Predicting Sovereign Fiscal Crises: High-Debt Developed Countries

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Abstract

High-debt countries risk solvency crises if debt is expected to exceed a fiscal limit – defined as the highest value of debt that the country could service and/or repay. Countries do not announce values for their fiscal limits. We argue that inferences from estimates of fiscal rules, in which surplus responsiveness to debt depends on the level of debt, yield either no estimate or an unreliable estimate of the fiscal limit. We estimate a fiscal rule in which surplus responsiveness is increasing in the interest rate. Although this specification does not yields an estimate for the fiscal limit, we combine the implied estimates for the long-run values of debt with historical maximum values to create a lower bound on the fiscal limit for each country. Using the estimated dynamic behavior of debt and the surplus together with the lower bound on the fiscal limit, we separate countries into two risk categories over the period 2008-2012. Both Greece and Portugal enter the high-risk category about one year prior to their financial crises. Italy is at high risk in 2012 and others are at low risk throughout the period.

JEL Classifications: E6, F5

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1 Introduction

Greece lost access to credit markets when debt exceeded 130% of GDP, whereas Belgium successfully handled identical debt relative to GDP. What determines the highest debt that a country could repay, defined as "fiscal limit?" Can we estimate the fiscal limit for a country which has never lost access to markets? We use historical data on primary surpluses (surpluses excluding interest payments) and debt relative to GDP for six high-debt, developed countries, two of which lost access to markets after the 2008 global financial crisis, to estimate the behavior of the primary surplus in response to changes in debt. The high-debt countries include Belgium, Canada, France, Greece, Italy, and Portugal.

The amount by which governments increase the primary surplus, using spending cuts and/or tax hikes, in response to an increase in debt, is a key behavior in determining whether fiscal policy is sustainable. As debt rises, the primary surplus must rise sufficiently to prevent debt from becoming explosive. Equivalently, the surplus responsiveness to debt must be large enough for the dynamic system in the surplus and debt to be stable. We can use regression estimates to determine whether this stability criterion is met. The problem is that the critical parameter, governing the responsiveness of the primary surplus to debt, appears to be time-varying. Ghosh et al. (2013) estimate primary surplus responsiveness to debt using a large panel model and find that responsiveness depends on the value of debt. They argue that at high values of debt, surplus responsiveness continues to fall as debt rises, a phenomenon that the authors termed "fiscal fatigue." Fiscal fatigue provides information about a fiscal limit since there is a value of debt beyond which surplus responsiveness becomes too small for fiscal sustainability.

There are two significant problems with this identification. First, responsiveness varies not only across time, but also across countries, something the panel methodology does not allow. Second, for most countries, the fiscal limit is well outside values of debt in the sample, requiring the inference that responsiveness continues to fall as debt rises to unobserved levels. Perhaps responsiveness falls as debt rises, but would not continue to fall with additional unobserved increases in debt. If responsiveness falls with high debt but remains above a critical value, then estimation of the surplus responsiveness does not identify a fiscal limit.

We estimate individual-country fiscal rules allowing surplus responsiveness to change discretely as debt crosses thresholds. This allows responsiveness to fall (or increase) as debt increases, but it does not require responsiveness to continue to fall were debt to continue to rise. We cut the sample in 2008 to exclude the financial crisis years, in which

some countries could have had difficulty following their fiscal rules. For all countries except Portugal, responsiveness changes as debt crosses thresholds. For the remaining five countries, responsiveness increases beyond a threshold, and for two countries, Belgium and Greece, there is a second threshold beyond which responsiveness falls. For Belgium, responsiveness remains high enough to imply stability, whereas for Greece it does not. Therefore, even in the threshold specification where responsiveness does not continue to fall with increases in debt, Greece experiences fiscal fatigue, whereas others, including Belgium, a country with comparably high debt, do not. The difference in behavior across countries highlights the importance of allowing parameter values to differ across countries.

Since the Greek surplus does not respond sufficiently to debt beyond the second debt threshold to yield stability, we use our estimates to identify a fiscal limit for Greece. We present a model in which debt is explosive with probability one whenever it is on or above the saddlepath relationship between debt and the surplus associated with parameter estimates for debt greater than the second threshold. We use estimates of the saddlepath and compare actual debt-surplus pairs with the saddlepath to determine years in which agents should have expected debt to be explosive, thereby violating the fiscal limit. We find many years in which agents should have refused to lend, casting doubt on the threshold model's ability to identify the fiscal limit.[1]

The ability to identify a fiscal limit from the responsiveness of the surplus to debt relies on the assumption that responsiveness depends on debt. However, it is possible that time variation in the surplus responsiveness to debt is not due to the level of debt, but to something else. If responsiveness is increasing in the interest rate and not in debt, then the reduced responsiveness at high debt, that we find for Greece and Belgium, and that Ghosh et al (2013) find in their panel, could occur because debt reached high values just as the interest rate fell. Our estimates confirm that responsiveness is increasing in the interest rate for all countries. Responsiveness is highest in the period when interest rates are highest. For five of the six countries, the interest rate falls later in the sample, and responsiveness falls for all, whether their debt rises or falls.

We cannot use our estimated fiscal rules to identify a fiscal limit on debt because, if responsiveness depends on interest and not on debt, there is no value of debt beyond which debt becomes explosive. However, even without an estimate of the fiscal limit, we can use our estimates and the concept of the fiscal limit to assess fiscal risk. First, we construct a lower bound on the fiscal limit. Our estimates yield a value for debt in the

[1] The second threshold for Greece is a debt of approximately 95% of GDP. Greek debt exceeds this threshold in 1993 and remains above it for the rest of the sample.

long-run. When a country is able to borrow, agents must believe that the fiscal limit is above the long-run value of debt. Second, since there were no fiscal crises in this set of countries between 1970 and 2008, then each country's fiscal limit must be above the maximum value of debt observed in the sample period. The largest of these two debt measures constitutes a lower bound on the fiscal limit. We use this lower bound, together with the projected path of debt from estimated fiscal rules, to separate countries into high and low risk categories in the period following the global financial crisis. We find that Greece and Portugal entered the "high risk" category in 2008 and 2010, respectively. Greece lost access two years later and Portugal lost access one year later. Fiscal limits on debt were between 112% and 130% of GDP for Greece and a little larger than 94% of GDP for Portugal. Italy entered the "high risk" category in 2012. In contrast, we find that Belgium, Canada, and France are at "low risk" for a debt crisis.

Next, we use our estimates to measure the difficulty countries faced over this financially turbulent period in adhering to their fiscal rules. Countries with adverse deviations from their surplus rules would have debt growing faster, implying greater risk. We find negative surplus deviations for both Greece and Portugal greater than five standard deviations in the year prior to each country's loss of access to credit markets. All other countries experienced negative deviations, but none were so large. Additionally, for both Greece and Portugal, debt had deviated from its projected path by 10% - 15% in the year prior to its fiscal crisis. These results reinforce our assessment of high risk for Greece and Portugal.

This paper is organized as follows. The next section derives the restrictions on the fiscal rule and on values for debt and the surplus implied by fiscal limits. Section 3 is empirical, with the first part providing estimates of fiscal rules where responsiveness varies with debt, and the second estimates with the responsiveness varying with the interest rate. Section 4 uses our estimates of the interest-rate fiscal rule to assess risk following the global financial crisis for our six high-debt developed countries. Section 5 provides conclusions.

2 Fiscal Limits and Fiscal Solvency

We assume that every country faces a fiscal limit on debt, defined as the value of debt beyond which a country could not raise primary surpluses sufficiently to satisfy its intertemporal budget constraint. It represents the highest debt that the country can repay and/or service. The fiscal limit on debt is partly due to a limit on the government's ability to raise tax revenue through distortionary taxation, perhaps through the Laffer curve (Bi

2012), or due to a limit on the ability to reduce government spending, perhaps through the need to provide public goods, or due to the political economy of the budget process (Bi, Leeper and Leith 2013), or due to tax evasion (Daniel 2014).

Consider the relationship between fiscal limits and fiscal solvency. Solvency is defined as satisfaction of the government's intertemporal budget constraint. Solvency with fiscal limits requires that debt not be explosive. If debt were explosive, then debt would be expected to exceed any finite value for a fiscal limit. Therefore, a key restriction imposed by solvency under fiscal limits is global stability of the dynamic system in the primary surplus and debt. Global stability rules out explosive behavior of debt.[2]

To determine stability criteria, it is necessary to specify fiscal policy. We assume that fiscal policy is characterized by a simple fiscal rule whereby the primary surplus as a fraction of GDP (s_t) reacts to its own lag (s_{t-1}) and to lagged debt as a fraction of GDP (d_{t-1}), given by

$$s_t = c + \beta s_{t-1} + \gamma d_{t-1} + \epsilon_t, \tag{1}$$

where c is a constant governing the long-run value of the primary surplus relative to GDP, and ϵ_t represents shocks, possibly from the business cycle. In the remainder of the paper we refer to the primary surplus as a fraction of GDP and debt as a fraction of GDP, more simply, as the surplus and debt, respectively.

The fiscal rule, given by equation (1), can be combined with an expression for the evolution of debt to yield a dynamic system of equations for the evolution of the surplus and debt over time. The debt accumulation equation is

$$d_t = (1 + r)\, d_{t-1} - s_{t,} \tag{2}$$

where r is the growth-adjusted real interest rate, which we refer to going forward simply as the interest rate. This is the risk-free rate, which, for now, we assume is constant.

The dynamic system in the surplus and debt, equations (1) and (2), is globally stable when the characteristic roots, $\lambda_{1,2}$, given by

$$\lambda_{1,2} = \frac{1 + \beta + r - \gamma \pm \sqrt{(1 + \beta + r - \gamma)^2 - 4\beta(1 + r)}}{2},$$

are both inside the unit circle. This yields a stability requirement as the first necessary

[2] In a world without fiscal limits Bohn (2007, 2008) argues that the stability requirement is too strong. A fiscal rule which allows debt to grow forever at a rate below the interest rate is sufficient to assure that the government's intertemporal budget constraint holds.

condition for solvency[3]

$$\frac{\partial s_t}{\partial d_{t-1}} = \gamma > r\left(1-\beta\right). \tag{3}$$

Equation (3) requires that the responsiveness of the surplus to an increase in debt, given by γ, be large enough to prevent an increase in debt from sending the system onto an explosive path. Shocks could push the economy away from its long-run values, but the large surplus responsiveness brings the economy back to its long-run equilibrium, given by

$$d^* = rs^* = \frac{-c}{\gamma - r\left(1-\beta\right)}, \tag{4}$$

where d^* is the long-run value of debt and s^* is the long-run value of the surplus. In the absence of global stability, debt could embark on an explosive path, thereby violating any finite value for the fiscal limit.

Global stability is necessary but not sufficient for solvency with fiscal limits. A country whose surplus and debt are globally stable can still loose access to credit markets. As a country approaches its fiscal limit, agents begin to anticipate that fiscal shocks could lead to insolvency. A standard response to insolvency is default, defined as a reduction in the value of debt that the government will actually honor. The expectation of default increases the interest rate to equate the country interest rate, inclusive of the expectation of capital loss due to default, with the market risk-free rate. The higher interest rate on the country's debt causes debt to grow faster, increasing the probability of reaching the fiscal limit. There is some combination of values for the surplus and debt, for which there is no interest rate at which agents would lend. For this combination, no interest rate can equate the expected rate of return with the market rate, while keeping the country below its fiscal limit. At this point, creditors refuse to lend, and the country faces a fiscal crisis. Daniel and Shiamptanis (2012) show that countries lose access to credit markets when the probability that debt will exceed its fiscal limit becomes high enough. This typically occurs before debt reaches the fiscal limit.

These concepts can be illustrated with a phase diagram for a globally stable system. Equations for the phase diagram can be obtained by subtracting the lagged value of the surplus from equation (1) and lagged value of debt from equation (2), yielding

$$\Delta s_t = s_t - s_{t-1} = \left(\beta - 1\right) s_{t-1} + \gamma d_{t-1} + c + \epsilon_t = 0$$

[3]Other combinations of parameter values can be consistent with global stability but they violate magnitudes of coefficients compatible with the estimates in the data.

$$\Delta d_t = d_t - d_{t-1} = (r-\gamma)d_{t-1} - [\beta s_{t-1} + c + \epsilon_t] = 0.$$

Setting the error to zero, the two equations for the phase diagram are given by

$$d_{t-1}|_{(\Delta s_t=0)} = \frac{-c + (1-\beta)\, s_{t-1}}{\gamma}$$

$$d_{t-1}|_{(\Delta d_t=0)} = \frac{c + \beta s_{t-1}}{r-\gamma}$$

The $\Delta s = 0$ curve has a positive slope and the $\Delta d = 0$ has a negative slope. Arrows of motion for both curves point toward each curve, yielding a globally stable model. The phase diagram is given in Figure 1.

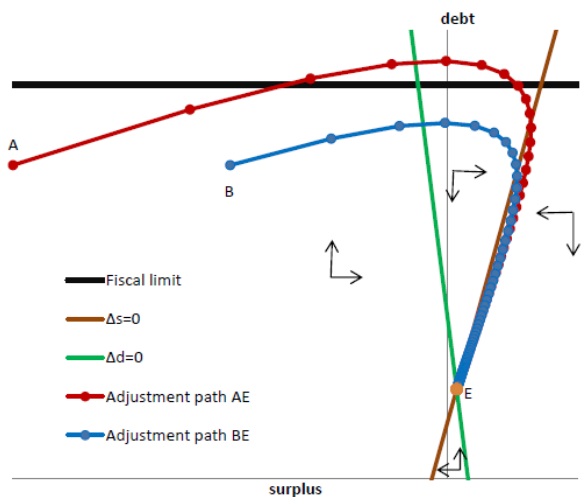

Figure 1

In the diagram, the fiscal limit is the horizontal black line at a high value of debt. Point E represents long-run equilibrium values for debt and the surplus, given by equations (4). Adjustment paths AE and AB reflect paths for the surplus and debt toward the long-run equilibrium when interest takes on its risk free value.

To determine the value of debt at which creditors refuse to repay, Daniel and Shiamptanis (2012) use the risk-free interest rate to create projections of debt from initial debt-surplus pairs toward the long-run equilibrium. These are the paths AE and BE in Figure 1. A critical projection is the one for which the peak value of debt is the fiscal limit. In Figure 2, we add this critical projection and label it FE. They show that for a debt-surplus pair that lies on or above FE, debt will travel above the fiscal limit in the

approach to the long-run equilibrium with probability one. This is because, for a debt-surplus pair that lies on FE, the default premium on the interest rate implies that debt is actually expected to rise faster than FE, thereby exceeding its fiscal limit in the approach to the long run. Therefore, whenever the debt-surplus pair lies on or above this critical projection, creditors refuse to lend and the country faces a fiscal crisis. The projection FE becomes the effective fiscal limit. A country whose debt-surplus pair is on or above the effective fiscal limit is insolvent. Equivalently, a debt-surplus pair for which the peak of the debt projection at the risk-free interest rate equals or exceeds the fiscal limit is a position of insolvency. The effective fiscal limit FE is not a single value for debt, but a range of values for the debt-surplus pair.

This implies that the second requirement for solvency is that debt must remain below the effective fiscal limit. Equivalently, the peak of the debt projection from an initial debt-surplus pair must not reach the fiscal limit. This requirement is most easily illustrated in Figure 2.

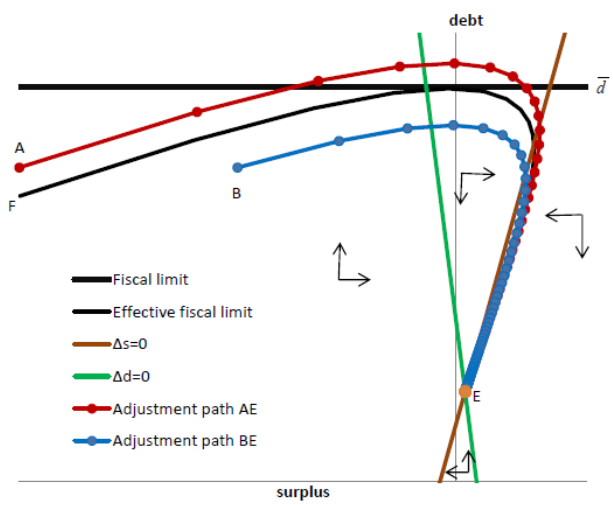

Figure 2

Consider an economy for which recent fiscal shocks have sent it to point A along the adjustment path AE. Point A is above the effective fiscal limit FE. Equivalently, the peak of the debt projection from point A exceeds the fiscal limit. Therefore, this debt-surplus pair represents a position of insolvency. An insolvent government cannot borrow to allow debt to rise along the adjustment path AE.

Alternatively, assume that this economy begins at point B. Points A and B both have identical initial debt, but point B has a higher surplus. Point B satisfies the solvency

requirement, but there could be one-period risk depending on its proximity to the effective fiscal limit. If point B is sufficiently below the effective fiscal limit, that the probability of a shock sending it above in the next period is zero, then the interest rate takes on its risk-free value, and the economy is expected to travel along the path BE toward its long-run equilibrium, with debt never exceeding the effective fiscal limit FE in transit. If point B is not sufficiently below the effective fiscal limit, then debt is expected to rise more quickly than indicated by the BE path, due to the default premium on the interest rate. In this case, an adverse shock next period could send the economy to a new projection, one along which debt would exceed the fiscal limit in transit. An economy beginning at point B is solvent, but depending on the distance to FE, the economy could face risk of insolvency in the next period. The initial values for both debt and the surplus play a critical role in determining solvency. Debt is identical at points A and B, but point A is not fiscally solvent, while point B is.

The presence of fiscal limits implies that the value of γ, which represents the responsiveness of the surplus to debt, takes on a greater importance than its role in guaranteeing stability. In Figure 3, a larger responsiveness rotates the $\Delta d = 0$ curve counterclockwise and the $\Delta s = 0$ curve clockwise, reducing the area for which debt is rising in favor of increasing the area for which debt is falling. A large enough rotation for the $\Delta d = 0$ could imply that an economy beginning at point A meets the second criterion for fiscal solvency. With this different responsiveness, the projection for debt at the risk-free interest rate never rises above the fiscal limit along its trajectory toward the long-run equilibrium. This does not imply that point A becomes risk-free, but its risk does fall since the vertical distance between the projection path and the fiscal limit is smaller. Therefore, an economy with a larger responsiveness has a lower probability of fiscal insolvency and is better prepared for large shocks.

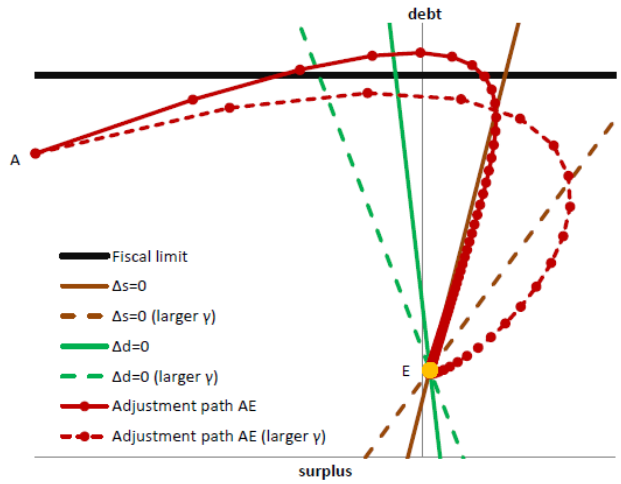

Figure 3

3 Estimates of Fiscal Rules

To compare debt projections with fiscal limits, we need estimates of fiscal rules and fiscal limits. There are special cases in which estimates of fiscal rules also provide estimates of an effective fiscal limit.

We estimate the responsiveness of the surplus to debt (γ) for six high-debt countries, Belgium, Canada, France, Portugal, Italy, and Greece using annual data during the period 1970-2008. Summary statistics are reported in Table 1.[4]

[4]The variables are from the OECD (Economic Outlook No. 94) and AMECO database. For s_t we use the general government primary balances relative to GDP, for d_t we use the gross public debt, Maastricht criterion, relative to GDP, for $\tilde{y}_t$ we use the economy's output gap and for $\tilde{g}_t$ we use the cyclical component of the log real government consumption expenditure obtained from the Hodrick-Prescott filter with the smoothing parameter set at 100, as in Mendoza and Ostry (2008).

Table 1 Summary Statistics

	Debt/GDP			surplus/GDP		
	Min	*Mean*	*Max*	*Min*	*Mean*	*Max*
Belgium	54.28	97.23	134.07	-8.83	0.91	6.42
Canada	44.08	71.54	109.40	-6.35	-0.60	5.95
France	19.82	40.37	68.21	-3.75	-0.72	1.10
Greece	15.74	63.67	112.90	-5.97	-1.11	3.54
Italy	37.25	85.57	121.25	-7.91	-1.08	5.99
Portugal	19.14	50.46	71.69	-4.93	-1.35	1.80

We cut our sample in 2008 because no country in our sample lost access to the markets prior to 2008, implying that countries were able to borrow to follow their fiscal rules over the sample.[5]

Equation (1) posits a constant linear response of the surplus to debt, but data reject this simple specification in favor of a time-varying responsiveness. We consider two alternative sources for the time-variation. The first follows others (Bohn 2007, 2008 and Ghosh et al. 2013) and allows responsiveness to vary with debt. With this approach, parameters can take on values which allow inference about the value of the effective fiscal limit. For the second, we present and test the hypothesis that responsiveness varies with interest rate. In this approach, estimates provide no evidence about the fiscal limit, although they do allow inference on the long-run value of debt, which we argue must be below the fiscal limit.

3.1 Surplus Responsiveness Changes with Debt

There are several ways to model the nonlinear surplus responsiveness as a function of the value of debt. One is to model the surplus as responding to powers of debt, and another is allow responsiveness to vary once debt crosses a threshold.

3.1.1 Powers of Debt Model

Ghosh et al. (2013) estimate a large panel model of 23 advanced economies using data prior to 2008. They utilize a cubic fiscal reaction function, which allows the surplus to

[5] For Portugal, primary surplus data begins after the Portuguese Revolution of 1975, limiting our sample.

respond to powers of debt given by

$$s_t = c + \gamma_0^p d_{t-1} + \gamma_1^p d_{t-1}^2 + \gamma_2^p d_{t-1}^3 + \epsilon_t.$$

Responsiveness is the derivative of the surplus with respect to debt, explicitly

$$\frac{\partial s_t}{\partial d_{t-1}} = \gamma_0^p + 2\gamma_1^p d_{t-1} + 3\gamma_2^p d_{t-1}^2. \tag{5}$$

They find that the responsiveness is negative at low levels of debt ($\gamma_0^p < 0$). It becomes positive as debt rises ($\gamma_1^p > 0$), and becomes negative again at high levels of debt ($\gamma_2^p < 0$). Equation (5) is a continuous function, which suggests that the responsiveness is low for both low and high levels of debt, and is high for intermediate values. Ghosh et al. (2013) label the fall in responsiveness at high levels of debt ($\gamma_2^p < 0$) "fiscal fatigue". Their cubic specification, together with the estimated negative value for γ_2^p, implies that as debt increases from observed levels, there is some level of debt, possibly well-outside the observed values in the data, for which responsiveness fails to satisfy the stability requirement.[6] This value of debt becomes an effective fiscal limit since debt becomes explosive from this value, violating any finite value for the fiscal limit. They use panel estimates of surplus-responsiveness together with individual-country interest rates to identify an effective fiscal limit on debt for each country.

A crucial assumption under the Ghosh et al. (2013) approach is that inference about the existence of the effective fiscal limit can be made, if and only if, γ_2^p is negative. A positive estimate for γ_2^p fails to provide any information about the effective fiscal limit. Since responsiveness falls only at very high levels of debt, they rely on observations from a few high-debt countries to obtain estimates of γ_2^p.[7]

The cubic specification with $\gamma_2^p < 0$ implies that as debt rises, responsiveness continues to weaken. With this assumption, there is always a value of debt high enough for responsiveness to fail the stability criterion such that debt becomes explosive. And this value for debt could be well outside the observed sample. The cubic specification rules out the possibility that responsiveness weakens at a high value of debt, but remains above the interest rate as debt rises.

[6] With $\beta = 0$ in their specification, global stability, equation (3), requires $\frac{\partial s_t}{\partial d_{t-1}} = \gamma_0^p + 2\gamma_1^p d_{t-1} + 3\gamma_2^p d_{t-1}^2 > r$. They find that debt must exceed 150% of GDP for responsiveness to become negative $\frac{\partial s_t}{\partial d_{t-1}} < 0$. This is a higher debt/GDP ratio than exists in our sample.

[7] Ghosh et al (2013) do provide some sensitivity analysis which they use to justify their assumption of identical responsiveness across countries. Our estimates reveal very different behavior across countries.

Finally, the estimated parameter values, together with individual-country interest rates, can imply that the dynamic model in debt is unstable, such that debt is never expected to reach a long-run equilibrium value. In this case, whenever debt is in the region for which it is expected to rise, agents should have refused to lend because they expected debt to exceed any fiscal limit. Ghosh et al obtain unstable systems for five of their twenty-three countries, including Greece, Italy, and Portugal, for some of their assumptions about interest rates. While Greece and Portugal do have a long-run equilibrium for one set of interest rates, the implied fiscal limits of 196.5% and 191.6% of GDP, respectively, are much too large, given that both countries lost access to markets with much lower debt levels. The fact, that these countries retained access to markets for many years for which debt was explosive, raises additional questions about the applicability of this approach in determining the fiscal limit and associated fiscal risk.

3.1.2 Threshold Model

Solvency Requirements in a Threshold Model An alternative way to model surplus responsiveness as a function of debt is to allow for the responsiveness to change as debt crosses a threshold. The threshold model differs from the cubic model in three respects. First, the responsiveness can weaken as debt rises, but it will not continue to weaken unless there is another threshold at higher debt with an even lower responsiveness. Second, it does not make inferences about how responsiveness changes for values of debt outside the sample. Third, a weaker responsiveness at high levels of debt yields information about the fiscal limit only if the model fails to satisfy global stability at these debt levels.

We consider the following threshold model, where the surplus responsiveness differs depending on whether debt has crossed a threshold. To avoid the assumption that countries have identical responsiveness to debt, we estimate individual-country fiscal rules. The surplus equation is given by

$$s_t = c + \beta s_{t-1} + \gamma_0^h d_{t-1} + \gamma_1^h \left(d_{t-1} - \tilde{d}_1 \right) + \gamma_2^h \left(d_{t-1} - \tilde{d}_2 \right) + \epsilon_t. \tag{6}$$

The terms $\tilde{d}_1$ and $\tilde{d}_2$ represent increasingly higher threshold values of debt. The coefficients γ_1^h and γ_2^h are non-zero only if the respective terms in parentheses are positive, and capture the changes in the surplus responsiveness as debt crosses the threshold levels $\tilde{d}_1$ and $\tilde{d}_2$, respectively. Responsiveness depends on the value of debt according to

$$\frac{\partial s_t}{\partial d_{t-1}} = \begin{cases} \gamma_0^h & \text{for} \quad d_{t-1} < \tilde{d}_1 \\ \gamma_0^h + \gamma_1^h & \text{for} \quad \tilde{d}_1 < d_{t-1} < \tilde{d}_2 \\ \gamma_0^h + \gamma_1^h + \gamma_2^h & \text{for} \quad \tilde{d}_2 < d_{t-1} \end{cases} .$$

This is a step function in the level of debt. The threshold model allows to differentiate between a case where the responsiveness weakens at high debt levels $\left(\gamma_2^h < 0\right)$ but $\frac{\partial s_t}{\partial d_{t-1}} = \gamma_0^h + \gamma_1^h + \gamma_2^h > r\left(1-\beta\right)$, and one where $\gamma_2^h < 0$ but $\frac{\partial s_t}{\partial d_{t-1}} = \gamma_0^h + \gamma_1^h + \gamma_2^h < r\left(1-\beta\right)$. If the model fails the stability criterion at high values of debt, then the second threshold can be used to infer an effective fiscal limit on debt, as in Ghosh et al (2013).

Figure 4 provides a phase diagram for a country which fails the stability criterion below the first debt threshold and above the second, but satisfies it in between. The time paths for surplus and debt can be used to express the saddlepath relationship between debt and surplus as

$$d_t = D + \left(\frac{\lambda_2 - \beta}{\gamma_0^h + \gamma_1^h + \gamma_2^h}\right)\left(s_t - rD\right), \tag{7}$$

where D is the level of debt at the intersection point between the $\Delta s = 0$ and $\Delta d = 0$ curve above the second threshold $\left(\tilde{d}_2\right)$, given by

$$D = \frac{-c + \gamma_1^h \tilde{d}_1 + \gamma_2^h \tilde{d}_2}{\gamma_0^h + \gamma_1^h + \gamma_2^h - r\left(1-\beta\right)} > \tilde{d}_2 \tag{8}$$

and λ_2 is the stable root of the dynamic system, given by equations (6) and (2).

Arrows of motion above the saddlepath imply that debt is expected to embark on an explosive path, eventually exceeding any finite limit. Along the saddlepath, expectations of default loss imply that debt rises faster than implied by the saddlepath, such that the probability of debt becoming explosive becomes unity. This makes the saddlepath the effective fiscal limit. Agents refuse to lend when debt reaches the saddlepath. Below the saddlepath, arrows of motion point downward toward the equilibrium with debt between the two threshold values. These arrows ignore the expectation of default loss in the neighborhood of the fiscal limit, implying that debt rises faster than these arrows imply. There is risk in the neighborhood below the saddlepath, but the probability of insolvency

remains less than unity and agents lend.

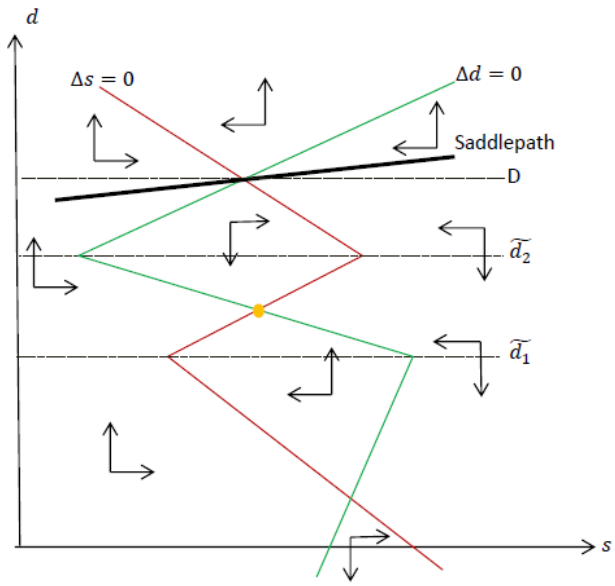

Figure 4

Figure 5 provides a phase diagram for a country with a second negative threshold $\left(\gamma_2^h < 0\right)$, but for which the stability criterion is satisfied at high values of debt. Since there are no explosive regions at high levels of debt, we cannot infer an effective fiscal limit on debt. Arrows of motion send the system toward the equilibrium, which is between the two threshold values. Even if debt is well above the second threshold $\left(\tilde{d}_2\right)$, the economy returns to its long-run equilibrium. This illustrates that a negative γ_2^h does not always provide evidence about the fiscal limit.

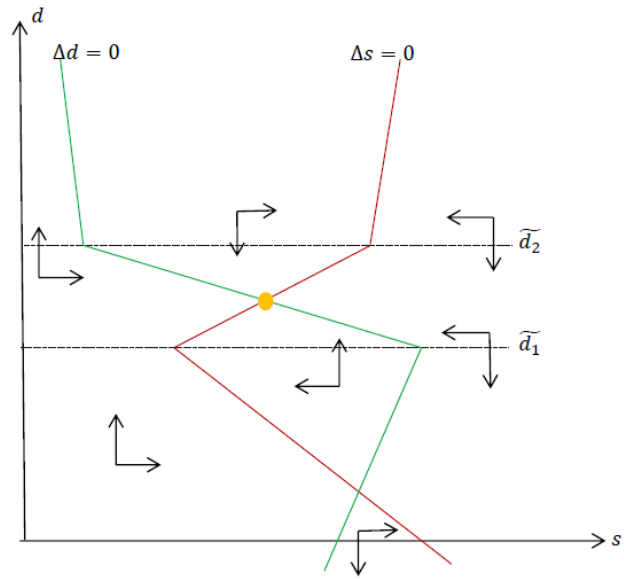

Figure 5

Estimation of the Threshold Model Before we proceed with the estimation of the fiscal policy rule parameters $\left(\beta, \gamma_0^h, \gamma_1^h, \gamma_2^h\right)$ and the threshold values of debt $\left(\tilde{d}_1, \tilde{d}_2\right)$, we verify that there is evidence for threshold effects in the data. We employ the double maximum tests, UD max and WD max, of Bai and Perron (1998, 2003), which do not require the pre-specification of the number of thresholds in the alternative hypothesis.[8] They test the null hypothesis of no thresholds against an unknown number of thresholds, given some maximum number. We allow for up to three thresholds and the results are reported in Table 2. For all countries except Portugal, both tests suggest the presence of thresholds in debt.

[8] Bai and Perron (2003) show that their structural changepoint tests can be applied to threshold models by first sorting the data based on the threshold variable and then applying their structural changepoint model.

Table 2 Test for Presence of Thresholds

	UDmax	*WDmax*
Belgium	12.64**	18.41***
Canada	17.78***	22.32***
France	11.37*	14.40**
Greece	41.31***	61.60***
Italy	15.13**	22.03***
Portugal	5.59	5.59

Note: The *, ** and *** indicate statistical significance at the 10, 5 and 1 percent level, respectively.

Given that the fiscal policy rules appear to have threshold effects, we estimate the threshold values of debt $\tilde{d} = \left[\tilde{d}_1 \ \tilde{d}_2\right]'$. We follow Hansen (2000) who recommends estimation of $\tilde{d}$ by minimizing the sum of squared errors (SSE) of equation (6), $\tilde{d} = \arg\min_{\tilde{d}} SSE\left(\tilde{d}\right)$. Hansen's procedure estimates one threshold at a time. The first threshold level is chosen as the one which allows the greatest reduction in the SSE. To identify the second threshold, we follow the sequential approach by Bai and Perron (1998, 2003). We test the null hypothesis of l thresholds against the alternative of $l+1$ thresholds using the $\sup F_T\left(l+1|l\right)$ statistic. If the overall minimum value of the SSE with $l+1$ thresholds is sufficiently smaller than the SSE from the model with l thresholds, then the null is rejected and the model with $l+1$ thresholds is chosen and the threshold selected is associated with this overall minimum SSE. This process is repeated by increasing l sequentially until the test fails to reject the null hypothesis.[9] The results, which are reported in Table 3, imply that Portugal has no thresholds, that Canada, France, and Italy each have one, and that Belgium and Greece each have two. We find no evidence for three thresholds.

Table 3 Debt Thresholds

	$\tilde{d}_1$	$\tilde{d}_2$
Belgium	65.66	98.36
Canada	62.27	-
France	39.72	-
Greece	56.39	94.86
Italy	84.17	-
Portugal	-	-

[9] Bai (1997) showed that it is possible to consistently estimate all breaks sequentially one at a time.

Next, consider estimation of the fiscal policy parameters $\left(\beta, \gamma_0^h, \gamma_1^h, \gamma_2^h\right)$. We use the threshold estimates from Table 3 to construct the spline terms in equation (6). After constructing the spline terms, we estimate the policy parameters using least squares as recommended by Hansen (2000) and Bai and Perron (1998, 2003). We use White robust standard errors to address potential concerns about heteroskedasticity. Following Bohn (1998, 2008), we include the output gap ($\tilde{y}_t$) and temporary outlays in government spending ($\tilde{g}_t$), where δ_1 and δ_1 are the parameters on $\tilde{y}_t$ and $\tilde{g}_t$, respectively.

Table 4 Responsiveness Varying with Debt Thresholds

	Belgium	Canada	France	Greece	Italy	Portugal
c	6.041	-0.430	0.375	0.385	-5.023***	-3.034***
	(4.412)	(1.361)	(0.546)	(0.712)	(1.121)	(1.094)
β	0.680***	0.566***	0.195	0.451***	0.306***	0.357***
	(0.075)	(0.050)	(0.122)	(0.153)	(0.105)	(0.128)
γ_0^h	-0.121	-0.016	-0.039**	-0.058*	0.030**	0.041*
	(0.077)	(0.025)	(0.016)	(0.031)	(0.013)	(0.021)
γ_1^h	0.208**	0.096***	0.063**	0.164**	0.126***	
	(0.100)	(0.034)	(0.025)	(0.073)	(0.037)	
γ_2^h	-0.073**			-0.273**		
	(0.035)			(0.117)		
δ_1	0.316**	0.453***	0.235**	-0.055	0.237	0.286**
	(0.182)	(0.069)	(0.114)	(0.084)	(0.184)	(0.130)
δ_2	-0.422**	-0.371***	-0.499***	-0.201**	-0.023	-0.228*
	(0.219)	(0.081)	(0.100)	(0.079)	(0.102)	(0.144)
$\bar{R}^2$	0.889	0.954	0.635	0.735	0.918	0.455
σ	1.322	0.752	0.655	1.402	1.083	1.244

Note: The *, ** and *** indicates statistical significance at the 10, 5 and 1 percent level, respectively.

We find that the responsiveness is low at low levels of debt and it increases once debt crosses its first threshold level. For Canada, France and Italy, we do not find evidence of a weaker responsiveness as debt increases. For Belgium and Greece, the coefficient on the second debt threshold is negative, implying that responsiveness weakens as debt crosses its second threshold level, with Belgium continuing to satisfy the stability criterion, but Greece failing for values of debt above the second threshold. The failure to satisfy the stability criterion above the second threshold implies that the threshold model can be

used to provide information about the effective fiscal limit for Greece.

To compute the effective fiscal limit for Greece, we use the saddlepath from equation (7), together with the fiscal policy parameters reported in Table 4, and an interest rate of 0.02%, which is the average interest rate in Greece over the period that debt is above the second threshold level (1993-2008). The phase diagram for debt beyond the second threshold, together with the saddlepath and the actual values for the surplus and debt over this period, are plotted in the left panel of Figure 6. The model implies that Greece should not have had access to credit markets whenever the actual value of debt was above the saddlepath. According to the model, Greece should have lost access to credit markets much earlier than 2010.

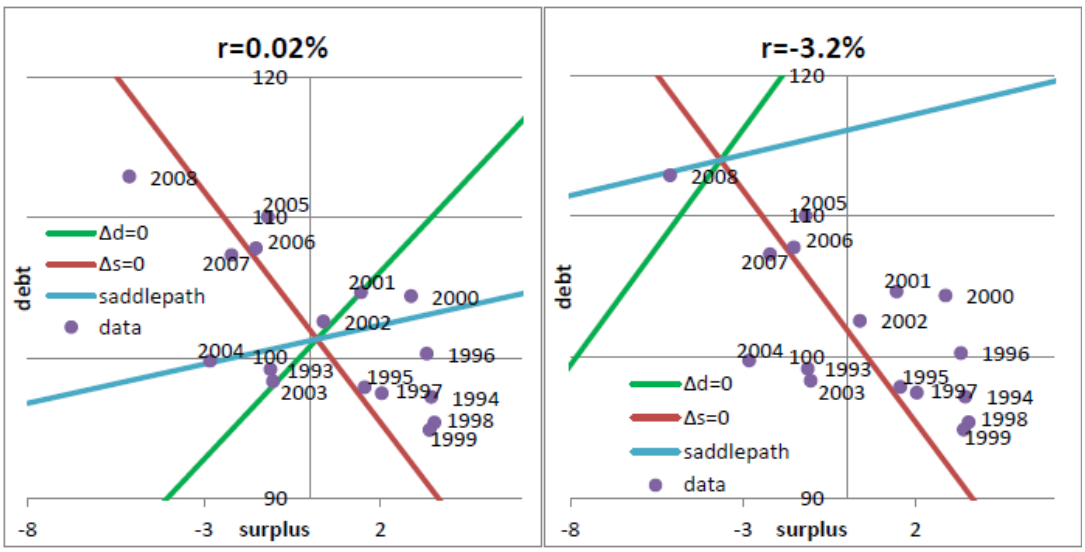

Figure 6

The value we chose for the interest rate in this assessment is critical. Although the behavior of the fical rule with the threshold specification (equation 6) is independent of the interest rate, the evolution of debt (equation 2) is not. Therefore, both the long-run value of debt, and the position of the saddlepath vary with the value of the interest rate. A lower interest rate yields a higher long-run value of debt and shifts the saddlepath upwards. The right panel of Figure 6 shows that the model is consistent with the data if the interest rate is as low as -3.2%, a value we view as unrealistically low.[10]

[10] Over this period in which the second threshold on debt applied, interest rates ranged in value from -5.30% to 9.48%, implying that inferences based on a constant value for the interest rate could be

These results imply that the threshold model, in which surplus responsiveness to debt varies with threshold levels of debt, does not explain Greece's ability to borrow in the years preceding its crisis, even though the estimates satisfy the criteria by which we can derive a fiscal limit. Our results are consistent with results in Ghosh et al (2013) where they find that the dynamic system for Greece is unstable, such that agents should not have been lending when they were.[11] These mis-predictions by models with surplus-responsiveness dependent on debt, lead us to question the assumption that the time variation in surplus-responsiveness is due to the value of debt. Additionally, low surplus-responsiveness to debt at both low and high debt, as implied by the powers-of-debt for all countries and by the debt-threshold models for Greece and Belgium, does not obviously make sense. Why would a country, faced with an increasing level of debt and a presumed desire to avoid a fiscal crisis, reduce its responsiveness with debt high, thereby raising the probability of a fiscal crisis. The reason cannot be that it has hit its effective fiscal limit, thereby experiencing a constrained response, because agents would not be lending at all at the effective fiscal limit (and would not just be lending less).

Perhaps responsiveness varies with the interest rate and not with debt. The level of debt could pick up interest rate behavior because at the beginning of the sample, both debt and interest rates were low, in the middle of the sample both debt and interest rates were high, and at the end of the sample the behavior of debt and interest rates diverged. Interest rates fell for most countries, while debt for some countries rose and for others fell. In the next section, we investigate the possibility that responsiveness varies with the interest rate, and not explicitly with the value of debt itself.

3.2 Surplus Responsiveness Changes with Interest Rate

For all countries in our sample, the growth-adjusted real interest rate took on values over a wide range during our sample. In the 1970's, interest rates were typically negative, rising in the early 1980's, and falling again in the late 1990's. We view these periods with different mean values of interest rates as different interest rate regimes and hypothesize that fiscal responsiveness could take on different values in different interest rate regimes.

misleading. We recompute the value of long-run debt and the position of the saddlepath for each year using that year's interest rate, and measure whether or not year-specific debt is below the year-specific saddlepath. Debt was above the saddlepath in 1993, the year that Greece reached its first threshold, and remained above until 1996. It again exceeded the saddlepath in 2005. In 2008, it was above the saddlepath and remained above until the end of the sample.

[11] Instabililty for the Greek system depends on the interest rates they use. They do find stability with one interest rate, but the implied fiscal limit, 196.5% of GDP, does not fit the data since Greece lost access with much lower debt.

Fiscal risk is affected by the value of the interest rate since a higher interest rate rotates the $\Delta d = 0$ curve clockwise, widening the space in debt and the surplus for which debt is rising. A rise in responsiveness reverses this rotation, offsetting this additional risk.

To identify different interest rate regimes for each country, we test for multiple break points using the sequential procedure of Bai and Perron (1998, 2003). We allow for up to 5 breaks and serial correlation in the errors. At the 5% significance, we find three separate interest rate regimes for all countries except Italy, which has only two. The dates for each regime in the period 1970-2008, together with the mean value of the growth-adjusted interest rates in each regime are given in Table 5.[12]

Table 5 Interest Rate Regimes

	Belgium	Canada	France	Greece	Italy	Portugal
first regime	-4.013	-3.457	-3.221	-6.333	-6.575	-7.268
	1970-1977	1970-1981	1970-1980	1970-1986	1970-1980	1970-1980
second regime	3.461	3.583	3.209	3.382	1.985	2.638
	1978-1996	1982-1996	1981-1997	1987-1997	1981-2008	1981-1997
third regime	0.353	-0.627	0.461	-1.767		-0.496
	1997-2008	1997-2008	1998-2008	1998-2008		1998-2008

Next, we reestimate the model allowing dummies for each regime on both the constant and on the responsiveness coefficient. The estimation requires the assumption that agents were able to follow their surplus rule, conditional on the interest rate regime, over the entire period. Greece experiences a very large fall in the surplus in 2008, while other countries experienced similar reductions in 2009. These reductions in the surplus were likely due to the financial crisis and could have reflected the country's inability to follow their fiscal rules in that year. Therefore, for each country, we cut the sample prior to the year with the large surplus reduction. For all countries except Greece, our sample ends in 2008, as before. For Greece, the sample ends in 2007.[13]

[12] The value for the growth adjusted interest rate is calculated as $r = \frac{1+i}{1+\rho} - 1$ where i is the long-run nominal interest rate on government bonds and ρ is the nominal GDP growth rate. We obtain almost identical results when we use the Bayesian Information Criterion and the modified Schwarz criterion of Liu et al. (1997).

[13] We tested this hypothesis by allowing the Greek sample to also end in 2008. We find very large differences in the coefficents for the third interest rate regime, implying that the attempt to fit this period substantially changes the inferred behavior. Differences are so large as to imply that Greece's fiscal rule becomes unstable.

We also repeated our threshold regressions ending the Greek sample in 2007, without significant differences from ending in 2008.

The surplus equation can be expressed as

$$s_t = c_i + \beta s_{t-1} + \gamma_i^r d_{t-1} + \epsilon_t, \tag{9}$$

where, i denotes the distinct interest rate regimes, and γ_i^r represents the responsiveness in that particular regime, not the additional responsiveness in that regime, as in the threshold model. Results are contained in Table 6.

Table 6 Responsiveness Varying with Interest Rate

	Belgium	Canada	France	Greece	Italy	Portugal
c_1	-4.641	-6.502*	-0.288	0.085	-5.718***	0.995
	(6.262)	(3.470)	(1.259)	(0.993)	(2.071)	(3.998)
c_2	-9.753***	-6.150***	-2.540***	-11.195***	-8.724***	-5.020**
	(3.549)	(0.956)	(0.582)	(2.249)	(2.630)	(2.107)
c_3	-4.476**	-5.378***	-3.797	-7.311		-3.503*
	(2.090)	(1.361)	(3.486)	(12.492)		(1.711)
β	0.393***	0.592***	0.284*	0.356**	0.422***	0.171*
	(0.075)	(0.077)	(0.144)	(0.152)	(0.114)	(0.096)
γ_1^r	0.053	0.0116	-0.004	-0.051	0.056	-0.136
	(0.114)	(0.070)	(0.016)	(0.044)	(0.042)	(0.125)
γ_2^r	0.091***	0.077***	0.047**	0.130***	0.091***	0.085**
	(0.031)	(0.011)	(0.016)	(0.027)	(0.027)	0.041
γ_3^r	0.067*	0.067***	0.047	0.066		0.035
	(0.136)	0.022	0.056	0.120		(0.027)
δ_1	0.246*	0.349***	0.267**	0.077	0.195	0.295*
	(0.136)	(0.065)	(0.084)	(0.285)	(0.204)	(0.163)
δ_2	-0.247	-0.403***	-0.633	-0.164**	-0.137	-0.148
	(0.224)	(0.086)	(0.116)	(0.073)	(0.106)	(0.157)
$\bar{R}^2$	0.908	0.958	0.650	0.779	0.905	0.517
σ	1.201	0.714	0.641	1.260	1.164	1.171

The interest rate regime model fits well, typically with adjusted R^2 measures slightly higher and standard errors slightly lower than for the threshold model.[14] The results also confirm our hypothesis that responsiveness increases in interest rates. In the first subperiod, when interest rates are negative for all countries, responsiveness is often negative

[14] Italy is an exception where the interest rate model has lower adjusted R^2 and a higher standard error.

and is always insignificantly different from zero. Even so, since stability requires comparison of responsiveness with the growth-adjusted real interest rate, which is negative in this period for all countries, these fiscal rules satisfy the stability criterion for all countries except Greece and Portugal. Responsiveness is highest in the middle period when interest rates are high, is strongly statistically significant for all countries and is high enough to satisfy the stability requirement. In the third period, as interest rates fall, responsiveness falls but remains higher than the interest rate, satisfying the stability requirement for all countries. Note, that even for Greece and Portugal, although responsiveness is not statistically different from zero in the third period, that zero exceeds the negative interest rate.

We confirm the existence of three distinct regimes for all countries except Italy with F tests for the equality of the regimes. Italy has two distinct interest rate regimes. Results are given in Table 7.

Table 7 F Tests on Regime Equality

	Belgium	Canada	France	Greece	Italy	Portugal
P(R1 = R2)	0.074	0.000	0.002	0.001	0.020	0.004
P(R2 = R3)	0.006	0.681	0.027	0.014	NA	0.028
P(R1 = R3)	0.443	0.005	0.538	0.258	NA	0.044

The very low interest rate regime, labeled R1 is always statistically different from the very highest one, labeled R2, often at greater than the 1 percent level. On the other hand, the two interest rate regimes with low rates are not statistically different for Belgium, France and Greece. The only anomaly in the table is that, for Canada, the high interest rate regime, labeled R2 is not statistically different from the lower interest rate regime labeled R3.

We experimented with many variants of the model, including letting the constant and the responsiveness be a linear function of the interest rate in each country, all with similar results. Responsiveness is increasing in the interest rate for all countries. The lower responsiveness in the third interest rate regime, when rates are low again, is robust to whether a country's debt actually falls or rises in the third regime. For Belgium and Canada, debt falls, while for France, Portugal, and Greece, it rises. Italy does not have a third interest-rate regime.

We use our estimated dynamic equation for the surplus, together with the debt accumulation equation, to solve for the value of long-run debt in each interest rate regime which satisfies the stability requirement. Long-run values of debt for each country in

each regime can be computed using equation (4). They are given in Table 8. Note that since the estimated values of the constant are negative, an increase in the interest rate tends to raise the long-run value of debt. This effect is mitigated, but not reversed by the accompanying increase in responsiveness, the rise in γ. And it is accentuated by the fact that the value for the constant often falls with an increase in the interest rate.

Table 8 Long Run Debt

	Belgium	Canada	France	Greece	Italy	Portugal
first regime	60.361	49.848	14.995	unstable	61.142	unstable
second regime	139.120	97.916	104.223	102.99	110.347	79.137
third regime	68.617	77.086	86.344	96.27	no third regime	90.293

It is interesting to compare our estimates for long-run debt with the Maastricht requirements in the European Monetary Union that debt be no more than 60% of GDP. Eichengreen and Panizza (2014) argue that the high-debt European countries are more risky than typically believed because they would have to run very high surpluses for very long periods of time, behaviors which are unprecedented for most of them, to bring debt back to the required 60% of GDP. Our estimates reveal that these countries have no plans to return debt to 60% of GDP with interest rates at their R3 values. And the estimated long-run values for debt require much lower surpluses for shorter periods of time. Even so, the trajectory toward the long-run value of debt can involve significant risk, which we investigate in the next section.

4 Fiscal Risk in Six High-Debt Developed Countries

The model states that a country is insolvent, implying that creditors will not lend, when the debt-surplus pair is on or above the effective fiscal limit. Equivalently, any debt-surplus pair, for which the peak of the debt projection at the risk-free rate exceeds the fiscal limit is a position of insolvency. When the debt-surplus pair is below but close to the effective limit, the country is at risk of receiving a shock which pushes it over and therefore is at risk of a fiscal crisis. However, the country is currently solvent and lending should occur with the interest rate adjusted to include a default premium.

Our model allows us to estimate fiscal limits for countries which have lost access to credit markets. The interest-rate model of fiscal rules provides parameter estimates which we can use to make projections of debt from any initial debt-surplus pair. Comparison

of these projections with the value for the fiscal limit determines whether any historical debt-surplus pair was deemed insolvent by creditors. Therefore, for a country, which has lost access to markets, we can use the debt-surplus pair at which they lost access, to project the path for debt and identify the peak along that path as the fiscal limit for that country.

We would like to do more. Both we and creditors would like to assess which high-debt countries are most at risk of a solvency crisis before the crisis occurs and, therefore, before we actually know the fiscal limit. We do not have a model which delivers reasonable estimates of either a fiscal limit or an effective fiscal limit for countries which have not lost access to markets. Creditors make an assessment of risk based on their information and we do the same. We do not use estimates for the top of the Laffer curve as in Bi (2012), Bi, Leeper & Leith (2013) for two primary reasons. First, it is not obvious that the political system could actually raise all tax rates to reach the peak, particularly the consumption tax rate. And, second, the ability to reduce government spending is also critically important in determining the maximum surplus, and the Laffer curve estimate has no information about the potential for government spending reduction.

Instead, we use estimates of the long-run value of debt and historical values for debt to construct a lower bound on the fiscal limit. We treat the lower bound as the fiscal limit and compare projections for debt using our estimated parameters with this lower bound. If our lower bound were the actual fiscal limit, then countries with projected debt exceeding the lower bound would have lost access to credit markets. When they retained access, we infer that creditors had more information on the fiscal limit than our lower bound. Even so, we judge countries which fail this second solvency criterion at the lower bound on the fiscal limit as at "high risk." We label countries which pass this criterion as at "low risk."[15] Given the absence of information on the value for a fiscal limit, we cannot make precise estimates of the risk of a financial crisis.

We divide countries into these two risk classes in the years following the global financial crisis and determine the extent to which the division predicts future fiscal crises. Greece and Portugal both receive the "high risk" classification one to two years prior to their crises. Italy receives the classification in 2012, while all other countries remain in the "low risk" class throughout the period.

These estimates assume that a country faces no additional fiscal limits in following its fiscal rule even following the upheavals after the global financial crisis. The large

[15] It would be possible to divide the "low risk" class into two sections, one with "low risk" and one with "minimal risk". The "low risk" category would be countries close enough to the effective fiscal limit to experience substantive default premia. We do not pursue this because we do not estimate default premia.

deviations of the primary surplus from the surplus rule in almost all countries over this period suggest otherwise. Countries with greater difficulties adjusting their surpluses to follow their rules could face greater risk of crises than our estimates suggest. Therefore, we compute two measures of a country's inability to follow its fiscal rule following the financial crisis.

4.1 Lower Bound on Fiscal Limit

Our estimates provide information on a lower bound for the effective fiscal limit. This is because they provide an estimate for the long-run value of debt in each regime. Given that agents were lending during each interest rate regime, their expectation must have been that the effective fiscal limit was above the long-run value of debt. Otherwise, they would not have lent in a regime for which debt had been expected to travel above its fiscal limit.

We have additional information on the lower bound, given by the historical experience with debt. All countries had access to financial markets over our sample, implying that all historical values for debt must have been below the fiscal limit. We use the historical values of debt to infer ability to pay for the same reasons that private credit markets use a household's history of borrowing and lending to set credit limits. Our assumption is that a country which has successfully serviced a particular level of debt could do so again.[16]

We use the estimated long-run values of debt in each interest rate regime, together with the historical values, to compute a lower bound on the value for the fiscal limit. Next, we use our estimated fiscal rule, together with the equation for the evolution of debt, to project the path of debt forward for each country using the average of the historical interest rate for the final interest-rate regime. Our projections for Greece begin in 2008, while those for others begin in 2009. Figure 7 contains phase diagrams for each country together with the projections.

[16] Objection to this assumption could be made on the grounds that ability to service a particular level of debt declines with an increase in the interest rate. However, in our sample maximum debt values tend to occur with interest rates higher or equal to those in the third-interest rate regime, which is our measure of current interest rates.

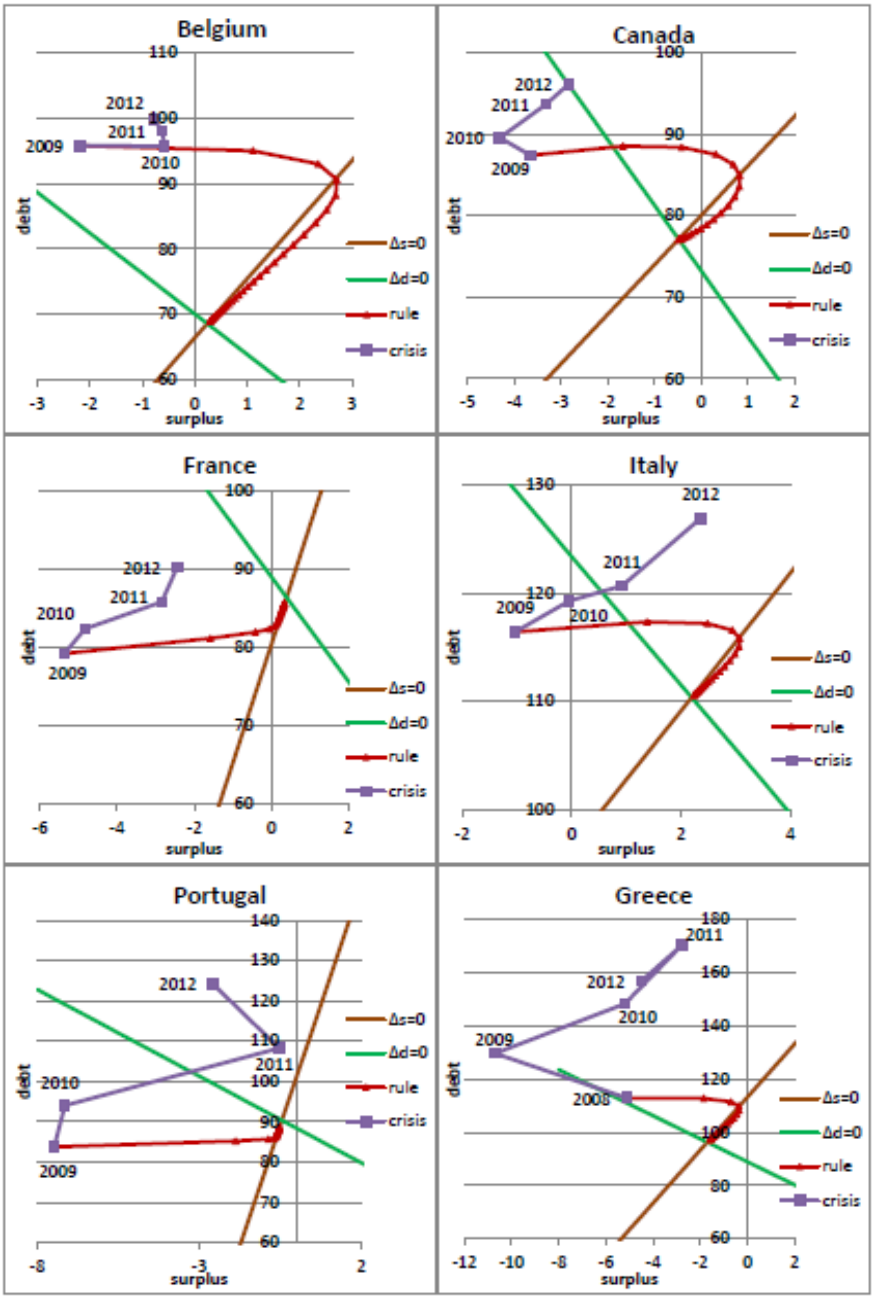

Figure 7

Table 9 contains both the lower bound on the fiscal limit and the maximum values for debt along the projected path beginning at each date. For Belgium and France, the lower bound is the long-run value of debt in the high interest-rate regime, for Portugal, it is the long-run value of debt in the third interest-rate regime, and for Canada and Italy,

the lower bound is a historical maximum on the value of debt over the period 1970-2008, and for Greece the historical maximum uses the dates 1970-2007.

Table 6 Fiscal Risk

	Belgium	Canada	France	Greece	Italy	Portugal
lower bound	139.12	109.4	104.22	110.03	121.25	90.29
2009	95.72	88.50	86.34	130.15	117.35	90.29
2010	95.66	90.87	86.34		119.60	94.99
2011	97.95	94.09	86.34		120.70	
2012	99.80	96.11	93.74		126.96	

Following the financial crisis, Belgium remains in the portion of the phase diagram for which debt is falling. Therefore, the maximum value of debt along the projected path is always its current value. Canada is just at the border separating the region for which debt is rising from that for which debt is falling. Therefore, for the first three years, debt is projected to rise by a small amount for one year, while for 2012, debt is projected to fall. France is initially in a position from which debt is projected to rise continuously toward its long-run equilibrium value without overshooting. However, that changes in 2012, when debt-surplus pair take on a value from which debt is projected to overshoot its long-run value, but to remain below the lower bound on the fiscal limit. Although for France, the space for which debt is rising is large, France is at a point low enough in that space such that debt is not expected to overshoot. For all three countries, debt is projected to remain well below the lower bound on the fiscal limit.

The other three countries did not weather the financial crisis as well. Italy has been at the boundary separating the space for which debt is increasing versus falling. In 2009, projected debt rises slightly for one year, while in the other years projected debt falls. However, in 2012, Italy reached in a position for which 2012 debt exceeds the lower bound on the fiscal limit, such that it meets our definition for a "high risk" country.

In 2009, Portugal looks something like France as it is in the lower portion of the space for which debt is projected to rise toward the long-run without overshooting. However, Portugal is riskier than France because its lower bound on the fiscal limit is the long-run value of debt for the third period. Although, debt is not projected to rise above the lower bound, since debt is projected to rise slowly toward the lower bound, even a small shock could change that. And that it exactly what happened in 2010. Debt rose above the upper bound and was projected to rise further, placing the fiscal limit around 95% of GDP. Portugal meets our definition for a "high risk" country at the end of 2010, and it

lost access to credit markets in 2011.

We assess the Greek risk of crisis beginning in 2008 since that was the year Greece experienced a large adverse surplus shock.[17] The sample for estimates of the fiscal rule ends in 2007. The 2008 adverse surplus shock was so large that debt exceeded the lower bound, placing Greece at "high risk" in 2008. Greece was able to borrow at the end of 2009 with debt at 130% of GDP but lost access early in 2010, placing the fiscal limit on Greece close to 130%.

We are not fully confident of our assessment that Greece entered the "high risk" category as early as 2008 or that its fiscal limit is as large as 130% of GDP. The problem is that Greek data over this period was substantially revised, implying that creditors could have been forming expectations based on misleading data. As of November 2009, OECD was projecting Greek debt to reach 112% of GDP by the end of 2009, with values of debt prior to 2009 all below the lower bound on the fiscal limit. Additionally, at this date, data implied that surpluses were all larger. Therefore, based on the data available at the time, Greece did not become at "high risk" of a fiscal crisis until 2009, and the crisis occurred shortly thereafter in early 2010.[18] Additionally, this data suggests that markets could have put Greece's fiscal limit much lower, since they were refusing to lend when data available at the time yielded a maximum debt projection of 112.6% of GDP.

4.2 Deviation from Fiscal Rule

Our second measure of risk takes account of the fact that countries might not have been able to follow their fiscal rules following the financial disruption created by the global financial crisis. Countries which faced greater constraints in adjusting their surpluses to the increase in debt, created by the global financial crisis, were more likely to see larger increases in debt. For countries whose debt was already near its fiscal limit, the difficulty raising the surplus to follow the fiscal rule increases the risk of a fiscal crisis. For each country, we compute two measures for the difficulty in following the fiscal rule.

For the first, we compute the deviation of the actual surplus from that predicted by the estimated surplus rule and report the deviations as a fraction of the standard error. We compute the error in 2009 and take the average error for 2010-2012. Results are contained in the first two rows of Table 10.

[17] We could assess the risk of a Greek crisis on earlier dates and would have identified 2005 as a year when Greece became at risk due to a large increase in debt. However, Greece was only "at risk" and not actually in crisis, and it brought debt down such that 2005 became the new historical maximum.

[18] Data revisions cannot account for the very early crisis prediction from the threshold model because the threshold model indicates debt above the fiscal limit as early as 1993 in one case and 2000 in another.

Table 10 Deviations from the Surplus Rule

	Belgium	Canada	France	Greece	Italy	Portugal
2009	-3.24	-1.59	-5.31	-5.46	-1.60	-4.32
2010-2012	-1.62	-1.53	-2.35	-2.64	-0.43	-2.49
debt projections	1%	4%	5%	15%	9%	10%

All of the countries experienced large negative shocks to the surplus in 2009 following the onset of the financial crisis, with shocks greater than one and a half standard deviations of the surplus residual. Four countries experienced surplus deviations greater than three standard deviations. For the next three years, the average deviation of the actual surplus from its predicted value was negative. For half of the countries, France, Greece, and Portugal, the deviation was greater than two standard deviations, implying that these countries faced greater difficulties than the rest. For Portugal, its 2010 deviation was -5.51 standard deviations, and for Greece, its 2009 deviation was -5.46, indicating extreme difficulty the year before each country lost access to credit markets. These countries are in the "high risk" category under the assumption that they were following their fiscal rule. Their inability to follow their fiscal rule provides more evidence for that categorization. France also performs poorly on this measure raising the possibility that its debt could rise sufficiently to place it in the "high risk" category in the near future.

For the second measure, we compute the percentage deviations of debt from their projected paths. For countries which retained access to markets, we project from 2009 to 2012, a three-year projection. For Portugal (Greece), the projection begins in 2009 (2008) and ends in 2010 (2009), the year prior to loss of access for each, one-year projections. Those deviations are contained in the final line of Table 10. For all countries which retained market access, the value of debt in 2012 relative to that projected using the fiscal rule from 2009 was less than ten percent higher. For Greece, debt was 15% higher in 2009, and for Portugal, debt was 10% higher in 2010, reflecting very large one-year increases in debt, relative to projections, the year prior to each country's loss of access to credit markets.

Therefore, for both Greece and Portugal, both measures of difficulty in following the fiscal rule reinforce the assessment that these countries belonged in the high risk category.

5 Conclusion

We propose and implement a simple data-based method for separating high-debt, developed countries into high and low risk categories. The method is based on the assumption that creditors do not lend if there is no interest rate which could both provide the expected risk-free rate of return on sovereign bonds while keeping the expected value of debt below the fiscal limit in its trajectory toward long-run equilibrium value. A necessary condition for the existence of an equilibrium interest rate consistent with lending, is that the path for debt, projected from the fiscal rule with interest fixed at the risk-free rate, remain below the fiscal limit in transit to the long-run equilibrium (Daniel and Shiamptanis 2012).

Implementation requires estimation of a fiscal rule, whereby the surplus adjusts to lagged debt. If surplus-responsiveness to debt is high enough, then the dynamic system, comprised of the fiscal rule and the debt accumulation equation, is globally stable, ruling out explosive debt. Agents would not lend if debt were explosive because for any finite value of a fiscal limit, debt would be expected to violate the limit in transit to infinity. We should observe an equilibrium with lending only when the fiscal authority is following a stable fiscal rule or is expected to switch to one in time to prevent debt from becoming explosive.

Empirically, the surplus-responsiveness to debt is time-varying. We follow Bohn (2007, 2008) and Ghosh et al (2013) and estimate a model in which surplus-responsiveness depends on the level of debt. This specification can provide an estimate of the fiscal limit if surplus-responsiveness falls in debt and eventually fails the stability criterion. This occurs only for Greece. However, the resulting model implies that debt was explosive for many years when Greece successfully borrowed, leading us to doubt the model specification.

We propose an alternative source of time-variation in surplus responsiveness to debt and allow the responsiveness to depend on the interest rate. The model yields surplus-responsiveness increasing in the interest rate. Additionally, the dynamic system in the surplus and debt is stable prior to the financial crisis, consistent with the lending that occurred. We estimate the fiscal rule through 2008 and use it to place the six high-debt countries into two risk categories following the global financial crisis.

We construct a lower bound on the fiscal limit by using the fact that agents who were lending over the sample did not expect debt to become explosive. Therefore, the equilibrium value of debt, toward which each economy was converging, must have been below the fiscal limit. Additionally, any historical value of debt, including the historical maximum, must have been below the fiscal limit. We let the lower bound on debt equal

the maximum of the equilibrium value and the historical maximum. We project debt forward from particular dates, using the risk-free interest rate and put the country into the "high risk" category if the projected debt exceeds the lower bound. The method predicts the Greek and Portuguese crises about one year before each occurred and warns of an Italian one. Additionally, we construct measures designed to capture a country's difficulty in following the fiscal rule after the global financial crisis and find that both Greece and Portugal had extreme difficulty compared to the other countries, reenforcing our assessment that these countries were at "high risk".

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