firms behave identically, aggregate output is given by

$$ Y_t = z_t K_t^{\theta_k} H_t^{\theta_h} (n_{w,t} L_t)^{1-\theta_k-\theta_h} h_{a,t}^{\phi}, \quad (1) $$

where we assume that $\theta_k + \theta_h + \phi < 1$ to ensure a balanced-growth equilibrium.\(^3\)

Letting small letters denote per-worker values, the per-worker value of aggregate output can be expressed as

$$ y_t = z_t k_t^{\theta_k} h_t^{\theta_h} (n_{w,t})^{1-\theta_k-\theta_h} h_{a,t}^{\phi}, \quad (2) $$

where $h_t = h_{a,t}$ in equilibrium.

The representative firm maximizes profit by solving the following problem

$$ \max_{k_t, n_{w,t}, h_t} y_t - d_t k_t - w_t n_{w,t} - w_{h,t} h_t, $$

where $d_t$ is dividends, $w_t$ is the wage paid per hour of work, and $w_{h,t}$ is the wage paid to the value of human capital per worker. In choosing human capital, the firm ignores the effect of his choice for human capital on aggregate human capital, implying that the firm chooses too little human capital.

At the optimum, returns to capital ($d_t$) and wages to both hours devoted to work ($w_t$) and to human capital formation ($w_{h,t}$) can be expressed, respectively, as

$$ d_t = \theta_k \frac{y_t}{k_t}, $$

$$ w_t = (1 - \theta_k - \theta_h) \frac{y_t}{n_{w,t}}, $$

\(^3\)Note that this specification is equivalent to specifying output as a function of effective labor, defined as a geometric index of human capital and hours by workers. Letting $Q_t^{1-\theta_k}$ denote effective labor,

$$ Q_t^{1-\theta_k} = H_t^{\theta_h} (n_{w,t} L_t)^{1-\theta_k-\theta_h}, $$

yields an expression for output as

$$ Y_t = z_t K_t^{\theta_k} Q_t^{1-\theta_k} h_{a,t}^{\phi}. $$
\[ w_{h,t} = \theta_h \frac{y_t}{h_t}. \]

The externality in human capital serves to augment payments to all factors in the same way that technology does, such that firm profits, after payment to all factors, are zero,

\[ \Pi_t = y_t - d_t k_t - w_t n_{w,t} - w_{h,t} h_t = 0. \] (3)

2.1.2 Household

The representative infinitely-lived household maximizes the expected discounted stream of utility. Labor endowment is normalized to unity such that leisure equals one minus hours worked \((n_{w,t})\) minus hours spent on education \((n_{h,t})\). Utility is separable in consumption \((c_t)\), leisure \((1 - n_{w,t} - n_{h,t})\), and utility-enhancing government consumption \((g_{c,t})\). Assuming log utility, the objective function can be expressed as

\[ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{ \log c_t + \alpha_n \log(1 - n_{w,t} - n_{h,t}) + \alpha_g \log g_{c,t} \}. \]

Letting \(\tau\) denote tax rates, the optimization occurs subject to the household’s budget constraint

\[
\begin{align*}
 c_t + (1 - s_t) e_t + i_t + b_{t+1} &= (1 - \tau_{n,t})(w_t n_{w,t} + w_{h,t} h_t) + [(1 - \tau_{k,t})(d_t - \delta) + \delta] k_t + TR_t + R_{b,t} b_t + m_t. \quad (4)
\end{align*}
\]

The household uses his after-tax earnings from labor hours \([(1 - \tau_{n,t})(w_t n_{w,t})\)] and human capital \([(1 - \tau_{n,t})(w_{h,t} h_t)\)], as well as his after-tax earnings on capital \([(1 - \tau_{k,t})(d_t - \delta)k_t + \delta k_t]\), plus the sum of government transfer payments \((TR_t)\), gross interest on the risk-free bond \((R_{b,t} b_t)\), and earnings from net foreign assets \((m_t)\) to purchase consumption \((c_t)\), subsidized education expenses \([(1 - s_t) e_t]\), investment \((i_t)\), and risk-free bonds \((b_{t+1})\). Capital taxes are imposed on dividends net-of-depreciation \((\delta)\) as in Prescott (2002, 2004) and Trabandt and Uhlig (2011). Firm profits are omitted since they are zero. We follow Trabandt and Uhlig (2011) in introducing \(m_t\) to represent earnings from net foreign assets. In a long-run equilibrium the current account is balanced so that net imports equal earnings from net foreign assets. This additional variable is necessary in
calibrating the model, since in an open economy, net imports drive a wedge between expenditure by domestic residents and production.

Accumulation equations for physical and human capital are given by

\[ k_{t+1} = (1 - \delta)k_t + i_t, \]  
\[ h_{t+1} = (1 - \delta_h)h_t + e_t^{\omega_h}h_t^{\omega_h}(z_t^{1/(1-\theta_h-\theta_k-\phi)}n_{h,t})^{1-\omega_e-\omega_h}. \]  

The accumulation equation for physical capital, equation (5), is standard, with capital depreciating at rate \( \delta \). There is no consensus in the literature on the correct specification for the human capital production function in equation (6), with authors including subsets of our three inputs, human capital \( (h) \), expenditures on education \( (e_t) \), and hours spent on education \( (n_{h,t}) \). Lucas (1988) introduces a human-capital-based endogenous growth model in which human capital itself, augmented by hours devoted to human capital accumulation, is the argument in the human capital production. Mankiw, Romer, and Weil (1992) also use a single input but with the assumption that human capital is fully produced by expenditures, measured as education enrollment. Klenow and Rodriguez-Clare (1997), Ben-Porath (1967), Rebelo (1991), Glomm and Ravikumar (1992) and Manuelli, Seshadri, and Shin (2012) include all three inputs.

An alternative way to model human capital accumulation is “learning by doing.” With this specification, employment is the argument in human capital production, and anything which boosts employment raises human capital. Undoubtedly, some human capital accumulates this way, but this specification ignores the large inputs directed toward education in the forms both of outright education expenditures and hours which students spend in school.

There is also no consensus on the appropriate returns to scale for the inputs. We assume that the production function for human capital is jointly constant returns to scale in these three factors and consider sensitivity analysis allowing increasing returns. We also assume that a technology growth factor \( (z_t^{1/(1-\theta_k-\theta_h-\phi)}) \), based on the same technology growth factor as output, augments hours, allowing a balanced growth path.
with constant hours, and with expenditures on education and human capital growing at
the rate of growth of the economy.\(^5\) Human capital depreciates at rate \(\delta_h\).

The household’s choice variables include \(c_t, i_t, e_t, b_{t+1}, k_{t+1}, h_{t+1}, n_{w,t}, \) and \(n_{h,t}\).

The household takes the hourly wage \(w_t\), the human capital wage \(w_{h,t}\), the dividend rate \(d_t\), labor and capital tax rates \(\tau_{n,t}, \tau_{k,t}\), the education subsidy rate \(s_t\), government consumption \(g_{c,t}\) and the gross interest rate \(R_{b,t}\) as exogenously given. Define \(\lambda_t, \zeta_t, \) and \(\mu_t\) as the Lagrange multipliers on the consumer’s budget constraint (equation 4) and the physical and human capital accumulation equations, (5) and (6), respectively.

The first order conditions with respect to \(c_t, i_t\) and \(e_t\), respectively, yield definitions of the multipliers according to

\[
\lambda_t = U_c(t), \tag{7}
\]

\[
\lambda_t = \zeta_t, \tag{8}
\]

\[
(1 - s_t)\lambda_t = \mu_t \omega_e \omega^{-1} (z_{t}^{1/(1-\theta_k-\theta_h-\phi)} n_{h,t})^{1-\omega_e-\omega_h}. \tag{9}
\]

Equation (7) states that the multiplier on the budget constraint is the marginal utility of consumption, as is standard. Equation (8) equates the marginal utility of capital with the marginal utility of consumption. The first order conditions with respect to expenditures on physical and human capital, equations (8) and (9), differ due to both the subsidy on education expenditures and the difference in production functions for human and physical capital. If \(s_t = 0, \omega_e = 1\) and \(\omega_h = 0\), the two first order conditions would be identical, and the marginal value of human capital would equal the marginal value of physical capital.

The first order conditions with respect to \(b_{t+1}, k_{t+1}\) and \(h_{t+1}\) yield Euler equations in each of the three assets according to

\[
\lambda_t = \beta E_t \{\lambda_{t+1} R_{b,t+1}\}, \tag{10}
\]

\[
\zeta_t = \beta E_t \{\lambda_{t+1} [(1 - \tau_{k,t+1})(\theta_k y_{t+1} k_{t+1}^{-1} - \delta) + \delta] + \zeta_{t+1} (1 - \delta)\}, \tag{11}
\]

\(^5\) We thank John B. Jones for suggesting this specification.
\[ \mu_t = \beta E_t \{ \lambda_{t+1} \left[ \theta_h (1 - \tau_{n,t+1}) \frac{y_{t+1}}{R_{t+1}} \right] + \mu_{t+1} \left[ 1 - \delta_h + \omega_h \epsilon_t \omega_h^{h_{t+1}} h_t^{-1} (z_t^{1/(1-\theta_k-\theta_h-\phi)} n_{h,t+1})^{1-\omega_e-\omega_h} \right] \}, \]  

(12)

where the agent takes \( h_{a,t+1} \) as exogenous in choosing \( h_{t+1} \). The first order condition on bonds is the standard Euler equation. Defining \( R_t \) as

\[ R_t = (1 - \tau_{k,t})(d_t - \delta) + 1, \]  

(13)

we can substitute into equation (11), using equation (8), to write the first order condition on capital as

\[ \lambda_t = \beta E_t \{ \lambda_{t+1} R_{t+1} \}. \]

It is useful to compare equations (11) and (12). The marginal values for physical capital and human capital have similar recursive expressions. Each equals the sum of the expected present value of marginal utility of income from the asset plus the marginal value of human/physical in the next period. The marginal value for human capital investment contains an additional positive term \( [\mu_{t+1} \omega_h \epsilon_{t+1} h_{t+1}^{1/(1-\theta_k-\theta_h-\phi)} n_{h,t+1})^{1-\omega_e-\omega_h} \] Since human capital is a factor of production in human capital, an additional unit adds to the productive capacity of human capital, raising the marginal product of human capital.

The first order conditions with respect to the allocation of hours, \( n_{w,t} \) and \( n_{h,t} \), can be expressed as

\[ \frac{\alpha_n}{1 - n_{w,t} - n_{h,t}} = \lambda_t w_t (1 - \tau_{n,t}), \]  

(14)

\[ \frac{\alpha_n}{1 - n_{w,t} - n_{h,t}} = \mu_t (1 - \omega_e - \omega_h) \epsilon_t \omega_h^{h_{t+1}} z_t^{1/(1-\theta_k-\theta_h-\phi)} n_{h,t}^{1-\omega_e-\omega_h}. \]  

(15)

The marginal cost of hours, which is the marginal cost of giving up leisure for both of the two uses of hours, must equal the marginal benefit. For labor hours, the later is determined by the after-tax wage multiplied by the marginal utility of consumption. For school hours, the marginal benefit is the addition to human capital created by the additional study hours multiplied by the marginal value of human capital.
2.1.3 Government

We assume that government purchases three types of goods and services: utility-enhancing government consumption \( (g_{c,t}) \), education subsidies \( (g_{h,t}) \), and something exogenous \( (g_t) \). The government budget constraint requires that total expenditures on goods and services plus transfer payments and interest on government debt equal tax revenue plus new government debt. The budget constraint is given by

\[
g_{c,t} + g_{h,t} + g_t + TR_t + R_{b,t}b_t = T_t + b_{t+1},
\]

where government spending on education \( (g_{h,t}) \) is given by

\[
g_{h,t} = e_t s_t,
\]

and tax revenues \( (T_t) \) are

\[
T_t = \tau_{n,t}(w_{tn_{w,t}} + w_{h,t}h_t) + \tau_{k,t}(d_t - \delta)k_t.
\]

We assume that exogenous government spending, transfer payments \( (TR_t) \), and government debt \( (b_t) \) all grow exogenously. Our assumptions on endogeneity are based on the following. We assume that the only reason government raises taxes is to provide benefits to agents who are being taxed. However, a representative-agent model does not capture some of these benefits very well. We set these benefits as exogenous. The primary purpose of government transfers is redistribution, implying that a representative-agent model can yield no information on their optimal level, justifying our assumption that they are exogenous. Additionally, government defense spending is chosen for reasons other than steady-state utility maximization. We set government debt as exogenous because government debt patterns are far from those predicted by the optimal tax literature.

Government subsidies to education \( (s_t) \) and utility-enhancing government consumption \( (g_{c,t}) \) are chosen jointly optimally with tax rates \( (\tau_{k,t}, \tau_{n,t}) \) to balance the government budget. This assumption differs the standard in which either transfer payments or government spending, either worthless or utility-enhancing and separable, adjust to balance the government budget.
We characterize the government’s optimal allocation problem by implicitly solving the system for the endogenous variables, derived from the agents’ optimization problems, as a function of the policy variables, \((s_t, g_{c,t})\). For any given tax rates, the benevolent government chooses the policy variables to maximize the consumer’s indirect utility, given by

\[
V(s_t, g_{c,t}) = E_0 \sum_{t=0}^{\infty} \beta^t \{ \log c_t(s_t, g_{c,t}) + \alpha_n \log(1 - n_{w,t}(s_t, g_{c,t}) - n_{h,t}(s_t, g_{c,t})) + \alpha_g \log g_{c,t} \},
\]

subject to the constraints in the economy

\[
g_{c,t} + g_{h,t}(s_t, g_{c,t}) + g_t + R_{b,t} b_t + TR_t = b_{t+1}(s_t, g_{c,t}) + \tau_{n,t}(1-\theta_k)g_t(s_t, g_{c,t}) + \tau_k \theta_k y_t(s_t, g_{c,t}) - \tau_{k,t} \delta k_t,
\]

\[
k_{t+1}(s_t, g_{c,t}) = (1 - \delta)k_t + i_t(s_t, g_{c,t}),
\]

\[
h_{t+1}(s_t, g_{c,t}) = (1 - \delta_h)h_t + e_t(s_t, g_{c,t})^{\omega_h} h_t^{\omega_h} n_{h,t}(s_t, g_{c,t})^{1-\omega_e-\omega_h},
\]

\[
y_t(s_t, g_{c,t}) + m_t = c_t(s_t, g_{c,t}) + i_t(s_t, g_{c,t}) + e_t(s_t, g_{c,t}) + g_t + g_{c,t},
\]

\[
g_{h,t}(s_t, g_{c,t}) = e_t(s_t, g_{c,t}) s_t.
\]

### 2.2 Stationary Equilibrium

Along the balanced growth path, \(n_{w,t}, n_{h,t}, R_t, R_{b,t}, \tau_{n,t}, \tau_{k,t}\) and \(s_t\) are stationary, and all other variables grow at the constant rate \(\psi = \xi^{1/(1-\theta_k-\theta_h-\phi)}\). We require that government bonds, the exogenous component of government spending, transfer payments, and income from net foreign assets all grow exogenously at the rate of growth of the economy. In order to obtain stationary solutions, we detrend all the growing variables before solving for the system. We denote detrended growing variables with tildes. Non-growing variables are not detrended and retain their original notation.

All variables, which grow at the rate of growth of the economy, are detrended by dividing by \(\psi^t\) to yield \(\tilde{x}_t = x_t/\psi^t\), where the tilde denotes detrended values. Lagrange multipliers grow at a different rate. Multiplying both sides of equation (7) by \(\psi^t\) implies that \(\tilde{\lambda}_t = 1/\tilde{c}_t = \psi^t \lambda_t\). Similarly, equations (8) and (9) imply that detrended multipliers are expressed as \(\tilde{\zeta}_t = \psi^t \zeta_t\) and \(\tilde{\mu}_t = \psi^t \mu_t\). Therefore all the Lagrange multipliers grow at
rate $\psi^{-1}$ along their balanced growth paths.

A competitive equilibrium is a set of plans \{$\bar{c}_t, \bar{\bar{c}}_t, \bar{\bar{b}}_t, \bar{\bar{k}}_{t+1}, \bar{\bar{h}}_{t+1}, n_{w,t}, n_{h,t}$\} satisfying the detrended equations of the model, given exogenous processes \{$\tau_{n,t}, \tau_{k,t}, s_t, TR_t, \bar{b}_t, \bar{\bar{m}}_t, \bar{\bar{g}}_{n,t}, \bar{\bar{g}}_{c,t}, \bar{\bar{g}}_{t}$\} and the initial condition \{$k_0, b_0, h_0, m_0, g_0, TR_0$\}. A detailed description of stationary equilibrium is provided in Appendix A1.

### 2.3 Steady State

In the steady state, $\bar{x}_{t+1} = \bar{x}_t$. We denote steady state values of the variables by dropping time subscripts to yield $\bar{x} = \bar{x}_{t+1} = \bar{x}_t$. The full system of steady state equations is described in Appendix A2. Here we present steady-state values of interest for analysis and calibration. First, the steady state production function is

$$\bar{y} = \bar{k}^{\theta_k} \bar{h}^{\theta_h} \mu^{1-\theta_h-\theta_k},$$

where $\bar{h}_t = \bar{h}_{a,t}$ in equilibrium. The steady-state equation for the accumulation of physical capital becomes

$$\left(\psi - 1 + \delta\right) \frac{\bar{k}}{\bar{y}} = \frac{\bar{i}}{\bar{y}}.$$  \hspace{1cm} (19)

The steady state Euler equation, with capital as the asset is given by

$$\psi/\beta - 1 = (1 - \tau_k)(\theta_k \bar{y} - \bar{k} - \delta).$$  \hspace{1cm} (20)

These two equations link the net-of-tax return to the capital stock and physical investment. Combining the two equations yields the steady state share of investment as a percentage of output

$$\frac{\bar{i}}{\bar{y}} = \frac{(\psi - 1 + \delta)(1 - \tau_k)\theta_k}{\psi/\beta - 1 + \delta(1 - \tau_k)}.$$  \hspace{1cm} (21)

The investment-to-output ratio is increasing in capital’s share ($\theta_k$) and decreasing in the capital tax rate ($\tau_k$).

The human capital Euler equation (12) in the steady state can be simplified by elim-
inating $\lambda$ and $\mu$, using equations (9) and (6), to yield

$$\hat{\lambda} = (1 - \tau_n) \left[ (\psi - 1 + \delta_h) \omega, \theta_h \right] \left[ \psi / \beta - 1 + \delta_h - \omega_h (\psi - 1 + \delta_h) \right].$$  \hspace{1cm} (22)

The long-run human capital expenditure-to-output ratio is increasing in human capital’s share ($\theta_h$) and in $1 - \tau_n$.

Combining equations (14) and (15) yields the steady state ratio of hours allocated to education and to work as

$$\frac{n_h}{n_w} = \frac{(1 - \tau_n)}{(1 - s)} \left[ \frac{1 - \omega_e - \omega_h}{\omega_e (1 - \theta_k - \theta_h)} \right] \frac{\hat{\lambda}}{\hat{y}}.$$  \hspace{1cm} (23)

This equation can be further simplified by eliminating $\frac{\hat{\lambda}}{\hat{y}}$, using equation (22), to yield

$$\frac{n_h}{n_w} = \frac{\theta_h}{1 - \theta_k - \theta_h} \left[ (\psi - 1 + \delta_h) (1 - \omega_e - \omega_h) \right] \left[ \psi / \beta - 1 + \delta (1 - \tau_k) \right].$$  \hspace{1cm} (24)

This equation implies that the relative time allocation between work and school does not depend on labor taxes or subsidies, and is proportionate to their relative shares $\frac{\theta_h}{(1 - \theta_k - \theta_h)}$.

The ratio of capital to output is derived by solving equation (20) to yield

$$\hat{k} = \frac{(1 - \tau_k) \theta_k}{\psi / \beta - 1 + \delta (1 - \tau_k)}. \hspace{1cm} (25)$$

The steady state value of human capital relative to output can be written as

$$\left( \psi - 1 + \delta_h \right) \hat{h} = (\hat{\lambda}/\hat{y}) \omega \left[ \frac{n_h}{\hat{y}} \right]^{1 - \omega_e - \omega_h}. \hspace{1cm} (26)$$

Given solutions for the great ratios and for labor hours, detrended output in the steady state can be expressed as

$$\hat{y} = \left( \frac{\hat{k}}{\hat{y}} \right)^{\theta_k/(1 - \theta_k - \theta_h - \phi)} \left( \frac{\hat{h}}{\hat{y}} \right)^{(\theta_h + \phi)/(1 - \theta_k - \theta_h - \phi)} \left( \frac{n_w}{\hat{y}} \right)^{(1 - \theta_k - \theta_h)/(1 - \theta_k - \theta_h - \phi)}. \hspace{1cm} (27)$$

The consumption-to-GDP ratio can be solved from the equation for the consumer’s
budget constraint in steady state, equation (4), yielding

\[
\frac{\tilde{c}}{\tilde{y}} = (1 - \tau_n)(\theta_n + \theta_h) + (1 - \tau_k)\theta_k + \tau_k\delta\tilde{k}/\tilde{y} + \tilde{TR}/\tilde{y} + (\psi/\beta - \psi)\tilde{b}/\tilde{y} - \tilde{i}/\tilde{y} - (1 - \sigma)\tilde{e}/\tilde{y} + \tilde{m}/\tilde{y}.
\] (28)

Hours allocated to work and schooling, respectively, in the steady state can be solved by combining the steady state versions of equations (14) and (15)

\[
n_w = \frac{(1 - \theta_k - \theta_h)}{(1 - \theta_k - \theta_h) + \alpha_n \frac{\tilde{c}}{\tilde{y}} + \frac{(1 - \sigma)\tilde{c}1 - \omega_e - \omega_h}{(1 - \tau_n)\tilde{y} \omega_e}}
\] (29)

\[
n_h = \frac{\frac{(1 - \sigma)\tilde{c}1 - \omega_e - \omega_h}{(1 - \tau_n)\tilde{y} \omega_e}}{(1 - \theta_k - \theta_h) + \alpha_n \frac{\tilde{c}}{\tilde{y}} + \frac{(1 - \sigma)\tilde{c}1 - \omega_e - \omega_h}{(1 - \tau_n)\tilde{y} \omega_e}}.
\] (30)

Note that taxes and subsidies distort the decision on hours only due to the distortion on consumption since \(\frac{(1 - \sigma)\tilde{c}}{(1 - \tau_n)\tilde{y}}\) is independent of both subsidies and taxes from equation (22).

The education subsidies in the steady state can be written as

\[
\tilde{g}_h = \tilde{e}s.
\] (31)

Substituting into equation (17), detrended long-run tax revenues can be expressed as

\[
\tilde{T} = \tau_n(1 - \theta_k)\tilde{y} + \tau_k\theta_k\tilde{y} - \tau_k\delta\tilde{k}.
\] (32)

### 2.4 Externalities and Subsidies

To facilitate understanding of the welfare implications of taxation decisions, we provide a solution to the planner’s problem in Appendix B. We denote steady-state values for the planner’s detrended optimal values with hats (\(\hat{x}\)). The planner’s optimal choice for \(k/y, e/y, n_h/n_w, n_h\) and \(n_w\) can be expressed as

\[
\frac{\hat{k}}{\hat{y}} = \frac{\theta_k}{\psi/\beta - 1 + \delta}
\] (33)

\[
\frac{\hat{e}}{\hat{y}} = \frac{\omega_e(\theta_h + \phi)(\psi - 1 + \delta_h)}{[\psi/\beta - 1 + \delta_h - \omega_h(\psi - 1 + \delta_h)]}
\] (34)
\[
\begin{align*}
\frac{n_h}{n_w} &= \frac{1 - \omega_c - \omega_h \hat{e}}{\omega_c (1 - \theta_k - \theta_h) \hat{y}} \\
n_w &= \frac{(1 - \theta_k - \theta_h)}{(1 - \theta_k - \theta_h) + \alpha_n \hat{c} \hat{y} + \frac{1 - \omega_c - \omega_h \hat{c}}{\omega_c \hat{y}}} \\
n_h &= \frac{1 - \omega_c - \omega_h \hat{e}}{(1 - \theta_k - \theta_h) + \alpha_n \hat{c} \hat{y} + \frac{1 - \omega_c - \omega_h \hat{c}}{\omega_c \hat{y}}}
\end{align*}
\]

where hours are understood to be the planner’s choices and are not given hat notation since they are not detrended.

We illustrate the distortions due to taxation and externalities by comparing the planner’s optimal choices with solutions in decentralized economies. Equations (25) and (33) can be used to demonstrate that the capital-to-output ratio in the decentralized economy is too low when capital taxes are positive.

From equations (22) and (34), the share of expenditures on education is proportional to its optimal value according to

\[
\frac{\hat{e}}{\hat{y}} = \frac{(1 - \tau_n)}{(1 - s)} \left[ \frac{\theta_h}{(\theta_h + \phi)} \right] \frac{\hat{e}}{\hat{y}}
\]

When subsidies and taxes are zero, the externality implies that agents choose too little expenditure on education. Taxes and subsidies can be chosen to offset this externality exactly. However, this leaves the consumption decision distorted, since from equation (28), consumption relative to income depends on the levels of taxes and subsidies. The distortion in consumption relative to income implies a distortion in hours worked and in hours allocated to human capital development from equations (36) and (37).

3 Calibration and Parameterization

The model is calibrated to annual data for the US economy. We follow Trabandt and Uhlig (2011) and choose the sample as 1995 to 2007. The data are from Federal Reserve Economic Database, the World Bank Database, the National Income and Product Accounts (NIPA), and the Digest of Education Statistics as described in Appendix C.

For tax rates, we use flat taxes, calculated using the methodology proposed by Mendoza, Razin, and Tesar (1994), instead of marginal tax rates. Calculations by Trabandt
and Uhlig (2011) yield average capital income tax and labor income tax rates of $\tau_k = 0.36$, $\tau_n = 0.28$, respectively.

The exogenous balanced growth factor ($\psi$) is set to 1.03, corresponding to the annual real GDP growth rate of 3% per capita over this period. For the interest rate on bonds, we average the annual real interest rate over 1995-2007 from the world bank database to yield $R_b = 1.048$. $\beta$ is calibrated from the steady state of equation (10) to yield $\beta = \psi/R_b = 0.9828$.

The capital depreciation rate is calibrated from the capital production function in steady state, equation (19), where capital and investment both include private and public values from NIPA. Substituting values into equation (19) yields a value for depreciation as $\delta = 0.04$.

We assume $\delta_h = \delta$, consistent with Mankiw, Romer, and Weil (1992) and Trabandt and Uhlig (2011).

The parameter on capital in the production function ($\theta_k$) is calibrated from equation (20), which can be rewritten as:

$$\theta_k = \frac{\psi/\beta - 1 + \delta(1 - \tau_k) \hat{k}}{1 - \tau_k} \frac{\hat{k}}{\hat{y}}$$

where $\frac{\hat{k}}{\hat{y}}$ is calibrated from NIPA data, averaged over the period. Substituting for values of parameters on the right hand-side yields $\theta_k = 0.33$.

Following Prescott (2002, 2004), McGrattan and Rogerson (2004), and Trabandt and Uhlig (2011), hours worked $n_w$ is calibrated at 0.25, consistent with evidence on hours worked per person aged 15-64 for the US. Schooling hours $n_h$ is calibrated to 0.06, approximately a quarter of $n_w$, following Trabandt and Uhlig (2011).

We split government spending into an exogenous component ($\hat{g}$), spending on education ($\hat{g}_h$), and a component which yields utility ($\hat{g}_c$). We let defense spending be the exogenous component and calibrate $\hat{g}$ to be the average of defense spending over the sample, yielding 4.3% of GDP. $\hat{g}_h$ is the average public spending on education, or 5.2% of GDP. $\hat{g}_c$ corresponds to the total government consumption expenditures subtracting government education expenditures and the defense expenses, yielding 5.7% of GDP. $\hat{c}/\hat{y}$ is set to match the expenditures of educational institutions as a percent of GDP, yielding
7.2%. Using equation (31), the subsidy rate \( s \), is the average of government education expenditures \( \tilde{g}_h \) relative to total expenditures by educational institutions \( \tilde{e} \), yielding 72%. Government debt relative to GDP \( \tilde{b} \) is calibrated as 38.2% of GDP, using federal debt held by the public. We set \( TR/\tilde{y} = 10.7\% \) which corresponds to the ‘implicit’ government transfer payments to GDP ratio in the data.\(^6\)

The ratio of net imports to GDP \( \hat{m}/\hat{y} \) is calibrated as the average value of imports relative to GDP, or 3.8%. Investment-to-GDP ratio \( \hat{i}/\hat{y} \) is calibrated using the aggregate investment relative to GDP at 19.7%. Consumption-to-GDP ratio \( \hat{c}/\hat{y} \) is set to match the average ratio of private consumption to GDP in the data.\(^7\)

We can find no literature that provides directly relevant methods for calibrating the parameters \( \theta_h \) and \( \phi \) in the human-capital production function. Related literature includes Mankiw, Romer, and Weil (1992), who estimate a constant returns-to-scale production function with human capital in a cross-country regression.\(^8\) They find that the coefficient on human capital is 0.28. Their estimate has been criticized for failing to account for country-specific effects. OLS estimates are biased when unobserved effects are correlated with the right-hand-side variables, in this case, the factors of production. Islam (1995) has argued that the extent of the bias could be large. Lucas (1988) proposed a different production technology and estimated the coefficient on labor hours multiplied by human capital as 0.75 and the externality term as 0.417. However, since the model framework is different, it has limited relevancy to our calibration.

Gao (2013) proposed a dynamic panel framework to capture the unobserved country-effects. After accounting for the unobserved country effects, the estimated coefficient on human capital is reduced from 0.28 in Mankiw, Romer, and Weil (1992) to 0.23. This finding is in line with other empirical results which find that the coefficient on human capital is much lower than one-third.\(^9\) Additionally, since there is an externality in this

\(^6\)We follow Trabandt and Uhlig (2011) and compute the government transfer-payment-to-income ratio that is consistent with the model using the steady state of government budget constraint (16),

\[ TR/\tilde{y} = \tilde{T}/\tilde{y} - (R_b - \psi)b/\tilde{y} - \tilde{g}_c/\tilde{y} - \tilde{g}/\tilde{y} - \tilde{g}_h/\tilde{y}. \]

We subtract government interest payments, government consumption, defense expenses, and public spending on education as a percentage of GDP in the data from the model-predicted tax revenue-to-income ratio in equation (32).

\(^7\)Private spending on education has been deducted from private consumption expenditures. It is calculated as the difference between total education expenditures and public spending on education.

\(^8\)Constant returns to scale requires the assumption that \( \phi = 0 \).

\(^9\)For example, see Islam (1995) and Caselli, Esquivel, and Lefort (1996).
model, the total coefficient on human capital is $\theta_h + \phi$ from equation (18) corresponds to the empirical estimate of human capital coefficient in Gao (2013). Therefore, we calibrate $\theta_h + \phi = 0.23$ and use this estimate in the benchmark analysis.\textsuperscript{10}

To calibrate parameter values for factor shares in the production function for human capital contained in equation (6), we use evidence provided by Kendrick (1976). He finds that the technology for producing human capital is intensive in labor. Approximately 50% of investment in human capital in the US represents the opportunity cost of student time, with the remaining 50% composed of human capital and physical resources. Hence, we follow this estimation and set the coefficient on schooling hours $(1 - \omega_e - \omega_h)$ to 0.5.

Given this condition, together with equations (22) (29) and (30), and the restriction $\theta_h + \phi = 0.23$, there are five equations with five unknown parameters $\{\omega_e, \omega_h, \theta_h, \phi, \alpha_n\}$. Solving this system of equations yields the benchmark calibration: $\omega_e = 0.12$, $\omega_h = 0.38$, $\theta_h = 0.20$, $\phi = 0.03$ and $\alpha_n = 1.41$. We conduct sensitivity analysis for alternative values of $\{\omega_e, \omega_h\}$ and $\{\theta_h, \phi\}$.

The final parameter to calibrate is the coefficient on government consumption in the utility function ($\alpha_g$). We do not have a closed-form solution for the steady-state ratio of government consumption to income from the government’s optimal allocation problem. Therefore, conditional on values for other baseline calibrations and the benchmark tax rates, we numerically compute the ratio obtained from the optimal allocation of government expenditure between government consumption and education subsidies for alternative values of $\alpha_g$. We choose the value of $\alpha_g$ for which the ratio most closely matches the value in the data of 5.7%. This yields a value of $\alpha_g = 0.06$, which we use in the benchmark calibration.

An overview of the benchmark model calibration is summarized in Tables 1 and 2. The calibrations allow us to match the moments in the data perfectly by construction.

\textsuperscript{10}The estimation is based on a sample of 89 countries excluding major oil-producers over 1970-2010. As a robustness check, we adopted the same methodology and estimated the coefficients over 1995-2007. The coefficients are almost identical. In particular, the estimated coefficient on human capital is 0.24.
Table 1: Part 1 of the Baseline Calibration, Parameter Values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
<th>Restriction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$</td>
<td>1.03</td>
<td>Exogenous growth rate</td>
<td>Data</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9828</td>
<td>Subjective discount rate</td>
<td>$\beta = \psi/R_b$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.04</td>
<td>Depreciation rate for capital</td>
<td>Data</td>
</tr>
<tr>
<td>$\delta_h$</td>
<td>0.04</td>
<td>Depreciation rate for human capital</td>
<td>$\delta_h = \delta$</td>
</tr>
<tr>
<td>$\theta_k$</td>
<td>0.33</td>
<td>Parameter on $k_t$ in production</td>
<td>Equation (38)</td>
</tr>
<tr>
<td>$\theta_h$</td>
<td>0.20</td>
<td>Parameter on $h_t$ in production</td>
<td>Equations (22) (29) and (30)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.03</td>
<td>Parameter on $h_{a,t}$ in production</td>
<td>Literature</td>
</tr>
<tr>
<td>$\omega_e$</td>
<td>0.12</td>
<td>Parameter on $e_t$</td>
<td>Equations (22) (29) and (30)</td>
</tr>
<tr>
<td>$\omega_h$</td>
<td>0.38</td>
<td>Parameter on $h_t$</td>
<td>Literature</td>
</tr>
<tr>
<td>$\alpha_n$</td>
<td>1.41</td>
<td>Parameter on leisure in utility</td>
<td>Equations (22) (29) and (30)</td>
</tr>
<tr>
<td>$\alpha_g$</td>
<td>0.06</td>
<td>Parameter on $g_{c,t}$ in utility</td>
<td>Data</td>
</tr>
</tbody>
</table>

Table 2: Part 2 of the Baseline Calibration, Great Ratios and Others

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
<th>Restriction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_n$</td>
<td>0.28</td>
<td>Labor Tax Rate</td>
<td>Data</td>
</tr>
<tr>
<td>$\tau_k$</td>
<td>0.36</td>
<td>Capital Tax Rate</td>
<td>Data</td>
</tr>
<tr>
<td>$n_w$</td>
<td>0.25</td>
<td>Hours Worked</td>
<td>Literature</td>
</tr>
<tr>
<td>$n_h$</td>
<td>0.06</td>
<td>Hours on Education</td>
<td>Literature</td>
</tr>
<tr>
<td>$R_b$</td>
<td>1.048</td>
<td>Real Interest Rate</td>
<td>Data</td>
</tr>
<tr>
<td>$\tilde{g}/\tilde{y}$</td>
<td>4.3%</td>
<td>Defense Expenses to GDP</td>
<td>Data</td>
</tr>
<tr>
<td>$g_{h}/\tilde{y}$</td>
<td>5.2%</td>
<td>Public Spending on Education to GDP</td>
<td>Data</td>
</tr>
<tr>
<td>$g_{c}/\tilde{y}$</td>
<td>5.7%</td>
<td>Government Non-defense Consumption to GDP</td>
<td>Data</td>
</tr>
<tr>
<td>$\tilde{c}/\tilde{y}$</td>
<td>7.2%</td>
<td>Education Expenses to GDP</td>
<td>Data</td>
</tr>
<tr>
<td>$s$</td>
<td>0.72</td>
<td>Subsidy Rate</td>
<td>Data</td>
</tr>
<tr>
<td>$\tilde{b}/\tilde{y}$</td>
<td>38.2%</td>
<td>Debt to GDP</td>
<td>Data</td>
</tr>
<tr>
<td>$\tilde{T}/\tilde{y}$</td>
<td>10.7%</td>
<td>Government Transfers to GDP</td>
<td>Implicit</td>
</tr>
<tr>
<td>$\tilde{m}/\tilde{y}$</td>
<td>3.8%</td>
<td>Net Import to GDP</td>
<td>Data</td>
</tr>
<tr>
<td>$\tilde{i}/\tilde{y}$</td>
<td>19.7%</td>
<td>Investment to GDP</td>
<td>Data</td>
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<tr>
<td>$\tilde{c}/\tilde{y}$</td>
<td>66.9%</td>
<td>Private Consumption to GDP</td>
<td>Data</td>
</tr>
</tbody>
</table>

4. Results

4.1 Steady State Laffer Curves

We characterize the Laffer curves for labor taxes and capital taxes. Laffer curves are obtained by varying the steady state labor (capital) tax rates, while holding the capital (labor) tax rate and other parameters fixed. As the tax rate changes, the benevolent
government optimally allocates additional tax revenue between public consumption and education subsidies to maximize the utility of the representative agent. We do not have an analytical solution for the optimal allocation. Therefore, we compare alternative allocations numerically and select the one with highest utility. We compute Laffer curves under the assumption of optimal allocation of marginal tax revenue and the assumption that subsidies are fixed at their benchmark rate, so that all marginal tax revenues are allocated to government consumption, and compare them. We demonstrate that optimal allocation substantially alters the shape of the Laffer curve, increasing its slope at low tax rates and raising its peak.

4.1.1 Labor Taxes

Figure 1 compares Laffer curves for labor income tax rates under the two alternative allocations of marginal tax revenue. The solid curve is the Laffer curve with optimal allocation while the dotted curve allocates all marginal revenue to government consumption. On the vertical axis, tax revenues at the benchmark tax rate of $\tau_n = 0.28$ are set equal to 100. Figure 1 illustrates a substantial increase in slope for low values of the tax rate and an increase in the peak. The peak of the Laffer curve with optimal allocation occurs with an additional 71% in tax revenues, while the peak yields only 49% more tax receipts with marginal revenue allocated toward government consumption. Under optimal allocation, the peak occurs at the labor tax rate equal to 0.70, implying that substantial increases in the tax rate would continue to yield increases in tax revenues; when the subsidy rate is fixed at the benchmark, the peak occurs at a labor tax rate of 0.65. Consideration of productive government spending implies that we are further from the Laffer curve peak.

Figure 1 inserted here

Figure 2 graphs the optimal subsidy rate and expenditures on human capital as a function of the tax rate. As the tax rate rises, the optimal subsidy rate for education expenditures initially increases rapidly. This implies that expenditures on human capital are increasing rapidly in the tax rate, peaking at an additional 159%. When the subsidy rate is fixed, expenditures on human capital decline in the tax rate since production is falling in the tax rate.
Figure 3 graphs output as a function of the labor tax rate. Under optimal allocation of marginal tax revenue, production increases as the labor tax rate rises from a low rate, eventually peaking and falling. Under full allocation of marginal revenue to government spending, production falls immediately as the tax rate increases, illustrating the importance of the allocation of marginal tax revenue.

Figure 4 illustrates the effect of increases in the labor tax rate on human and physical capital stocks and on hours allocated to schooling and to work. Human and physical capital stocks both have hump shapes with optimal revenue allocation, while both decline in the tax rate with full allocation to government consumption. Since physical capital is complementary to human capital, physical capital mimics the hump shape for human capital. The peak for human capital is about 7% higher than its current value while the peak in physical capital is only about 0.1% higher. Hours allocated toward either non-leisure activity are downward-sloping in the tax rate and are not affected by the allocation of tax revenue. The higher labor tax rate discourages hours allocated to both work and school because the ultimate return on both depends on the after-tax wage.

Finally, Figure 5 graphs the effect of an increasing labor tax on human capital subsidies and on government consumption. As the labor tax rate rises, both the value of subsidies to human capital and expenditures on utility-enhancing government spending exhibit a hump shape under optimal allocation. Under full allocation to utility-enhancing government consumption, government subsidies fall as the tax rate falls due to the output response to tax rates. Government consumption exhibits a hump shape with a higher peak.
4.1.2 Capital Taxes

The Laffer curves for capital taxes are graphed in Figure 6. The capital-tax Laffer curve with the optimal allocation of spending has a less pronounced increase in slope at lower tax rates and a smaller increase in peak compared with the labor-tax curve. With optimal allocation of marginal tax revenue, the government could raise an additional 12% in tax revenues by increasing capital taxes, while the peak yields only 8% more tax receipts with marginal revenue allocated toward government consumption. Under optimal allocation, the peak occurs at a capital tax rate equal to 0.67, compared with a capital rate of 0.63 at the peak with the subsidy rate fixed at the benchmark.

[Figure 6 inserted here]

Figure 7 illustrates effects of the increase in the capital tax rate on the optimal subsidy rate and on education expenditures. As the capital tax rate increases, the government optimally raises the subsidy rate, similar to the case with labor taxes.

[Figure 7 inserted here]

Figure 8 illustrates the effect of an increase in the capital tax rate on production. There is a slight hump shape with a peak at a tax rate lower than the current rate. Figure 9 reveals that human and physical capital initially move in opposite directions in response to an increase in the capital tax rate. Human capital increases initially, responding to the higher subsidy rate, while physical capital responds negatively to the reduced return implied by the higher capital tax rate. The response of hours is independent of the allocation of marginal tax revenue, as for the case with labor taxes.

[Figure 8 inserted here]

[Figure 9 inserted here]

Figure 10 illustrates additional government expenditures on education and on government consumption allowed by the increase in the tax rate. Both are hump-shaped with optimal allocation, while government expenditures on education fall continuously with allocation only to government consumption. The pattern is similar to that for labor taxes.

[Figure 10 inserted here]
Finally, Figure 11 depicts the Laffer curve ‘surface’ along both the capital and labor tax rate dimensions. The curve is steeper along the labor tax direction; after the peak has been reached, the curve falls sharply in almost any direction. The peak of the Laffer curve surface occurs at $\tau_n = 0.69$ and $\tau_k = 0.44$. An additional 73% in tax revenues can be achieved at the peak.

4.2 Sensitivity Analysis

4.2.1 Human Capital Externalities

There is no consensus among economist about either the existence or magnitude of human capital externalities (Moretti (2004b)). Therefore, we conduct sensitivity analysis allowing for different magnitudes for the externality.

Arguing for a larger elasticity, Rauch (1993) estimates a log wage equation including average schooling in cities and own schooling, in addition to other terms. Defining the total labor income as $W_t \equiv w_t n_{w,t} + w_{h,t} h_t$, our wage equation in logarithms can be expressed as

$$
\log W_t = \log(1 - \theta_k) + \log z_t + \theta_k \log k_t + \theta_h \log h_t + (1 - \theta_k - \theta_h) \log n_{w,t} + \phi \log h_{a,t}.
$$

Rauch (1993) finds that coefficient on average schooling in cities is 0.033 while the coefficient on own education is 0.048.

We cannot directly use his estimates since there are different conditioning variables in his equation and ours. We use his estimates to reflect the relative magnitudes of the effects of average and own education on wages, subject to the constraint that the sum of the coefficients equals 0.23. Rauch’s estimates imply that the relative strength of externalities is 0.033/0.048, or roughly 0.69, which corresponds to $\theta_h/\phi$ in (39). This ratio implies that $\theta_h = 0.14$ and $\phi = 0.09$. In Lucas (1988), the relative strength of externalities is 0.417/0.7, approximately 0.60. Ciccone and Peri (2002) and Moretti (2004a), also find significant positive spill-overs with similar magnitudes. Therefore, we use the relative
strength of the elasticities from Rauch (1993) as an alternative by considering Laffer curves with \( \theta_h = 0.14, \phi = 0.09 \). To focus on the effect of the externality term alone, we retain benchmark values of all other parameter values.

In contrast, Acemoglu and Angrist (1999) find no significant positive human capital spill-overs. Therefore, we also consider Laffer curves with \( \theta_h = 0.23, \phi = 0 \) with all other parameters at benchmark values.

Figures 12 and 13 depict labor-tax and capital-tax Laffer curves with alternative values for the externality parameter \( \phi \). The patterns are very similar to those in our original specification. Whether externalities are stronger or weaker compared with our original specification, Laffer curves with optimal allocation exhibit higher peaks that occur at larger tax rates, compared with the case of subsidies fixed at the optimal rate conditional on the current tax rates.\(^{11}\) Subsidies to human capital expenditures allow higher tax revenue even in the absence of externalities because they offset some of the distortions created by the labor tax.

\[^{11}\text{The benchmark subsidy rate is higher with stronger externalities, since the optimal allocation of revenue from existing tax rates allocates more toward education when externalities are higher.}\]

4.2.2 Technology for Human Capital Production

Calibration of \( \omega_e, \omega_h \) is implemented by matching the model-predicted great ratios to the data, with a restriction of \( \omega_e + \omega_h = 0.5 \) from external evidence in the literature. This restriction is essential, without which the system of equations (22) (29) and (30) would be under-identified with respect to the coefficients \( \omega_e, \omega_h, \theta_h, \) and \( \alpha_n \). There is no consensus in the literature on the appropriate technology for human capital production. Our calibration assumes constant returns to scale in the three inputs of education expenditure, labor hours, and human capital. As an alternative specification, Gao (2013) estimates an increasing returns to scale human capital production function

\[ h_{t+1} = (1 - \delta_h)h_t + \varepsilon_t^e (h_t n_{h,t})^{1-\omega}, \]
and concludes that education expenditures explain approximately one-third of human
capital production, with the remaining explained by the combination of two other inputs.
If we keep the relative strength of factor shares unchanged, and impose constant returns
to scale, the human capital production function would be

\[ h_{t+1} = (1 - \delta_h)h_t + \epsilon_t^{0.2}h_t^{0.4}h_{h,t}^{0.4}. \]

This finding is similar to Klenow and Rodriguez-Clare (1997), who estimate a constant
returns to scale human capital production function based on the factor compensations,
and find the shares on expenditures, human capital and students’ time are 0.2, 0.3 and 0.5, respectively.

Following the methodology by Gao (2013), we adopt the constant returns to scale
technology and reestimate the coefficients using data over 1995-2007. We find that stu-
dents’ time has a much smaller coefficient than our baseline specification, with \( \omega_e = 0.11 \)
and \( \omega_h = 0.61 \). The details are explained in Appendix D.

Therefore, we consider two alternative sets of \( \{\omega_e, \omega_h\} \) based on the above empirical
evidence: \( \{\omega_e = 0.20, \omega_h = 0.40\} \) and \( \{\omega_e = 0.11, \omega_h = 0.61\} \). All other parameters
remain the same as in the baseline calibration.

We compute Laffer curves for labor and capital taxes with alternative values for \( \omega_e \)
and \( \omega_h \), and graph them in Figures 14 and 15. For comparison, we also graph Laffer
curves under the assumption that subsidies are fixed at their benchmark rate, with the
benchmark rate set at optimal allocation under the current tax rates. Patterns are
similar to our benchmark results. Laffer curves with optimal allocation exhibit higher
peaks which occur at larger tax rates than those with subsidies fixed at the optimal rate
conditional on the current tax rates.

4.3 Welfare

In this section, we consider the constrained optimal values for labor and capital tax
rates. The constraint is that the government must raise taxes through distortionary labor
and capital taxes to finance exogenous transfer payments, the exogenous component of government spending, and government debt, all of which are growing at the rate of growth of the economy. From this minimum value for taxes, the government could choose to raise additional tax revenue. If so, we assume that the marginal tax revenue is allocated optimally between utility-enhancing government spending and subsidies to education.

First, we present welfare curves, which we define as welfare as a function of the tax rates, under the two alternative assumptions about the allocation of marginal tax revenue. Figure 16 graphs the welfare curve for the labor tax rate when the capital tax rate is fixed at 0.36. Welfare is maximized at a labor tax rate of 0.22, smaller than the current rate of 0.28. Figure 17 shows that welfare is maximized at capital tax rate of 0.10, given the labor tax of 0.28. Note that under optimal allocation of marginal tax revenue, a small change in either tax rate, from an initially low value, creates a very small change in welfare. A reduction in the labor tax rate from 0.28 to its constrained optimal value of 0.22, a 21% decrease in the labor tax rate, increases welfare by only 1.5% in terms of consumption.\footnote{Following Lucas (1987), the magnitude of welfare costs is measured as the percentage decrease in consumption.}

To further understand the welfare implications of labor and capital taxes with productive government spending, we find the combination of labor and capital tax rates that maximizes welfare of the representative agent in the steady state. We emphasize that this is not an optimal taxation calculation because government debt, transfers, and exogenous government spending, all as a fraction of GDP, are held constant. The results are summarized in Table 3. We find that the optimal capital tax is zero, as in the optimal tax literature, and the optimal labor tax is higher at 0.32. Productive government spending does not change the basic result from the optimal tax literature that the capital tax should be zero. The gain in welfare, moving from the current set of taxes to the optimal set is 5% in terms of consumption. At optimal tax rates and optimal allocation, the ratios of aggregates to GDP are lower, except for capital and investment. However, the lower
consumption relative to GDP is due to the rise in GDP since consumption actually rises. The remaining aggregates are lower under optimal taxation.

The last column in Table 3 displays the optimal tax rates and aggregate ratios when the government makes no subsidies to education, equivalently when all government spending is non-productive. The optimal labor tax rate falls from 0.32 to 0.29. In the absence of government subsidies to education, total expenditure on education falls sharply to 2.0%, and output falls relative to the that with optimal subsidies. Consumption-to-GDP rises to 68.2% with consumption falling proportionately with the fall in income. Welfare is substantially lower with no subsidies to education.

### 5 Concluding Remarks

We calculate steady-state Laffer curves for the US under the assumption that marginal tax revenue is optimally allocated between utility-enhancing government consumption and productive subsidies to human capital investment. We find that Laffer curves for both labor and capital taxes have steeper slopes at low tax rates and higher peaks, compared with Laffer curves along which all marginal tax revenue is allocated to government consumption. Therefore, if government optimally allocates marginal tax revenue between productive spending and utility-enhancing spending, then a government’s ability to raise taxes is less constrained than in the absence of productive spending.

For labor taxes, these numbers are large. The peak of the labor-tax Laffer curve
occurs at an additional 79% in tax revenue. This additional revenue is 45% higher than could be achieved with all marginal spending on public consumption. The Laffer curve for the capital tax is relatively flat. Therefore, even though tax revenues are 50% higher with optimal allocation than with allocation to consumption, they are only 12% higher than their current level at the peak of the Laffer curve.

Sensitivity analysis with varying values for the externality associated with human capital confirm the general results about the changes in the shape of the Laffer curve with productive government spending. And our analysis includes only one type of productive government spending, implying that Laffer curve slopes at low tax rates and peaks could be even higher.

The ability to raise additional tax revenue does not mean that such behavior would increase welfare. Under the assumption that government debt, transfer payments, and defense spending are exogenous, we compute optimal values for the labor and capital tax rates with marginal tax revenue allocated optimally to government consumption and to education subsidies. At the optimum, the government should reduce the capital tax rate to zero, consistent with the optimal tax literature, and raise the labor tax rate to provide subsidies to human capital and utility-enhancing government spending. The inclusion of human capital subsidies substantially raises the optimal labor tax. The optimum labor tax rate is 14.3% higher with productive government spending than without in our calibration, while the optimal capital tax remains at zero.

This analysis highlights the importance of productive government spending in determining both the government’s ability to raise tax revenue and the welfare effects of such behavior. The standard assumption, that government spending is not productive, leads to a smaller government than is optimal when productive government spending is allowed. Additionally, since subsidies to education are only one form of productive government spending, we view our calibrations as a lower bound on the effects of including productive government spending on the Laffer curve and welfare.
References


Appendix A. Technical Appendix

Appendix A1. Stationary Equilibrium

Along a balanced growth path, \( n_{w,t}, n_{h,t}, R_t, R_{b,t}, s_t, \tau_{n,t}, \tau_{k,t} \) and \( s_t \) are constant. All other variables grow at rate

\[
\psi = \xi^{1/(1-\theta_k-\theta_h-\phi)}.
\]

In order to obtain stationary solutions, all the growing variables are detrended. Define
Figure 3: Production - Labor Taxes

Figure 4: Effects of Labor Taxes
Figure 5: Government Expenditures - Labor Taxes

Figure 6: Laffer Curves for Capital Taxes

Figure 7: Optimal Subsidy Rate - Capital Taxes
Figure 8: Production - Capital Taxes

Figure 9: Effects of Capital Taxes
Figure 10: Government Expenditures - Capital Taxes

Figure 11: Laffer Curve Surface
Figure 12: Sensitivity Analysis for Externalities: Laffer Curves for Labor Taxes

Figure 13: Sensitivity Analysis for Externalities: Laffer Curves for Capital Taxes

Figure 14: Sensitivity Analysis for Human Capital: Laffer Curves for Labor Taxes
Figure 15: Sensitivity Analysis for Human Capital: Laffer Curves for Capital Taxes

Figure 16: Welfare for Labor Taxes

Figure 17: Welfare for Capital Taxes
the detrended variables along the balanced growth path as \( \hat{x} = x_t/\psi^t \). Since variables are constant along the balanced growth path, we drop the time subscripts.

In the economy with the representative agent, \( h_{a,t} = h_t \), implying that the aggregate per worker production function is given by

\[
y_t = z_t k_t^{\theta_k} n_{w,t}^{1-\theta_k} h_t^{\theta_h} + \phi.
\]

Dividing both sides of the production function by \( z_t = \psi^t \) yields

\[
\tilde{y}_t = \tilde{h}_t^{\theta_h} n_{w,t}^{1 - \theta_h} h_t^{\theta_h} + \phi.
\] (40)

The detrended consumer budget constraint is obtained by dividing equation (4) by \( \psi^t \) to yield

\[
\tilde{c}_t + (1 - s_t) \tilde{c}_t + \tilde{\hat{e}}_t + \tilde{\hat{b}}_{t+1} = (1 - \tau_{n,t})(\hat{w}_t n_{w,t} + w_{h,t} \hat{h}_t) + [(1 - \tau_{k,t})(d_t - \delta) + \delta] \tilde{k}_t + \tilde{T}_t + R_{b,t} \hat{b}_t + \tilde{\hat{m}}_t.
\]

Dividing equations (5), (6), and (4) by \( \psi^t \) yields detrended versions of the physical capital and human capital accumulation equations and the government budget constraint

\[
\psi \tilde{k}_{t+1} = (1 - \delta) \tilde{k}_t + \tilde{\hat{i}}_t,
\] (41)

\[
\psi \tilde{h}_{t+1} = (1 - \delta_h) \tilde{h}_t + \tilde{\hat{e}}_t \tilde{h}_t^{\omega_h} n_{h,t}^{1 - \omega_h - \omega_h},
\] (42)

\[
\tilde{y}_t + \tilde{\hat{g}}_{c,t} + \tilde{T}_t R_t + s_t \tilde{e}_t + R_{b,t} \tilde{b}_t = \tilde{T}_t + \psi \tilde{\hat{b}}_{t+1},
\] (43)

with detrended tax revenues from equation (17) given by

\[
\tilde{T}_t = \tau_{n,t}(1 - \theta_k) \tilde{y}_t + \tau_{k,t} \theta_k \tilde{y}_t - \tau_{k,t} \delta \tilde{k}_t.
\] (44)

Adding the consumer and government budget constraints and imposing equation (3) yields the detrended resource constraint as

\[
\tilde{y}_t + \tilde{\hat{m}}_t = \tilde{c}_t + \tilde{\hat{i}}_t + \tilde{\hat{e}}_t + \tilde{\hat{g}}_{c,t} + \tilde{\hat{y}}_t.
\] (45)
Along the balanced growth path, the Lagrange multipliers shrink at the rate of growth. Therefore, define \( \tilde{\lambda}_t, \tilde{\mu}_t \) as the detrended Lagrange multiplier along the balanced growth path, where

\[
\tilde{\lambda}_t = \lambda_t \psi^t, \quad \tilde{\mu}_t = \mu_t \psi^t.
\]

Since the detrended multipliers must be constant along the balanced growth path, \( \lambda_{t+1}/\lambda_t = \mu_{t+1}/\mu_t = \psi^{-1} \).

The detrended first order conditions with respect to \( \tilde{c}_t, \tilde{i}_t, \tilde{e}_t, \tilde{b}_{t+1}, \tilde{k}_{t+1}, \tilde{h}_{t+1} \) are derived by multiplying equations (7), (8), (9), (10), (11), and (12) by \( \psi^t \) to yield

\[
\tilde{\lambda}_t = \frac{1}{\tilde{c}_t}, \quad \tilde{\lambda}_t = \tilde{\zeta}_t, \quad (1 - s_t)\tilde{\lambda}_t = \tilde{\mu}_t \omega_e \tilde{c}_t^{\omega_e} \tilde{h}_t^{\omega_h} n_{h,t}^{1-\omega_e-\omega_h}, \quad \tilde{\lambda}_t = \beta E_t \{ \psi^{-1} \tilde{\lambda}_{t+1} R_{b,t+1} \},
\]

\[
\tilde{\zeta}_t = \beta \psi^{-1} E_t \{ \tilde{\lambda}_{t+1} [(1 - \tau_{k,t+1}) (\theta_k \frac{\tilde{y}_{t+1}}{\tilde{k}_{t+1}} - \delta) + \tilde{\zeta}_{t+1} (1 - \delta)] \}, \quad \tilde{\mu}_t = \beta \psi^{-1} E_t \{ \tilde{\lambda}_{t+1} \theta_h (1 - \tau_{n,t+1}) \frac{\tilde{y}_{t+1}}{\tilde{h}_{t+1}} + \tilde{\mu}_{t+1} [1 - \delta_h + \omega_h \tilde{c}_t^{\omega_e} \tilde{h}_t^{\omega_h} n_{h,t}^{1-\omega_e-\omega_h}] \}. \]

Next, detrending the first order conditions with respect to \( n_{w,t} \) and \( n_{h,t} \), equations (14) and (15), yields

\[
\frac{\alpha_n}{1 - n_{w,t} - n_{h,t}} = \tilde{\lambda}_t (1 - \theta_k - \theta_h) (1 - \tau_{n,t}) \tilde{y}_t/n_{w,t}, \quad \frac{\alpha_n}{1 - n_{w,t} - n_{h,t}} = \tilde{\mu}_t (1 - \omega_e - \omega_h) \tilde{c}_t^{\omega_e} \tilde{h}_t^{\omega_h} n_{h,t}^{1-\omega_e-\omega_h}.
\]

A competitive equilibrium is a set of plans \( \{\tilde{c}_t, \tilde{i}_t, \tilde{e}_t, \tilde{b}_{t+1}, \tilde{k}_{t+1}, \tilde{h}_{t+1}, n_{w,t}, n_{h,t}\} \) satisfying the equations (40) to (53), given exogenous processes \( \{\tau_{n,t}, \tau_{k,t}, s_t, \tilde{TR}_t, \tilde{b}_t, \tilde{m}_t, \tilde{g}_{h,t}, \tilde{g}_{c,t}, \tilde{g}_t\} \) and the initial condition \( \{k_0, b_0, m_0, g_0, TR_0\} \).
Appendix A2. Steady State

We can solve for the steady-state equilibrium by recognizing that in the steady state all detrended variables must be constant. Therefore, we drop time subscripts and denote steady state values as unsubscripted variables. Using equations (41) and (42), the physical and human capital accumulation equations in steady state are given by

\[(\psi - 1 + \delta)\tilde{k} = \tilde{\iota},\]  
\[(\psi - 1 + \delta_h)\tilde{h} = e^{\omega_e}\tilde{h}^{\omega_h}n_{\tilde{h}}^{-1}\tilde{\omega}_w - \omega_h.\]  

From equation (40), the steady state production function is given by

\[\tilde{y} = \tilde{k}^{\theta_k}n_{\omega}^{-\theta_k}\tilde{h}^{\theta_h}\tilde{\theta} + \phi.\]  

Equation (45) can be used to derive the steady state resource constraint as

\[\tilde{y} + \tilde{m} = \tilde{c} + \tilde{\iota} + \tilde{e} + \tilde{g}_c + \tilde{g}.\]  

Using equation (43), the steady state budget constraint becomes

\[\tilde{g}_c + \tilde{g}_h + \tilde{g} + T\tilde{R} + R_b\tilde{b} = \tilde{T} + \psi\tilde{b},\]  

where, from equation (44), taxes are given by

\[\tilde{T} = \tau_n(1 - \theta_k)\tilde{y} + \tau_k\theta_k\tilde{y} - \tau_k\delta\tilde{k},\]  
\[\tilde{g}_h = s\tilde{e}.\]  

The steady state values for equations (47)-(51) are given by

\[\tilde{\lambda} = \frac{1}{\tilde{c}},\]  
\[\psi = \beta R_b = \beta R,\]  

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\[ \hat{\lambda} = \tilde{\zeta}, \]  
\[ (1-s)\hat{\lambda} = \tilde{\mu}_w e^{\omega_c - 1} \tilde{h} \omega_n^{1-\omega_c - \omega_h}. \]

The steady state equation for \( R \) is obtained by dropping time subscripts from equation (13)
\[
R = (1 - \tau_k)(\theta_k \tilde{y} - \delta) + 1. \tag{65}
\]

Simplification of equation (51) using equations (55) and (64) yields
\[
\frac{\tilde{e}}{\tilde{y}} = \frac{(1 - \tau_n)\omega_c \theta_h (\psi - 1 + \delta_h)}{(1-s)[\psi/\beta - 1 + \delta_h - \omega_h (\psi - 1 + \delta_h)]}. \tag{66}
\]

The steady state versions of equations (52) and (53), after substituting from equation (64) for \( \tilde{\mu}_t \), become
\[
\frac{\alpha_n}{1 - n_w - n_h} = (1 - \theta_k - \theta_h)(1 - \tau_n) \frac{\tilde{y} \tilde{\lambda}}{n_w}, \tag{67}
\]
\[
\frac{\alpha_n}{1 - n_w - n_h} = (1 - s) \frac{1 - \omega_c - \omega_h}{\omega_e} \frac{\tilde{e}}{n_h} \tilde{\lambda}. \tag{68}
\]

**Appendix B. Central Planner’s Problem**

The representative agent fails to account for the effect of his choice of human capital on average human capital, implying an externality. Prices do not incorporate the externalities, resulting in underproduction of human capital. The planner is able to internalize this externality. The planner chooses optimal paths of \( c_t, g_{c,t}, e_t, n_{w,t}, n_{h,t}, i_t, k_{t+1}, h_{t+1} \) to maximize the expected discounted sum of utility, subject to the resource constraint and physical and human capital accumulation equations. We assume that \( z_t = \xi^t \), and \( m_t \) and \( g_t \) grow exogenously along the balanced growth path.
\[
\max_{c_t, g_{c,t}, e_t, n_{w,t}, n_{h,t}, i_t, k_{t+1}, h_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{ \log c_t + \alpha_n \log(1 - n_{h,t} - n_{w,t}) + \alpha_g \log g_{c,t} \}
\]
subject to
\[
c_t + e_t + g_{c,t} + g_t + k_{t+1} - (1 - \delta)k_t = y_t + m_t,
\]

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\[ y_t = z_t k_t^{\theta_k} n_{w,t}^{1-\theta_k-\theta_h} h_t^{\theta_h+\phi}, \]

\[ k_{t+1} = (1 - \delta) k_t + i_t, \]

\[ h_{t+1} = (1 - \delta_h) h_t + \epsilon_t^{\omega_e h} (z_t^{1/(1-\theta_k-\theta_h-\phi)} n_{h,t})^{1-\omega_e-\omega_h}. \]

Letting \( \mu_t \) and \( \lambda_t \) be Lagrange multipliers on the human capital equation and the resource constraint, respectively, the first order conditions with respect to \( c_t, g_{c,t}, e_t, k_{t+1}, h_{t+1}, n_{w,t}, n_{h,t} \) yield

\[ 1/c_t = \lambda_t, \]

\[ \alpha_g/g_{c,t} = \lambda_t, \]

\[ \lambda_t = \mu_t \omega_e \epsilon_t^{\omega_e-1} h_t^{\omega_h} n_{h,t}^{1-\omega_e-\omega_h}, \]

\[ \lambda_t = \beta E_t[\lambda_{t+1}(1 - \delta + \theta_k y_{t+1}/k_{t+1})], \]

\[ \mu_t = \beta E_t\{\mu_{t+1}[1 - \delta_h + \omega_h \epsilon_t^{\omega_e} h_{t+1}^{\omega_h} n_{h,t+1}^{1-\omega_e-\omega_h}] + \lambda_{t+1}(\theta_h + \phi) \frac{y_{t+1}}{h_{t+1}}\}, \]

\[ \frac{\alpha_n}{1 - n_{h,t} - n_{w,t}} = \lambda_t (1 - \theta_k - \theta_h) y_t / n_{w,t}, \]

\[ \frac{\alpha_n}{1 - n_{w,t} - n_{h,t}} = \mu_t (1 - \omega_e - \omega_h) \epsilon_t^{\omega_e} h_t^{\omega_h} z_t^{(1-\omega_e-\omega_h)/(1-\theta_k-\theta_h-\phi)} n_{h,t}^{1-\omega_e-\omega_h}. \]

Along the balanced growth path \( \lambda_t \) and \( \mu_t \) grow at rate \( \psi^{-1} \). All other variables, except \( n_{w,t} \) and \( n_{h,t} \), grow at rate \( \psi \). Denote the central planner’s detrended variables as \( \hat{x}_t \), and their steady states as \( \hat{x} = \hat{x}_{t+1} = \hat{x}_t \). In the steady state, the human capital accumulation equation becomes

\[ (\psi - 1 + \delta_h) \hat{h}^{1-\omega_h} = \epsilon_t^{\omega_e} n_h^{1-\omega_e-\omega_h}. \]

The resource constraint in steady state is

\[ \hat{c} + \hat{i} + \hat{e} + \hat{g}_c + \hat{g} = \hat{y} + \hat{m}, \]

where

\[ \hat{y} = \hat{n}_w^{\theta_k} n_{h}^{1-\theta_k-\theta_h} \hat{h}^{\theta_h+\phi}. \]
The steady state of the first order conditions can be written as

\[ 1/\hat{c} = \hat{\lambda}, \]

\[ \alpha_g/\hat{g}_c = \hat{\lambda}, \]

\[ \hat{\lambda} = \hat{\mu}\hat{\omega}_e\hat{\omega}_c^{-1}\hat{h}^{\omega_h}n_h^{1-\omega_e-\omega_h}, \]

\[ \psi/\beta = 1 - \delta + \theta_k \hat{y}_k, \]

\[ \hat{\mu}\psi/\beta = \hat{\mu}[1 - \delta_h + \omega_h\hat{\omega}_h^{-1}n_h^{1-\omega_e-\omega_h}] + \hat{\lambda}(\theta_h + \phi)\hat{y}_h, \]

\[ \frac{\alpha_n}{1 - n_w - n_h} = \hat{\lambda}(1 - \theta_k - \theta_h)\hat{y}_n, \]

\[ \frac{\alpha_n}{1 - n_w - n_h} = \hat{\mu}(1 - \omega_e - \omega_h)\hat{\omega}_e\hat{h}^{\omega_h}n_h^{-\omega_e-\omega_h}. \]

We can obtain the steady state values of \( k/y, e/y, n_w \) and \( n_h \) using the equations above to yield

\[ \hat{k}/\hat{y} = \frac{\theta_k}{\psi/\beta - 1 + \delta}, \]

\[ \hat{e}/\hat{y} = \frac{\omega_e(\theta_h + \phi)(\psi - 1 + \delta_h)}{[\psi/\beta - 1 + \delta_h - \omega_h(\psi - 1 + \delta_h)]}, \]

\[ \frac{n_h}{n_w} = \frac{1}{1 - \theta_k - \theta_h - \omega_e - \omega_h \hat{e}/\hat{y}}, \]

\[ n_w = \frac{(1 - \theta_k - \theta_h)}{\alpha_n \hat{e}/\hat{y} + 1 - \omega_e - \omega_h \hat{e}/\hat{y}}, \]

\[ n_h = \frac{1 - \omega_e - \omega_h \hat{e}/\hat{y}}{(1 - \theta_k - \theta_h)} + \alpha_n \hat{e}/\hat{y} + \frac{1 - \omega_e - \omega_h \hat{e}/\hat{y}}{\omega_e}. \]

The steady-state equations for the planner differ from those of the agent due to both the distortionary taxes and the externality.

**Appendix C. Data Details**

The data sets for calibration are from National Income and Product Accounts Tables (NIPA) by the Bureau of Economic Analysis, Federal Reserve Economic Data (FRED),
the World Bank Data (WBD), and the Digest of Education Statistics (DES). All the data
below except for real interest rates are denominated in US dollars.

**GDP**: Gross domestic product (Table 1.1.5, NIPA)

**Investment**: Gross domestic investment (Table 5.1, NIPA)

**Capital Stock**: Fixed assets (private and public) (Table 1.1, Fixed Assets Accounts Tables, NIPA)

**Consumption**: Personal consumption expenditures (Table 1.1.5, NIPA)

**Government Debt**: Federal debt held by the public (FYGDPUN, FRED)

**Total Government Consumption Expenditures**: Government consumption expenditures (Table 3.1, NIPA)

**Government Defense Expenditures**: Defense expenditures (Table 3.15.5, NIPA)

**Government Subsidies to Education**: Government education expenditures (Table 3.15.5, NIPA)

**Net Import**: Net exports of goods and services (Table 1.1.5, NIPA)

**Education Expenditures (% of GDP)**: Expenditures of educational institutions as a percent of GDP - all institutions (Table 28, 2011 Tables and Figures, DES)

**Real GDP Growth Rate**: Percent change from preceding period in real gross domestic product (Table 1.1.1, NIPA)

**Real Interest Rate**: Real interest rate, World Bank Data, available at:

http://data.worldbank.org/indicator/FR.INR.RINR

### Appendix D. Estimation of Human Capital Technology

As an alternative to the benchmark calibration, we follow Gao (2013) to estimate the coefficients on education expenditures ($\omega_e$) and human capital ($\omega_h$)

First, Gao (2013) performs a cross-country regression to estimate the coefficients of the production function in a Solow model augmented with human capital. The aggregate production function is given by

$$Y_t = z_t K_t^{\theta_k} H_t^{\theta_h} L_t^{1-\theta_k-\theta_h} h_{a,t}^\phi,$$  \hspace{1cm} (69)
where $L_t$ denotes the aggregate labor force. Due to lack of data availability in a large cross section of countries for labor hours and for labor force participation, we assume that labor hours are fixed and compute per capita values instead of per worker values. In order to obtain estimated coefficients consistent with our model, we assume there is an externality in the human capital sector. $h_{a,t}$ is per capita human capital with a positive exponent $\phi$. Dividing equation (69) by $L_t$ yields the per capita production function

$$y_t = z_t h_t^{\theta_k} k_t^{\theta_k} h_{a,t}^{\phi}.$$  

In equilibrium, per capita human capital is equal to the agent’s own human capital, $h_t = h_{a,t}$. Therefore,

$$y_t = z_t k_t^{\theta_k} h_t^{\theta_k + \phi}. \quad (70)$$

After rearrangement, equation (70) can be rewritten as

$$y_t = z_t^{\frac{1}{1-\omega_k-\omega_h}(k_t/y_t)\frac{\theta_k}{1-\omega_k-\omega_h} (h_t/y_t)\frac{\theta_h+\phi}{1-\omega_k-\omega_h}}.$$

(71)

Per capita output is now a function of an unobserved factor, $z_t^{\frac{1}{1-\omega_k-\omega_h}}$, and two capital intensities.

Next, Gao (2013) uses capital and human capital accumulation technologies from equations (5) and (6) to derive expressions to substitute for $k_t/y_t$ and $h_t/y_t$ in equation (71). To obtain coefficients consistent with our model, we assume that the two capitals depreciate at the same rate: $\delta = \delta_h$. Define $A_t = z_t^{\frac{1}{1-\omega_k-\omega_h}}$. Assume that $A_t$ grows exponentially at rate $\psi$ and that labor grows exponentially at rate $n$, such that $A_t = A_0 e^{\psi t}$ and $L_t = L_0 e^{nt}$. In steady state,

$$k/y = \frac{i/y}{\psi + \delta + n}, \quad (72)$$

$$h/y = (e/y)^{\omega_k/(1-\omega_h)}\left(\frac{n_h}{y_t/A_t}\right)^{(1-\omega_k-\omega_h)/(1-\omega_h)}(\psi + \delta + n)^{-1/(1-\omega_h)}.$$  

(73)

where we have dropped the time subscript to denote the steady state. Note that along the balanced growth path, $y_t/A_t$ is the detrended output per capita and, therefore, is constant.
Next, substitute the expressions for \( k/y \) and \( h/y \), from equations (72) and (73), into equation (71) and rearrange to yield

\[
y_t = A_t (\psi + \delta + n) \frac{(1 - \omega_h)\theta_k + \theta_h + \phi}{\Upsilon} \frac{(1 - \omega_e)\theta_k}{i/y} \frac{\omega_e (\theta_h + \phi)}{n_h} \frac{(1 - \omega_e - \omega_h)(\theta_h + \phi)}{e/y} \omega_e (\theta_h + \phi) \Upsilon \log(\psi + \delta + n) + (1 - \omega_e - \omega_h)(\theta_h + \phi) \Upsilon \log n_h,
\]

where \( \Upsilon = (1 - \omega_h)(1 - \theta_k) - \omega_e (\theta_h + \phi) \). Next, take the logarithm to yield

\[
\log y_t = \log A_t - \frac{(1 - \omega_h)\theta_k + (\theta_h + \phi)}{\Upsilon} \log(\psi + \delta + n) + \frac{(1 - \omega_e)\theta_k}{\Upsilon} \log(i/y) + \frac{\omega_e (\theta_h + \phi)}{\Upsilon} \log(e/y) + \frac{(1 - \omega_e - \omega_h)(\theta_h + \phi)}{\Upsilon} \log n_h.
\]

Following Mankiw, Romer, and Weil (1992), we assume that \( A_t \) is randomly distributed across countries. We estimate parameters using a single cross-country regression for the log of each country’s per capita output regressed on logs of each country’s investment-to-GDP ratio, education expenses-to-GDP ratio, education enrollment, and the sum of exogenous rates. We exclude oil producers, and use the largest possible remaining sample of countries, 78 countries over our sample period of 1995 - 2007. Details of the cross-country regression are in Gao (2013). The parameters \( \omega_e \) and \( \omega_h \) are calculated from the estimated coefficients. The regression results are summarized in Table 4.
**Table 4: Single Cross-Country Regression**

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>log ( y ) in 2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Countries:</td>
<td>78</td>
</tr>
<tr>
<td>Constant</td>
<td>2.80</td>
</tr>
<tr>
<td></td>
<td>(2.02)</td>
</tr>
<tr>
<td>( \log(i/y) )</td>
<td>0.83***</td>
</tr>
<tr>
<td></td>
<td>(0.28)</td>
</tr>
<tr>
<td>( \log(\psi + \delta + n) )</td>
<td>-3.96***</td>
</tr>
<tr>
<td></td>
<td>(0.58)</td>
</tr>
<tr>
<td>( \log(n_h) )</td>
<td>0.86***</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
</tr>
<tr>
<td>( \log(e/y) )</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>(0.23)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.77</td>
</tr>
</tbody>
</table>

**Implied Coefficients:**

- Implied \( \omega_e \): 0.11
- Implied \( \omega_h \): 0.61

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*a* The standard errors are in parentheses immediately below.

*b* *, **, *** indicate significance at 10%, 5% and 1% level, respectively.

c \( \log(y_t) \): log real GDP per working-age population in 2007, calculated by dividing real GDP by working-age population (15-64), from Summers-Heston PWT 7.1 and World Development Indicators, respectively.

d \( n_h \): the ratio of secondary and tertiary school enrollment to the working-age population, from UNESCO.

e \( e/y \): public spending on education-to-GDP ratio, World Bank Data

f \( i/y \): Investment-to-GDP ratio, from Summers-Heston PWT 7.1.

g \( n \): working-age population growth rate, from World Development Indicators.

h The investment, working-age population growth rates and education enrollment rates are averaged over 1995-2007; \( (\psi + \delta) \) is assumed to be 0.05.