Sticky-Price Exchange Rate Models
Obstfeld and Rogoff, Chapter 9
1 Empirical Motivation for Sticky Prices

- Variability of the real exchange rate \( \frac{E_P^*}{P} \) is very similar to variability of the nominal exchange rate \( E \)

- The real exchange rate is much more volatile over floating rate periods

- Engel demonstrated that the relative price of similar goods between the US and Canada is much more volatile than the relative price of different goods within a country

- Why would a country deliberately depreciate its currency to shift demand toward their goods if prices immediately adjusted, eliminating the effect?
1.1 Mundell-Fleming-Dornbusch model

1.2 Equations of the model

- Small letters denote logarithms with the exception of the interest rate

- Uncovered interest rate parity with fixed foreign interest rate

\[ i_{t+1} = i^* + e_{t+1} - e_t \]

- Money demand

\[ m_t - p_t = -\eta i_{t+1} + \phi y_t \]
• Real exchange rate fluctuates in the short run and has a fixed long-run value (fixed foreign price level at unity implying \( p^* = 0 \))

\[
\bar{q} = q_t = e_t + p^* - p_t
\]

• Demand is increasing in the real exchange rate relative to its long-run value

\[
y_t^d = \bar{y} + \delta (e_t + p^* - p_t - \bar{q})
\]

\[
y_t^d - \bar{y} = \delta (q_t - \bar{q})
\]

• Phillips Curve describing price adjustment

\[
p_{t+1} - p_t = \psi \left( y_t^d - \bar{y} \right) + \tilde{p}_{t+1} - \tilde{p}_t
\]
- prices rise in response to excess demand and to inflation which would occur in full equilibrium

- $\tilde{p}_t$ is defined as the price level that would prevail if the goods market cleared

$$\tilde{p}_t \equiv e_t + p^* - \bar{q}$$
1.3 Solution of the Model with Phase Diagrams

- Write the model as two difference equations in the real and nominal exchange rates, $q_t$ and $e_t$

- Difference equation in $q_t$
  
  - Substitute excess demand into the Phillips Curve
    \[ p_{t+1} - p_t = \psi \delta (q_t - \bar{q}) + \tilde{p}_{t+1} - \tilde{p}_t \]
  
  - Use equilibrium price to substitute for equilibrium inflation
    \[ \tilde{p}_{t+1} - \tilde{p}_t = e_{t+1} - e_t \]
– Substitute into the inflation equation

\[ p_{t+1} - p_t = \psi \delta (q_t - \bar{q}) + e_{t+1} - e_t \]

– Solve for the rate of change of the real exchange rate

\[ p_{t+1} - e_{t+1} - (p_t - e_t) = \psi \delta (q_t - \bar{q}) \]

\[ q_{t+1} - q_t = -\psi \delta (q_t - \bar{q}) \]
• Difference equation in $e_t$

  - Money demand equals money supply

    $$m_t - p_t = -\eta i_{t+1} + \phi y_t$$

  - Simplify by setting

    $$\bar{y} = i^* = 0$$

  * Substitute definition of real exchange rate for price

    $$p_t = e_t - qt$$

  * Interest rate parity for the interest rate

    $$i_t = e_{t+1} - e_t$$
* And aggregate demand for output

\[ y_t = \delta (q_t - \bar{q}) \]

* to yield money demand as

\[ m_t - e_t + q_t = -\eta (e_{t+1} - e_t) + \phi \delta (q_t - \bar{q}) \]

– Solve for the rate of change of the nominal exchange rate

\[ e_{t+1} - e_t = \frac{1}{\eta} \left[ -m_t + e_t - q_t + \phi \delta (q_t - \bar{q}) \right] \]
Phase diagram with $e$ on vertical axis and $q$ on horizontal

$$\Delta q_{t+1} = q_{t+1} - q_t = -\psi \delta (q_t - \bar{q})$$

$$\Delta e_{t+1} = e_{t+1} - e_t = \frac{1}{\eta} [ -m_t + e_t - q_t + \phi \delta (q_t - \bar{q}) ]$$

$-\Delta q_{t+1} = 0$ requires

$$q_t = \bar{q}$$

* $\Delta q_{t+1} = 0$ is vertical at $q_t = \bar{q}$

* When $q_t$ increases, $\Delta q_{t+1}$ falls

$$\frac{\partial \Delta q_{t+1}}{\partial q_t} = -\psi \delta < 0$$

• implying that arrows of motion point towards $\Delta q_{t+1} = 0$
\[ \Delta e_{t+1} = 0 \] requires

\[ e_t = m_t + qt - \phi \delta (qt - \bar{q}) \]

* Slope of \( \Delta e_{t+1} = 0 \) is \( 1 - \phi \delta \)

* Assume that money is constant and assume that output is not too responsive to relative price so that

\[ 1 - \phi \delta > 0 \]

  · such that the slope of \( \Delta e_{t+1} = 0 \) is positive

* When \( e_t \) increases, \( \Delta e_{t+1} \) increases

\[ \frac{\partial \Delta e_{t+1}}{\partial e_t} = \frac{1}{\eta} > 0 \]

  · implying that arrows of motion point away from \( \Delta e_{t+1} = 0 \)
– System has an upward sloping saddlepath and a steady state with

\[ \tilde{e} = \tilde{m} + \tilde{q} \]
• Comparative Statics: Unanticipated increase in $m_t$

  – $\Delta e_{t+1} = 0$ curve shifts vertically upwards by the increase in the money supply

  – new saddlepath shifts upwards to go through the new steady state

  – system must jump to the saddlepath immediately or dynamics will not take it to the steady state

  – price cannot jump, but the exchange rate can

  – an increase in $e_t$ increases $e_t$ and $q_t$ by equal quantities, moving the system along a 45 degree line from the initial equilibrium toward the saddlepath
– the exchange rate overshoots its long-run equilibrium value, and therefore begins falling after the initial increase
• Intuition for overshooting

– money market equilibrium

\[ m_t - p_t = -\eta i_{t+1} + \phi y_t \]

– increase in the money supply with prices fixed requires an increase in money demand

– if output is not too responsive, get increase in money demand with a fall in the nominal interest rate

– interest rate parity implies

\[ i_{t+1} = i^* + e_{t+1} - e_t \]

* therefore, for the nominal interest rate to fall \( e_t \) must rise by more than \( e_{t+1} \) rises, that is, overshooting
– if output is more responsive, the nominal interest rate could rise and overshooting would not occur

– implies that exchange rates are forecastable
1.4 Analytical Solution

- Solve equation for real exchange rate first

\[ q_{t+1} - q_t = -\psi \delta (q_t - \bar{q}) \]

\[ q_{t+1} - \bar{q} = (1 - \psi \delta) (q_t - \bar{q}) \]

- Solve forward

\[ q_{t+2} - \bar{q} = (1 - \psi \delta) (q_{t+1} - \bar{q}) = (1 - \psi \delta)^2 (q_t - \bar{q}) \]

\[ q_s = \bar{q} + (1 - \psi \delta)^{s-t} (q_t - \bar{q}) \]
• Solve equation for nominal exchange rate

\[ e_t = \frac{\eta}{1 + \eta} e_{t+1} + \frac{1}{1 + \eta} \left[ m_t + q_t (1 - \phi \delta) + \phi \delta \bar{q} \right] \]

- Subtract \( \bar{q} \)

\[ e_t - \bar{q} = \frac{\eta}{1 + \eta} (e_{t+1} - \bar{q}) + \frac{1}{1 + \eta} \left[ m_t + (q_t - \bar{q}) (1 - \phi \delta) \right] \]

- Solve forward

\[ e_{t+1} - \bar{q} = \frac{\eta}{1 + \eta} (e_{t+2} - \bar{q}) + \frac{1}{1 + \eta} \left[ m_{t+1} + (q_{t+1} - \bar{q}) (1 - \phi \delta) \right] \]
- Substitute for \((e_{t+1} - \bar{q})\)

\[
e_t - \bar{q} = \frac{\eta}{1 + \eta} \left[ \frac{\eta}{1 + \eta} (e_{t+2} - \bar{q}) \right] \\
+ \frac{\eta}{1 + \eta} \left[ \frac{1}{1 + \eta} [m_{t+1} + (q_{t+1} - \bar{q}) (1 - \phi \delta)] \right] \\
+ \frac{1}{1 + \eta} [m_t + (q_t - \bar{q}) (1 - \phi \delta)] \\
= \left( \frac{\eta}{1 + \eta} \right)^2 (e_{t+2} - \bar{q}) \\
+ \frac{1}{1 + \eta} [m_t + (q_t - \bar{q}) (1 - \phi \delta)] \\
+ \frac{1}{1 + \eta} \left[ \frac{\eta}{1 + \eta} [m_{t+1} + (q_{t+1} - \bar{q}) (1 - \phi \delta)] \right]
\]
\[ e_t - \bar{q} = \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left( \frac{\eta}{1+\eta} \right)^{s-t} \left[ m_s + (q_s - \bar{q}) (1 - \phi \delta) \right] \]

\[ + \lim_{T \to \infty} \left( \frac{\eta}{1+\eta} \right)^T (e_{t+T} - \bar{q}) \]

- Setting the limit equal to zero places the system on the saddlepath by ruling out speculative bubbles.

- Set \( m_t = \bar{m} \) and substitute for \( q_s - \bar{q} \) from above yields the equation for the saddlepath.
\[ e_t - \bar{q} = \bar{m} + \frac{1}{1 + \eta} \sum_{s=t}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^{s-t} \left( 1 - \psi \delta \right)^{s-t} (q_t - \bar{q}) (1 - \phi \delta) \]

\[ = \bar{m} + \frac{(1 - \phi \delta)}{1 + \eta} (q_t - \bar{q}) \sum_{s=t}^{\infty} \left( \frac{\eta (1 - \psi \delta)}{1 + \eta} \right)^{s-t} \]

\[ = \bar{m} + \frac{(1 - \phi \delta)}{1 + \eta \psi \delta} (q_t - \bar{q}) \]

- Saddlepath relates the value of the current nominal exchange rate to the current real exchange rate

- Substitute for \((q_t - \bar{q})\) to solve for the exchange rate going forward

\[ e_s - \bar{q} = \bar{m} + \frac{(1 - \phi \delta)}{1 + \eta \psi \delta} (1 - \psi \delta)^{s-t} (q_t - \bar{q}) \]
- Solve for jumps in \( e_t \) and \( q_t \) due to the increase in \( \tilde{m} \)

\[
\frac{de_t}{d\tilde{m}} = 1 + \frac{(1 - \phi \delta) dq_t}{1 + \eta \psi \delta d\tilde{m}}
\]

- recognize that

\[
\frac{de_t}{d\tilde{m}} = \frac{dq_t}{d\tilde{m}}
\]

- substitute

\[
\frac{de_t}{d\tilde{m}} = 1 + \frac{(1 - \phi \delta) de_t}{1 + \eta \psi \delta d\tilde{m}}
\]

\[
\frac{de_t}{d\tilde{m}} = \frac{1 + \eta \psi \delta}{\phi \delta + \eta \psi \delta}
\]

- yields overshooting if \( \phi \delta < 1 \)
• For effect on interest rate need effect on $e_{t+1}$

$$\frac{de_{t+1}}{d\bar{m}} = 1 + \frac{(1 - \phi \delta) (1 - \psi \delta)}{1 + \eta \psi \delta} \frac{dq_t}{d\bar{m}}$$

– Substitute

$$\frac{de_t}{d\bar{m}} = \frac{dq_t}{d\bar{m}} = \frac{1 + \eta \psi \delta}{\phi \delta + \eta \psi \delta}$$

$$\frac{de_{t+1}}{d\bar{m}} = 1 + \frac{(1 - \phi \delta) (1 - \psi \delta)}{\phi \delta + \eta \psi \delta}$$
\[ \frac{de_{t+1}}{d\tilde{m}} - \frac{de_t}{d\tilde{m}} = 1 + \frac{(1 - \phi\delta)(1 - \psi\delta) - (1 + \eta\psi\delta)}{\phi\delta + \eta\psi\delta} \]

\[ = \frac{\phi\delta - 1 + (1 - \phi\delta)(1 - \psi\delta)}{\phi\delta + \eta\psi\delta} \]

\[ = \frac{-\psi\delta(1 - \phi\delta)}{\phi\delta + \eta\psi\delta} < 0 \]
Consider model with monetary growth

\[ m_s = \bar{m}_t + \mu (s - t) \]

- Substitute into solution for exchange rate

\[ e_t - \tilde{q} = \frac{1}{1 + \eta} \sum_{s=t}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^{s-t} \left[ \bar{m}_t + \mu (s - t) + (q_s - \tilde{q}) (1 - \phi \delta) \right] \]

\[ \frac{1}{1 + \eta} \sum_{s=t}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^{s-t} [\bar{m} + \mu (s - t)] = \bar{m} + \mu \eta \]

\[ e_t - \tilde{q} = \bar{m}_t + \mu \eta + \frac{(1 - \phi \delta)}{1 + \eta \psi \delta} (q_t - \tilde{q}) \]
– compute effect on $e_t$ of increase in money growth $\mu$

$$
\frac{de_t}{d\mu} = \eta + \frac{(1 - \phi \delta) de_t}{1 + \eta \psi \delta} d\mu
$$

$$
\frac{de_t}{d\mu} = \frac{\eta (1 + \eta \psi \delta)}{\phi \delta + \eta \psi \delta}
$$

– solve generally for $e_s$

$$
e_s - \bar{q} = \bar{m}_s + \mu \eta + \frac{(1 - \phi \delta)}{1 + \eta \psi \delta} (q_s - \bar{q})
$$

– substitute for $q_s - \bar{q}$ and for $\bar{m}_s$

$$
e_s - \bar{q} = \bar{m}_t + \mu (s - t) + \mu \eta + \frac{(1 - \phi \delta)}{1 + \eta \psi \delta} (1 - \psi \delta)^{s-t} (q_t - \bar{q})
$$
- solve for $e_{t+1}$

$$e_{t+1} - \bar{q} = \bar{m}_t + \mu + \mu \eta + \frac{(1 - \phi \delta)}{1 + \eta \psi \delta} (1 - \psi \delta)(q_t - \bar{q})$$

$$\frac{de_{t+1}}{d\mu} = 1 + \eta + \frac{(1 - \phi \delta)(1 - \psi \delta)de_t}{1 + \eta \psi \delta} \frac{d\mu}{d\mu}$$

$$= 1 + \eta + \frac{\eta (1 - \phi \delta)(1 - \psi \delta)}{\phi \delta + \eta \psi \delta}$$
– subtracting effect of \( \mu \) on future and current exchange rates

\[
\frac{de_{t+1}}{d\mu} - \frac{de_t}{d\mu} = 1 + \eta + \frac{\eta (1 - \phi \delta) (1 - \psi \delta)}{\phi \delta + \eta \psi \delta} - \frac{\eta (1 + \eta \psi \delta)}{\phi \delta + \eta \psi \delta}
\]

\[
= 1 + \eta \frac{[\phi \delta + \eta \psi \delta - 1 - \eta \psi \delta]}{\phi \delta + \eta \psi \delta} + \frac{\eta (1 - \phi \delta) (1 - \psi \delta)}{\phi \delta + \eta \psi \delta}
\]

\[
= 1 + \frac{\eta [\phi \delta - 1]}{\phi \delta + \eta \psi \delta} + \frac{\eta (1 - \phi \delta) (1 - \psi \delta)}{\phi \delta + \eta \psi \delta}
\]

\[
= \frac{\phi \delta + \eta \psi \delta - \eta \psi \delta (1 - \phi \delta)}{\phi \delta + \eta \psi \delta} > 0
\]

– implying that the interest rate rises when money growth increases
1.5 Real shocks

- Increase in $\bar{q}$

$$\bar{e} = \bar{m} + \bar{q}$$

- Exchange rate adjusts immediately to long-run equilibrium value
  - No need for price to adjust so no delay and no overshooting
1.6 Correlations between money shocks, real interest rates and nominal interest rates

- Permanent increase in money supply
  
  - Future exchange rate \((e_{t+1})\) rises less than current exchange rate \((e_t)\) implying the nominal interest rate falls from IRP

\[
i_t = i^* + e_{t+1} - e_t
\]

- Increase in money growth rate
  
  - Future exchange rate rises more than current exchange rate implying the nominal interest rate rises

  - Fisher relation
• Real interest rate equals the nominal rate plus the rate of depreciation of the real exchange rate

\[ i_t - (p_{t+1} - p_t) = i^* + e_{t+1} - e_t - (p_{t+1} - p_t) \]

\[ = i^* + q_{t+1} - q_t \]

– Solution for \( q_{t+1} \)

\[ q_{t+1} - \bar{q} = (1 - \psi \delta) (q_t - \bar{q}) \]

– Solving for \( q_{t+1} - q_t \)

\[ q_{t+1} - q_t = -\psi \delta (q_t - \bar{q}) \]

– Real interest rate is high when the real exchange rate is low

\[ i_t - (p_{t+1} - p_t) = i^* - \psi \delta (q_t - \bar{q}) \]
Empirical evidence

- Strong dollar in 1980’s when US nominal interest rates were high

- Countries which adopt tight monetary policy, raising nominal interest rates, experience an increase in real interest rates and currency appreciation

- Empirically, do not get tight relationship between the real exchange rate and real interest rate over short horizons

- Monetary model of the nominal exchange rate
  * Meese and Rogoff show that a random walk model predicts better
    - Asset price prediction is generally difficult and exchange rates are asset prices
More recent empirical work is more supportive of monetary model as a long-run equilibrium relationship
2 Gold Standard

2.1 Assumptions

- Two-country model

- Money supply in each country is proportional to gold

\[ M_t = \alpha Z_t G_t \]

\[ M_t^* = \alpha^* Z_t^* G_t^* \]

- \( \alpha \) is money multiplier
– \( Z_t \) is fixed domestic-currency price of gold

– \( G_t \) is quantity of gold

– superscript * denotes foreign country

• World supply of gold is fixed

\[
G_t + G^*_t = \bar{G}
\]
2.2 Prices satisfy PPP

- Purchase red wine from France under the gold standard
  - sell $1 for $\frac{1}{Z}$ units of gold
    - sell $\frac{1}{Z}$ units of gold for $\frac{Z^*}{Z}$ francs
    - trade $\frac{Z^*}{Z}$ francs for $\frac{1}{P^*}$ units of wine

- Alternatively, purchase identical quality red wine from California
  - sell $1 for $\frac{1}{P}$ units of wine
• Assumption of identical wine, arbitrage requires that the $1 purchase the same quantities of wine

\[
\frac{1}{P} = \frac{Z^*}{Z} \frac{1}{P^*}
\]

\[
\frac{P^*}{P} = \frac{Z^*}{Z} = \frac{1}{E}
\]

– Relative gold prices determine relative price levels and the nominal exchange rate
2.3 Distribution of gold (relative money supplies) is endogenous

- Money demand for each country in logs
  \[ m_t - p_t = \phi y_t - \eta t_{t+1} \]
  \[ m^*_t - p^*_t = \phi y^*_t - \eta^* t_{t+1} \]

- Subtract foreign from domestic
  \[ m_t - m^*_t - (p_t - p^*_t) = \phi (y_t - y^*_t) - \eta (i_{t+1} - i^*_{t+1}) \]
  - money difference
  \[ m_t - m^*_t = \alpha - \alpha^* + z_t - z^*_t + g_t - g^*_t \]
- PPP

\[ p_t - p_t^* = z_t - z_t^* \]

- IRP with fixed exchange rates

\[ i_{t+1} - i_{t+1}^* = 0 \]

- Substituting

\[ \alpha - \alpha^* + g_t - g_t^* = \phi(y_t - y_t^*) \]

Solve for distribution of gold

\[ g_t - g_t^* = \phi(y_t - y_t^*) - (\alpha - \alpha^*) \]

- money market equilibrium determines the world distribution of gold and therefore the world distribution of money supplies
– gold flows into countries with relative higher money demand (higher income) and with a relatively lower gold multiplier ($\alpha$)

* need more gold to meet money demand to make up for the smaller multiplier
2.4 Great Depression

- Began in the US with bank failures reducing money multiplier, $\alpha$
  - Gold flows into US and out of the rest of the world
  - Tight money in US, due to bank failures, was transmitted to ROW through gold standard

- Hypothesis: if money has real effects, then countries which abandoned the gold standard more quickly should have recovered from the Great Depression more quickly
  - Verified
  - Considered one of the major pieces of evidence for real effects of money
3 Choice between Fixed and Flexible Exchange Rates

3.1 Real Effects of Money

- If money has no real effects, both systems are equivalent

- Choice can depend on why and how money has real effects
3.2 Keynesian Sticky Prices

- Model
  - Money demand with an error
    \[ m_t - p_t = \phi y_t - \eta i_{t+1} + \epsilon_t \]
  - Demand for goods with an error
    \[ y^d_t = \bar{y} + \delta (e_t + p^* - p_t - \bar{q}_t) - \lambda i_{t+1} + \nu_t \]
  - Let prices be fixed in the short run

- Fixed exchange rates
– Money demand disturbances do not affect interest rate and are not transmitted to goods market
  * Money demand disturbances only affect endogenous quantity of money
– Aggregate demand disturbances directly affect aggregate demand without any offset from the exchange rate

• Flexible exchange rates
  – Increase in aggregate demand appreciates the exchange rate ($e_t$ falls) offsetting the effect of the disturbance on output
  – Increase in money demand affects the interest rate, transmitting the effect of the disturbance to the goods market.
Choice between fixed and flexible exchange rates depends on where most disturbances originate

- money market ⇒ fixed rates are better
- goods market ⇒ flexible rates are better
- optimal flexibility could be based on fraction of disturbances originating in each market
3.3 Other Considerations

- Flexible exchange rates are volatile and imply volatile real exchange rates
  - Volatility could increase uncertainty, thereby reducing trade, and the gains from trade
  - Countries which borrow in foreign currency could see large fluctuations in indebtedness

- Fixed exchange rates require that the pegging country give up independent monetary policy
  - Currency crisis when country needs to regain independent monetary policy
Country could have trouble maintaining low inflation under flexible exchange rates and independent monetary policy.

- Fixed rates might provide discipline
4 Optimum Currency Areas (Mundell)

4.1 Monetary Union

- Adopt a common currency
- Common monetary policy
- Equivalently, irrevocably fixed exchange rates
4.2 Benefits

- Reduction in transactions costs for trades within the monetary union
  - Increased trade and increased gains from trade
  - Gains from real and financial integration

- Reduction in volatility of real exchange rates due to fixed nominal exchange rate

- Credibility for a monetary policy with low inflation

- Eliminate currency crises and associated speculation (really?)
4.3 Costs

- Each country loses independent monetary policy
  - Loss is more serious
    * the greater the real effects of money perhaps due to sticky prices
    * the less mobile is labor across countries
  - Consider a switch in demand from country A to country B
    * Labor mobility
      - If labor is mobile, labor moves from country A to country B
      - If not, and if prices are sticky, employment falls in country A and rises in country B
* Price flexibility
  
  - If prices are flexible, prices rise in country A relative to country B and output and employment are unchanged

* Monetary policy can substitute for labor mobility and price flexibility
  
  - Increase in money supply in country A raises demand
  
  - Decrease in money supply in country B lowers demand

  - Southern European countries in 2012

  * Stimulate economies if they could use expansionary monetary policy

  * Depreciate independent currencies and switch demand toward their goods
• Each country loses right to determine its own reliance on seigniorage as a method of financing government spending
  – Each receives a share of seigniorage determined by the union monetary authority
  – For countries with previously high inflation, represents a reduction in revenue, to which they must adjust by raising other taxes or reducing spending

• Financial crisis in a single country shared by others
5 Second Generation Currency Crises

5.1 Models with Multiple Equilibria

- One equilibrium in which exchange rate remains fixed
- Another in which fixed exchange rate fails
- System which prevails depends on beliefs by agents
- Role for speculators’ beliefs to determine whether fixed rate succeeds or fails
- Monetary expansion increases domestic inflation and output
5.2 Model

- Government loss function

\[ L = (y_t - \bar{y} - k)^2 + \chi \pi^2 + c(\pi_t) \]

- Government wants output higher than full employment (\(\bar{y}\)) by \(k\)

- Government cares about deviations of output from goal and about inflation

- Loss is quadratic in deviations

- Political cost of abandoning the fixed exchange rate

* If country depreciates, \(c(\pi_t) = \bar{c}\)
* If country appreciates, \( c(\pi_t) = \tilde{c} \)

- Inflation is initially zero under fixed rates

- Output depends on Lucas inflation surprises with a disturbance \( z_t \)

\[
y_t = \bar{y} + \pi_t - \pi_t^e - z_t
\]
5.3 Monetary Policy

- One-shot game
  - Government chooses $\pi_t$ after observing $\pi_t^e$ and $z_t$

- Solve for minimum loss conditional on abandoning fixed exchange rate and choosing $\pi_t$ to minimize loss
  - Loss with output substituted
    \[
    L = (\pi_t - \pi_t^e - z_t - k)^2 + \chi \pi_t^2 + c(\pi_t)
    \]
– Minimize with respect to $\pi_t$

$$\frac{\partial L}{\partial \pi_t} = 2 (\pi_t - \pi^e_t - z_t - k + \chi \pi_t) = 0$$

$$\pi_t = \frac{\pi^e_t + z_t + k}{1 + \chi}$$

– When $\pi^e_t = 0$, consistent with confidence in the fixed exchange rate, and $z_t$ takes on its mean value of zero, the incentive to inflate depends on a positive value for $k$

– Minimum loss under flexible exchange rates is

$$L^{flex} = \left( \frac{\pi^e_t + z_t + k}{1 + \chi} - \pi^e_t - z_t - k \right)^2 + \chi \left( \frac{\pi^e_t + z_t + k}{1 + \chi} \right)^2 + c(\pi_t)$$

$$= \frac{\chi}{1 + \chi} (\pi^e_t + z_t + k)^2 + c(\pi_t)$$
• Loss under fixed exchange rates sets $\pi_t = 0$

\[ L^{fix} = (\pi_t^e + z_t + k)^2 \]

• Compare the loss under maintaining the fixed rate with the loss from abandoning

\[ L^{fix} - L^{flex} = (\pi_t^e + z_t + k)^2 - \left[ \frac{\chi}{1 + \chi} (\pi_t^e + z_t + k)^2 + c(\pi_t) \right] \]

\[ = \frac{1}{1 + \chi} (\pi_t^e + z_t + k)^2 - c(\pi_t) \]

– If the cost of abandoning the fixed rate were zero, the country would generally choose flexible rates due to $k > 0$

– Incentive to abandon the fixed rate depends on
* Preferences towards inflation, $\chi$

  - The more tolerant the country is toward inflation, the lower the $\chi$, and the more willing it is to abandon the fixed rate

* Economic fundamentals, $z_t$

  - Demand is low when $z_t$ is high
  
  - Cost of maintaining the fixed exchange rate is higher in recession

* Expectations of inflation depend on the cost of maintaining the fixed rate

  - When demand is low, cost are higher
  
  - Therefore, $\pi^e_t$ will be high when $z_t$ is high
• Multiple equilibria

   – Solve for shock values which create an exchange rate crisis

   * Raise exchange rate (devalue) if

     \[
     \frac{1}{1 + \chi} (\pi_t^e + z_t + k)^2 > \bar{c}
     \]

     \[z \geq \bar{z} = [\bar{c} (1 + \chi)]^{\frac{1}{2}} - k - \pi^e\]

   * Reduce exchange rate (revalue) if

     \[
     \frac{1}{1 + \chi} (\pi_t^e + z_t + k)^2 > \tilde{c}
     \]

     \[z \leq \tilde{z} = -[\tilde{c} (1 + \chi)]^{\frac{1}{2}} - k - \pi^e\]
* Retain fixed rate if

\[ z \in [\tilde{z}, \bar{z}] \]

* Monetary authorities defend the fixed exchange rate for all but large (in absolute value) shocks
- Expectations must be rational

\[ E\pi = E[\pi \mid z < \tilde{z}] \Pr(z < \tilde{z}) + E[\pi \mid z > \tilde{z}] \Pr(z > \tilde{z}) \]

* Assume a uniform distribution for \( z \) on \([-Z, Z]\)

* Inflation is given by

\[ \pi_t = \frac{\pi_t^e + z_t + k}{1 + \chi} \]

* Expected inflation

\[ E\pi = \frac{\pi_t^e + k}{1 + \chi} + \frac{E[z \mid z < \tilde{z}] \Pr(z < \tilde{z}) + E[z \mid z > \tilde{z}] \Pr(z > \tilde{z})}{1 + \chi} \]

  - very non-linear function and can equal \( \pi^e \) at multiple values

  - multiple equilibria possible only for extreme values of \( z \)
– when \( z \) takes on moderate values, do not get possibility of multiple equilibria