Sovereign Debt and Default
Obstfeld and Rogoff, Chapter 6
Schmitt-Grohe-UrIBE, Chapter 13
1 Sovereign Debt

1.1 Why Do Sovereign Countries Pay International Debts?

- No legal enforcement in a world court

- Sanctions designed to punish defaulting country
  - Seizures of assets located abroad
  - Trade embargoes, import tariffs and quotas
  - Not declaration of war
• Reputation – country with reputation for default loses access to world capital markets
  – Countries will not make loans to a sovereign with a reputation for default
  – Countries repay to maintain their access to international financing
2 Default

- Failure to meet principal or interest payments on the due date
- Exchange old debt for new debt with lower value

2.1 Stylized Facts (Schimtt-Grohe and Uribe)

- Data - Nine emerging markets with at least one default or restructuring of external debt between 1824 and 1999
  - Default frequency
* One credit event every 33 years
* Implying empirical probability of default of 3 percent per year
  - Length of a default episode is about 11 years
* Before default resolved limited access to new external financing
* Financial autarky, but might not be complete if country can find new sources of credit

- Data - 1975 - 2014 for 93 sovereigns who have defaulted once
  - Default probability of 4% per year
  - Average length has fallen to 8 years
• Data - 1970-2000 for emerging countries that defaulted at least once between 1824 and 1999

  – Debt/GDP ratios at the onset of a default or restructuring are on average 14 percentage points above normal

    * Due to fall in output?

    * Rise in debt?

  – Interest rate premium (country spread)

    * Average of 5.5 percentage points

• Most evidence implies that default occurs when output is cyclically low
2.2 Cost of Default

- Output reduction
  - Growth regression has a negative coefficient on default
    * correlation not causation
  - Capital-output ratio peaks and then falls after default
    * correlation not causation

- Disruptions in international trade
  - Paris Club is an informal association of creditor-country finance ministers and central bankers that negotiates bilateral debt rescheduling agreements with debtor-country governments
– Gravity equation by Rose

\[
\ln T_{ijt} = \beta_0 + \beta X_{ijt} + \sum_{m=0}^{M} \phi_m R_{ijt-m} + \epsilon_{ijt}
\]

* Average real value of bilateral trade between countries \(i\) and \(j\)

  · \(R_{ijt-m}\) is a binary variable equal to unity if countries \(i\) and \(j\) renegotiated debt using Paris Club in period \(t\)

  · \(X_{ijt}\) includes distance, combined output, combined population, combined area, sharing a common language, sharing land borders, cosigners of a free trade agreement, having a colonial relationship, etc.

* Data
- All bilateral trade between 217 countries between 1948 and 1997 at annual frequency

- Includes 283 Paris-Club debt-restructuring deals
* Transactions cost variables
  
  · Distance reduces trade
  
  · High-income country pairs trade more
  
  · Countries trade more if they share a common currency, language, border, or membership in regional free trade agreement
  
  · Colonial relationships increase trade
  
  · Landlocked countries and islands trade less

* Financial stress variable
  
  · Inception of IMF program reduces trade by about 10% over three years
* Results on default dummy

  - default has a significant and negative effect on bilateral trade
  - effect persists for about 16 years
  - cumulative effect is about one year’s worth of GDP
  - trade reduction could justify repayment of debt
– Does bilateral dummy capture trade sanctions or economic distress associated with default?

* Add dummy with a value of one if one country of the pair is renegotiating debt with any country

  · if creditor country is sanctioning the debtor, negative coefficient on original dummy

  · if general economic distress associated with default reduces trade, coefficient on new dummy should be negative

  · original dummy becomes positive and new dummy takes the negative sign

  · suggests general distress reduces trade, but perhaps countries collectively apply sanctions against the debtor country
* Additional dummies designed to capture collective sanctions
  
  · Original sanctions dummy is significant only at long lags (15 years)
  
  · General distress dummy is significant at shorter lags
3 Sovereign Risk in Endowment Economies (Obstfeld and Rogoff)

3.1 Model with Sanctions

- Assumptions
  - Small open economy
  - Risk averse representative agent
  - 2 periods
  - Date 1 endowment is zero
– date 2 output is stochastic

\[ Y_2 = \bar{Y} + \epsilon \quad \bar{\epsilon} \leq \epsilon \leq \bar{\epsilon} \quad E(\epsilon) = 0 \]

– date 1 consumption yields no utility

\[ U_1 = E_1 u(C_2) \]

– in period 1 make state-contingent contracts with foreign insurers to reduce second period uncertainty
• State-contingent contract

- country promises to pay $P(\epsilon)$ to foreign insurers on date 2

- risk-neutral foreign insurers are willing to sign a contract for which expected payouts are zero

  \[ \sum_{i=1}^{N} \pi(\epsilon_i) P(\epsilon_i) \]

- pay positive amounts in some states and negative amounts in others

- the foreigner is completely credible – can commit to pay in states requiring payment

- sovereign’s credibility is an issue in states for which $P(\epsilon) > 0$
• Benchmark case: full insurance

  – assume the country can commit to any payment for which

    \[ P(\epsilon) \leq Y_2 \]

  – equilibrium is

    \[ P(\epsilon) = \epsilon \]

  – implies consumption is fixed at mean output, yielding full insurance and complete diversification

    \[ C_2 = Y_2 - \epsilon = \bar{Y} \]

  – country pays \( \epsilon \) whenever \( \epsilon > 0 \) and receives \( -\epsilon \) whenever \( \epsilon < 0 \)
equivalent to a forward sale of uncertain future output where willing to pay what you expect to receive

\[ \sum_{i=1}^{N} \pi(\epsilon_i) (\epsilon_i + \bar{Y}) = \bar{Y} \]

Possibility of default

* what is the country’s incentive to pay when \( \epsilon > 0 \)?
• Optimal incentive-compatible contract with sanctions

  – Three features

    * Cannot call on sovereign to make payments larger than the sanction cost
      \[
      P(\epsilon_i) \leq \eta(\bar{Y} + \epsilon_i)
      \]

    * Competition among risk-neutral insurers results in an equilibrium with expected profits of zero

    * Contract is optimal for sovereign
Optimal contract solves

\[
\max_{C_2(\epsilon), P(\epsilon)} \sum_{i=1}^{N} \pi(\epsilon_i) u[C_2(\epsilon_i)]
\]

subject to

* incentive compatibility constraint

\[
P(\epsilon_i) \leq \eta(\bar{Y} + \epsilon_i)
\]

* zero profit constraint

\[
\sum_{i=1}^{N} \pi(\epsilon_i) P(\epsilon_i) = 0
\]

* \(N\) budget constraints

\[
C_2(\epsilon_i) = \bar{Y} + \epsilon_i - P(\epsilon_i)
\]
– Lagrangian

\[ L = \sum_{i=1}^{N} \pi(\epsilon_i) u [\bar{Y} + \epsilon_i - P(\epsilon_i)] \]

\[-\lambda(\epsilon_i) \sum_{i=1}^{N} [P(\epsilon_i) - \eta(\bar{Y} + \epsilon_i)] + \mu \sum_{i=1}^{N} \pi(\epsilon_i) P(\epsilon_i)\]

* FO conditions

  \[ P(\epsilon_i) \]
  \[ \pi(\epsilon_i) u'(C_2(\epsilon_i)) + \lambda(\epsilon_i) = \mu \pi(\epsilon_i) \]

  \[ \text{when } \lambda(\epsilon_i) > 0, \text{ consumption will not be equal across states} \]

  \[ \text{Kuhn-Tucker condition} \]
  \[ \lambda(\epsilon_i) \left[ P(\epsilon_i) - \eta(\bar{Y} + \epsilon_i) \right] = 0 \]
\cdot \lambda (\epsilon_i) = 0 \text{ when strict inequality holds}

\cdot \text{consumption is equal across states for which strict inequality holds}

\quad u' (C_2(\epsilon_i)) = \mu
– Characteristics of contract
  
  * low values of $\epsilon_i$ such that strict inequality holds, $\lambda(\epsilon_i) = 0$
    
    - consumption is constant across states
      
      \[ u'(C_2(\epsilon_i)) = \mu \]
    
    - implies payments take form of
      
      \[ P(\epsilon_i) = P_0 + \epsilon_i \]
    
    - yielding consumption of
      
      \[ C_2(\epsilon_i) = \bar{Y} + \epsilon_i - P(\epsilon_i) = \bar{Y} - P_0 \]
    
    - marginal utility of consumption for low $\epsilon_i$
      
      \[ u'(\bar{Y} - P_0) = \mu \]
* high values of $\epsilon_i$ such that $\lambda(\epsilon_i) \neq 0$

- divide by $\pi(\epsilon_i)$ to write first order condition as

\[
u'(C_2(\epsilon_i)) + \frac{\lambda(\epsilon_i)}{\pi(\epsilon_i)} = \mu\]

- strict equality in incentive compatibility contract implies $P(\epsilon_i) = \eta(\bar{Y} + \epsilon_i)$

\[
u'(\bar{Y} - P_0) - \nu'(C_2(\epsilon_i)) = \nu'(\bar{Y} - P_0) - \nu'(\bar{Y} + \epsilon_i - P(\epsilon_i))
= \nu'(\bar{Y} - P_0) - \nu'[(1 - \eta)(\bar{Y} + \epsilon_i)]
= \mu - \mu + \frac{\lambda(\epsilon_i)}{\pi(\epsilon_i)}
= \frac{\lambda(\epsilon_i)}{\pi(\epsilon_i)} \geq 0\]
where last inequality occurs because marginal utility of consumption in low state must be at least as great as marginal utility in the high state

- as $\epsilon$ falls, $\lambda(\epsilon_i)$ must fall eventually reaching zero
* define \( e \) as the value of \( \epsilon \) for which \( \lambda (e) = 0 \)
  
  - for \( \epsilon > e \), \( \lambda (\epsilon) > 0 \), and \( P(\epsilon) = \eta (\bar{Y} + \epsilon) \)
  
  - for \( \epsilon < e \), \( \lambda (\epsilon) = 0 \), and \( P(\epsilon) = P_0 + \epsilon \)

  - at \( \epsilon = e \), two expressions for \( P(\epsilon) \) must be equal implying
    \[
    \eta (\bar{Y} + e) - e = P_0
    \]

  * repayment schedule for values of \( \epsilon < e \), rises one-for-one with increases in \( \epsilon \)
    \[
    P(\epsilon) = \eta (\bar{Y} + e) - e + \epsilon
    \]

  * repayment schedule for values of \( \epsilon > e \), rises only by \( \eta \) as \( \epsilon \) rises
* determine value for \( e \) with zero profit condition

- assume uniform density for \( \varepsilon \)

\[
\begin{align*}
\int_{-\bar{\varepsilon}}^{\varepsilon} \left[ \eta \left( \bar{Y} + \varepsilon \right) - \varepsilon + \varepsilon \right] \frac{d\varepsilon}{2\bar{\varepsilon}} + \int_{\varepsilon}^{\bar{\varepsilon}} \eta \left( \bar{Y} + \varepsilon \right) \frac{d\varepsilon}{2\bar{\varepsilon}} &= 0 \\
&= \left[ \eta \bar{Y} + (\eta - 1) \varepsilon \right] \frac{e + \bar{\varepsilon}}{2\bar{\varepsilon}} + \left( \eta \bar{Y} \right) \frac{-e + \bar{\varepsilon}}{2\bar{\varepsilon}} \\
&\quad + \left( \frac{\varepsilon^2}{2} \right) \left( \frac{1}{2\bar{\varepsilon}} \right) \bigg|_{-\bar{\varepsilon}}^{\varepsilon} + \left( \frac{\eta\varepsilon^2}{2} \right) \left( \frac{1}{2\bar{\varepsilon}} \right) \bigg|_{\varepsilon}^{\bar{\varepsilon}} = 0 \\
e^2 + 2e\bar{\varepsilon} + \bar{\varepsilon}^2 - 4 \frac{\eta \bar{Y}}{1 - \eta} &= 0 \\
e &= -\bar{\varepsilon} + 2 \left[ \frac{\eta \bar{Y}}{1 - \eta} \right]^{\frac{1}{2}}
\end{align*}
\]
• if

\[ \frac{\eta \bar{Y}}{1 - \eta} \leq \bar{\epsilon} \]

then

\[ e = -\bar{\epsilon} + 2 \left( \frac{\eta \bar{\epsilon} \bar{Y}}{1 - \eta} \right)^{\frac{1}{2}} \leq -\bar{\epsilon} + 2\bar{\epsilon} = \bar{\epsilon} \]

• criteria requires \( \eta \left( \bar{Y} + \bar{\epsilon} \right) \leq \bar{\epsilon} \), such that country prefers default with sanctions to full insurance payment, in best state, implying that sanctions are not strong enough to provide full insurance
– Consumption

\[ C_2(\epsilon) = \tilde{Y} + \epsilon - P(\epsilon) \]

* in low income states

\[ P(\epsilon) = \eta (\tilde{Y} + e) - e + \epsilon \]

\[ C_2(\epsilon) = \tilde{Y} + \epsilon - [\eta (\tilde{Y} + e) - e + \epsilon] = (\tilde{Y} + e)(1 - \eta) \]

- the more severe the sanctions, the more states over which consumption-smoothing is possible implying that agents are better off with more severe sanctions

- sanctions are not exercised in equilibrium

- serve only to obtain commitment to repay
in high income states there is less consumption-smoothing

\[ P(\epsilon) = \eta (\bar{Y} + \epsilon) \]

\[ C_2(\epsilon) = (\bar{Y} + \epsilon) (1 - \eta) \]

- Intuition

* In low income states, no enforcement problem since agents receive payments, implying that agents can smooth consumption

* In higher income states, country would prefer default to transferring \( \epsilon \) to creditor as required under full insurance, so optimal contract requires that borrower transfer only a fraction \( \eta \) to creditor

* since transfer less to insurers in good states (compared with full insurance), zero profit criterion requires that transfer more to insurers
in bad states yielding

\[(\bar{Y} + e) (1 - \eta) < \bar{Y}\] and \[P_0 = \eta (\bar{Y} + e) - e > 0\]

* can even make positive payments to insurers for low values of \(\epsilon\), when full insurance would allow negative payments

* expected consumption equals \(\bar{Y}\) due to zero profits condition, but contract fails to smooth consumption over states

- Default

* danger of default only in good states because in bad states, insurers pay agents
• Role of savings

  – Utility

    \[ U_1 = u(C_1) + \beta u(C_2) \]

  – Output

    * period 1

      \[ Y_1 = \bar{Y} \]

    * period 2

      \[ Y_2 = \bar{Y} + \epsilon \]
– Penalties to default in period 2

  * insurer can seize all assets accumulated in first period up to value of default

  * if country still owes insurer and does not repay, insurer can seize $\eta$ of output

– Savings in period 1 provides collateral which insurer can seize in the event of default and can replace sanctions if large enough

– Country distorts its intertemporal consumption profile to reduce variability of period-2 consumption
• Observability

  – If cannot observe all contracts, then the aggregate of all promises to repay could exceed the aggregate of all sanctions, implying that aggregate of contracts fails the incentive compatibility condition
3.2 Model of Reputation (Obstfeld and Rogoff)

- Assumptions

  - A country has a reputation for repayment if it has never defaulted
  
  - Failure to repay (default) is punished with permanent exclusion from world capital markets
  
  - Output is stochastic with mean-zero iid shocks

\[ Y_s = \bar{Y} + \epsilon_s \]

  - \[ \beta (1 + r) = 1 \]
• Representative agent problem

  – Maximize utility

  \[ U_t = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \right\} \]

  – subject to the budget constraint with non-state-contingent bonds and insurance payments contingent on the state

  \[ B_{s+1} = (1 + r)B_s + \bar{Y} + \epsilon_s - C_s - P_s(\epsilon_s) \]

  – constraint that insurance payments satisfy the zero profit constraint

  \[ \sum_{i=1}^{N} \pi(\epsilon_i) P_i(\epsilon_i) = 0 \]
• Optimal contract

  – If agents can commit to pay, then full insurance is optimal

\[ P_s (\epsilon_s) = \epsilon_s \quad C_s = \bar{Y} \quad B_s = 0 \]

• Optimal contract is feasible only if punishment for default exceeds gains from default

  – Gains from default are utility with default minus utility with repayment

\[ \text{Gain} (\epsilon_t) = u (\bar{Y} + \epsilon_t) - u (\bar{Y}) \]

  – Cost of default is difference between the present value of utility with full insurance and utility under autarky

\[ \text{Cost} = \sum_{s=t+1}^{\infty} \beta^{s-t} [u (\bar{Y}) - E_t u (\bar{Y} + \epsilon_s)] \]
* write the cost as time invariant

\[
\text{Cost} = \frac{\beta}{1 - \beta} \left[ u(\bar{Y}) - Eu(\bar{Y} + \epsilon) \right]
\]

* cost is positive due to concave utility yielding

\[
u > Eu
\]

* cost of default does not depend on the size of the default

  - trigger strategy defined as any infraction pulls the trigger

  - agents will never choose partial default since gains smaller and costs the same
• Compare gains to default to cost of default
  
  – Full insurance is sustainable only if the gain in every state is less than the cost, requiring the gain in the best state to be less than the cost
  
  – Temptation to default is greatest in high endowment state
  
  – Incentive compatibility constraint

\[
\text{Gain}(\bar{e}) \leq \text{Cost}
\]

\[
u(\bar{Y} + \bar{e}) - u(\bar{Y}) \leq \frac{\beta}{1 - \beta} \left[ u(\bar{Y}) - E u(\bar{Y} + e) \right]
\]

– If $\beta$ is close to unity, then the criteria holds

– Permanent autarky is not worth the utility gain for a single period
• Consider finite horizon with terminal date at $T$

  – Gains to repaying at $T - 1$ are zero, implying default with probability one at $T - 1 \implies$ no lending at $T - 1$

  – Since no contracts at $T - 1$, gains to repaying at $T - 2$ are zero $\implies$ no lending at $T - 2$

  – With a finite horizon, reputation cannot support an equilibrium
• Partial insurance $P(\epsilon) \neq \epsilon$
  
  – What if full insurance not possible

  * $\beta$ is too small

  – One-period contracts to share next period’s output risk with competitive risk-neutral foreign insurer

  – Optimization problem identical to one before

  – Gain to default is utility of extra consumption if fail to make payment

  \[ \text{Gain}(\epsilon_t) = u(\bar{Y} + \epsilon_t) - u(\bar{Y} + \epsilon_t - P(\epsilon_t)) \]
- Cost is expected present value of utility with future contracts less utility in autarky

\[
\text{Cost} = \frac{\beta}{1 - \beta} \left[ Eu \left( \bar{Y} + \epsilon - P(\epsilon) \right) - Eu \left( \bar{Y} + \epsilon \right) \right]
\]

- incentive compatibility constraint requires gain to be less than cost

\[
u \left( \bar{Y} + \epsilon_t \right) - u \left( \bar{Y} + \epsilon_t - P(\epsilon_t) \right) \\
\leq \frac{\beta}{1 - \beta} \left[ Eu \left( \bar{Y} + \epsilon - P(\epsilon) \right) - Eu \left( \bar{Y} + \epsilon \right) \right] \\
= \frac{\beta}{1 - \beta} \sum_{j=1}^{N} \pi(\epsilon_j) \left[ u \left( \bar{Y} + \epsilon_j - P(\epsilon_j) \right) - u \left( \bar{Y} + \epsilon_j \right) \right]
\]
- Optimization problem will be to choose \( P(\epsilon) \) to maximize expected one-period utility subject to incentive compatibility constraint and zero profit constraint.

- Lagrangian

\[
L = \sum_{i=1}^{N} \pi(\epsilon_i) u(\bar{Y} + \epsilon_i - P(\epsilon_i)) + \mu \sum_{i=1}^{N} \pi(\epsilon_i) P(\epsilon_i) \\
- \sum_{i=1}^{N} \lambda(\epsilon_i) \left\{ u(\bar{Y} + \epsilon_i) - u(\bar{Y} + \epsilon_i - P(\epsilon_i)) \right\} \\
+ \sum_{i=1}^{N} \lambda(\epsilon_i) \left\{ \frac{\beta}{1 - \beta} \sum_{j=1}^{N} \pi(\epsilon_j) \left[ u(\bar{Y} + \epsilon_j - P(\epsilon_j)) - u(\bar{Y} + \epsilon_j) \right] \right\}
\]
- FO conditions

* \( P(\epsilon_i) \)

\[
\left\{ \pi(\epsilon_i) + \lambda(\epsilon_i) + \pi(\epsilon_i) \sum_{j=1}^{N} \lambda(\epsilon_j) \frac{\beta}{1 - \beta} \right\} u'(C(\epsilon_i)) = \mu \pi(\epsilon_i)
\]

* Kuhn-Tucker conditions from inequality constraint

\[
0 = \lambda(\epsilon_i) \left\{ u(\bar{Y} + \epsilon_i) - u(\bar{Y} + \epsilon_i - P(\epsilon_i)) \right\}
\]

\[
-\lambda(\epsilon_i) \frac{\beta}{1 - \beta} \sum_{j=1}^{N} \pi(\epsilon_j) \left[ u(\bar{Y} + \epsilon_j - P(\epsilon_j)) - u(\bar{Y} + \epsilon_j) \right]
\]
- Interpretation

* For low values of $\epsilon$, the incentive compatibility constraint does not bind and $\lambda(\epsilon_i) = 0$

- FO condition implies that the marginal utility of consumption is fixed across these values for $\epsilon$

$$\left\{ \pi(\epsilon_i) + \pi(\epsilon_i) \sum_{j=1}^{N} \lambda(\epsilon_j) \frac{\beta}{1-\beta} \right\} u'(C(\epsilon_i)) = \mu \pi(\epsilon_i)$$

$$u'(C(\epsilon_i)) = \frac{\mu}{1 + \sum_{j=1}^{N} \lambda(\epsilon_j) \frac{\beta}{1-\beta}}$$

- Implying that consumption is fixed for low values of $\epsilon$, stabilizing consumption for the worst downside risks
· When $\lambda (\epsilon) = 0$, can write

$$P (\epsilon) = P_0 + \epsilon \quad C (\epsilon) = \bar{Y} - P_0$$

* For higher values of $\epsilon$, the incentive compatibility constraint is an equality, and $\lambda (\epsilon_i) > 0$

$$\frac{dP (\epsilon)}{d\epsilon} = \frac{u' (\bar{Y} + \epsilon - P (\epsilon)) - u' (\bar{Y} + \epsilon)}{u' (\bar{Y} + \epsilon - P (\epsilon))}$$

* Concavity of utility implies numerator is positive so that

$$0 < \frac{dP (\epsilon)}{d\epsilon} < 1$$

* As $\epsilon$ rises, payment rises by less than one to give agent the incentive to repay
* Consumption for large $\epsilon$

\[ C(\epsilon) = \bar{Y} + \epsilon - P(\epsilon) \]

- Need solution for $P(\epsilon)$

- Solve for multiplier $\mu$

\[ C(\epsilon) = \bar{Y} - P_0 \]

\[ u'(\bar{Y} - P_0) = \frac{\mu}{1 + \sum_{j=1}^{N} \lambda(\epsilon_j) \frac{\beta}{1-\beta}} \]

\[ \mu = \left[ 1 + \sum_{j=1}^{N} \lambda(\epsilon_j) \frac{\beta}{1-\beta} \right] u'(\bar{Y} - P_0) \]
· Eliminate $\mu$ from FO condition

$$\left\{ \pi (\epsilon_i) + \lambda (\epsilon_i) + \pi (\epsilon_i) \sum_{j=1}^{N} \lambda (\epsilon_j) \frac{\beta}{1 - \beta} \right\} u' (C (\epsilon_i)) = \mu \pi (\epsilon_i)$$

$$\left\{ \pi (\epsilon_i) + \lambda (\epsilon_i) + \pi (\epsilon_i) \sum_{j=1}^{N} \lambda (\epsilon_j) \frac{\beta}{1 - \beta} \right\} u' (C (\epsilon_i))$$

$$= \left[ 1 + \sum_{j=1}^{N} \lambda (\epsilon_j) \frac{\beta}{1 - \beta} \right] u' (\bar{Y} - P_0) \pi (\epsilon_i)$$

$$\frac{\lambda (\epsilon_i) u' (C (\epsilon_i))}{\pi (\epsilon_i)} = \left[ 1 + \sum_{j=1}^{N} \lambda (\epsilon_j) \frac{\beta}{1 - \beta} \right] [u' (\bar{Y} - P_0) - u' (C (\epsilon_i))]$$
* Consumption

  - $C'(\epsilon)$ falls as $\epsilon$ falls for $\lambda(\epsilon) > 0$

  - rhs falls as $\epsilon$ falls until $\epsilon = e$, where $u'(\bar{Y} - P_0) - u'(C'(e)) = 0$ and $\lambda(\epsilon) = 0$

  - consumption is continuous at $e$

  - $P(\epsilon)$ is rising in $\epsilon$ for $\epsilon > e$, but by less than one for one

  - agents get to keep some of the high $\epsilon$, thereby reducing the gains to default
• Allow savings

  – Assume creditor can seize foreign assets

    * Incentive to save because assets provide additional collateral that insurer can seize in the event of default allowing more insurance

    * Continue to save until assets with interest are just equal to the highest $\epsilon$

    * In that case, with default country loses assets equal to highest $\epsilon$ and gains $\epsilon$, yielding non-positive net gains

    * Equilibrium supports full insurance
– Assume creditor can seize foreign assets and that country has a reputational contract

* State $\bar{e}$ occurs, country defaults and uses $\bar{e}$ to buy a foreign bond

* It obtains another identical reputational contract by pledging the foreign bond as collateral

* Consumption is higher by the interest on the bond

* Since the country would default in the best state, the contract cannot exist because the zero profit condition is violated
• General Equilibrium Model of Reputation: No party can pre-commit

  – Assumptions

    * Large number of countries (j)

    * Utility

      \[
      U_t^j = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u(C_s^j) \right\}
      \]

    * Endowment output has idiosyncratic shock (\( \epsilon_t^j \)) and global shock (\( \omega_t \))

      \[
      Y_t^j = \bar{Y} + \epsilon_t^j + \omega_t
      \]
• Aggregated idiosyncratic shock is zero

\[ \sum_{j} \epsilon_{t}^{j} = 0 \]

• Aggregate shock is iid and bounded by \( \tilde{\omega} \) and \( \bar{\omega} \)

• Idiosyncratic shock is iid and bounded by \( \tilde{\epsilon} \) and \( \bar{\epsilon} \)
– Efficient (full risk-sharing) allocation across countries is

\[ C_t^j = \bar{Y} + \omega_t \]

* Countries sell off positive idiosyncratic shocks and insure themselves against negative shocks at actuarially fair prices

– Can the efficient equilibrium be supported with only reputation (no direct sanctions)?
* Specific trigger strategy

  - Any country \( j \) that defaults is completely and permanently cut off from world markets

  - Country \( j \) loses its reputation for repayment, so everyone believes it will default given the opportunity

  - All other countries lose their reputation for repaying country \( j \)

  - Under these assumptions, after a default, no country lends to country \( j \) and country \( j \) lends to no country (autarky for country \( j \))
* Compare gains and costs to default
  
  - Gain to default is short term
    \[ \text{Gain} \left( \epsilon_t^j, \omega_t \right) = u \left( \bar{Y} + \epsilon_t^j + \omega_t \right) - u \left( \bar{Y} + \omega_t \right) \]
  
  - Gains are largest when \( \epsilon_t^j \) is high \( \omega_t \) is low
  
  - Cost is present value of expected utility with full insurance less expected utility in autarky
    \[ \text{Cost} = \frac{\beta}{1 - \beta} \left[ E_t u \left( \bar{Y} + \omega \right) - E_t u \left( \bar{Y} + \epsilon_t^j + \omega \right) \right] \]
  
  - Strategy works if
    \[ \text{Gain} \left( \bar{\epsilon}, \bar{\omega} \right) \leq \text{Cost} \]
  
  - A value of \( \beta \) close to unity implies more likely to work
· If full insurance is not supported, partial insurance might be
Permanent exclusion from world capital markets after a default is not consistent with the historical record

* Could be optimal to reopen negotiations with a country in default and agree on another insurance contract

In equilibrium, there will be no default if there is a contract because we have assured that the costs to default are always at least as large as the benefits
4 Sovereign Risk with Investment (Obstfeld and Rogoff)

4.1 Model with Sanctions

- Assumptions
  - two periods
  - no uncertainty
  - small country
- utility

\[ U_1 = u(C_1) + \beta u(C_2) \]

- production function for period 2 output

\[ Y_2 = F(K_2) \]

- period 1 budget constraint where \( D_2 \) is borrowing from the rest of the world and \( Y_1 \) is endowment

\[ K_2 = Y_1 + D_2 - C_1 \]

- period 2 budget constraint

\[ C_2 = F(K_2) + K_2 - \mathcal{R} \]

where \( \mathcal{R} \) is repayment of loans
– repayments are the minimum of debt with interest or sanctions
\[ R = \min [(1 + r) D_2, \eta(F(K_2) + K_2)] \]

– If country could commit to repay, then equilibrium looks like previous models
\[ F'(K_2) = r \]
\[ u'(C_1) = \beta(1 + r)u'(C_2) \]

– Without commitment
\[ R \leq \eta(F(K_2) + K_2) \]
Discretion over investment: borrower does not have to use $D$ for investment

- Creditor asks if we lend $D_2$ today, will the borrower buy enough capital to assure

$$\eta (F (K_2) + K_2) \geq (1 + r) D_2$$

* diminishing marginal productivity is important

* define $\bar{D}$ as the maximum debt which does not trigger default, equivalently the debt ceiling
– Country’s optimization problem after receiving $D_2$

* Choose $K_2$ to maximize

$$
u (Y_1 + D_2 - K_2) + \beta u \left\{ F(K_2) + K_2 - \min\{(1 + r) D_2, \eta (F(K_2) + K_2)\}\right.$$  

– Consumption possibilities frontier

* GDP=$F(K_2) + K_2$ and does not include any net factor payments (no debt repayments)

  • intercept for horizontal axis ($C_2 = K_2 = 0$) has $C_1 = Y_1 + D_2$

  • as $K_2$ increases, $C_1$ falls and $C_2$ increases at a decreasing rate
* GNP Default

\[ C_2^D = GNP^D = (1 - \eta) (F(K_2) + K_2) \]

* GNP No default shifts GDP vertically down by \((1 + r) D_2\)

\[ C_2^N = GNP^N = F(K_2) + K_2 - (1 + r) D_2 \]

* Consumption possibilities frontier is outermost envelope of two GNP's such a kink occurs where

\[ (1 + r) D_2 = \eta (F(K_2) + K_2) \]

* Consumption possibilities frontier is not concave such that two levels of investment, one yielding default and one yielding repayment, could provide same utility
− Simplify problem to get analytical solution

* utility

\[ U_1 = \log C_1 + \beta \log C_2 \]

* production function is linear with marginal product of capital (\( \alpha \))
greater than world interest rate (\( r \))

\[ Y_2 = \alpha K_2 \]

\[ \alpha > r \]

* assume that higher debt makes default more attractive

\[ 1 + r > \eta (1 + \alpha) \]
* compute maximum utility subject to no default
  
  - period 1 budget constraint
    \[ K_2 = Y_1 + D_2 - C_1 \]

  - period 2 budget constraint
    \[ C_2 = (1 + \alpha) K_2 - (1 + r) D_2 \]

  - intertemporal budget constraint (IBC), eliminating \( K_2 \)
    \[ C_2 = (1 + \alpha)(Y_1 + D_2 - C_1) - (1 + r) D_2 \]
    \[ C_1 + \frac{C_2}{1 + \alpha} = Y_1 + \frac{(\alpha - r)}{1 + \alpha} D_2 \]
* FO conditions: max utility subject to IBC

  - period 1 consumption
    \[ \frac{1}{C_1} = \lambda \]

  - period 2 consumption
    \[ \beta \frac{1}{C_2} = \lambda \left( \frac{1}{1 + \alpha} \right) \]

  - Euler equation
    \[ C_2 = \beta (1 + \alpha) C_1 \]
* Substitute Euler equation into IBC

\[ C_1 = \frac{1}{(1 + \beta)} \left[ Y_1 + \frac{(\alpha - r)}{1 + \alpha}D_2 \right] \]

\[ C_2 = \frac{\beta (1 + \alpha)}{(1 + \beta)} \left[ Y_1 + \frac{(\alpha - r)}{1 + \alpha}D_2 \right] \]

* Substitute consumption into utility function and write expression for utility maximized subject to no default

\[ U^N = \log \left\{ \frac{1}{(1 + \beta)} \left[ Y_1 + \frac{(\alpha - r)}{1 + \alpha}D_2 \right] \right\} \]

\[ + \beta \log \left\{ \frac{\beta (1 + \alpha)}{(1 + \beta)} \left[ Y_1 + \frac{(\alpha - r)}{1 + \alpha}D_2 \right] \right\} \]
Next, compute maximum utility with default

- period 1 budget constraint

\[ K_2 = Y_1 + D_2 - C_1 \]

- period 2 budget constraint

\[ C_2 = (1 - \eta)(1 + \alpha)K_2 \]

- intertemporal budget constraint (IBC)

\[ C_2 = (1 - \eta)(1 + \alpha)(Y_1 + D_2 - C_1) \]

\[ C_1 + \frac{C_2}{(1 + \alpha)(1 - \eta)} = Y_1 + D_2 \]
* FO conditions: max utility subject to IBC

  - period 1 consumption
    \[
    \frac{1}{C_1} = \lambda
    \]

  - period 2 consumption
    \[
    \beta \frac{1}{C_2} = \lambda \left( \frac{1}{(1 + \alpha)(1 - \eta)} \right)
    \]

  - Euler equation
    \[
    C_2 = \beta (1 + \alpha) (1 - \eta) C_1
    \]
* Substitute Euler equation into IBC

\[ C_1 = \frac{1}{(1 + \beta)} [Y_1 + D_2] \]

\[ C_2 = \frac{\beta (1 + \alpha)(1 - \eta)}{(1 + \beta)} [Y_1 + D_2] \]

* Substitute consumption into utility function and write expression for utility maximized subject to no default

\[ U^D = \log \left\{ \frac{1}{(1 + \beta)} [Y_1 + D_2] \right\} + \beta \log \left\{ \frac{\beta (1 + \alpha)(1 - \eta)}{(1 + \beta)} [Y_1 + D_2] \right\} \]
* Utility difference

\[
U^D - U^N = \log \left\{ \frac{1}{(1 + \beta)} [Y_1 + D_2] \right\} \\
+ \beta \log \left\{ \frac{\beta (1 + \alpha) (1 - \eta)}{(1 + \beta)} [Y_1 + D_2] \right\} \\
- \log \left\{ \frac{1}{(1 + \beta)} \left[ Y_1 + \frac{\alpha - r}{1 + \alpha} D_2 \right] \right\} \\
- \beta \log \left\{ \frac{\beta (1 + \alpha)}{(1 + \beta)} \left[ Y_1 + \frac{\alpha - r}{1 + \alpha} D_2 \right] \right\} \\
= (1 + \beta) \log \frac{[Y_1 + D_2]}{[Y_1 + \frac{(\alpha - r) D_2}{1 + \alpha}]} + \beta \log (1 - \eta) \\
= (1 + \beta) \log \frac{[1 + \frac{D_2}{Y_1}]}{[1 + \frac{(\alpha - r) D_2}{1 + \alpha} \frac{Y_1}{Y_1}]} + \beta \log (1 - \eta)
\]
\[
\text{Note } \beta \log (1 - \eta) < 0, \text{ implying that for very low levels of debt, utility with no default is higher}
\]

* Solve for level of debt at which just indifferent to default

\[
\exp(U_D - U^N) = 1
\]

\[
\left[ \frac{1 + \frac{D_2}{Y_1}}{1 + \frac{(\alpha - r)D_2}{1 + \alpha Y_1}} \right]^{1+\beta} (1 - \eta)^\beta = 1
\]

\[
\tilde{D} = \left[ \frac{\left( \frac{1}{1-\eta} \right)^{\frac{\beta}{1+\beta}} - 1}{1 - \frac{\alpha - r}{1 + \alpha} \left( \frac{1}{1-\eta} \right)^{\frac{\beta}{1+\beta}}} \right] Y_1 > 0
\]

* debt limit is increasing in sanctions (\(\eta\)) and in capital productivity (\(\alpha\))
Intuition on the debt limit from optimal capital under default and no default

\[ K_2 = Y_1 + D_2 - C_1 \]

* No default

  - consumption

\[ C_1^N = \frac{1}{(1 + \beta)} \left[ Y_1 + \frac{(\alpha - r)}{1 + \alpha} D_2 \right] \]

  - capital

\[ K_2 = \frac{\beta}{(1 + \beta)} (Y_1 + D_2) + \frac{(1 + r)}{(1 + \beta)(1 + \alpha)} D_2 \]
Default

- consumption

\[ C_1^D = \frac{1}{(1 + \beta)} [Y_1 + D_2] \]

- capital

\[ K_2 = \frac{\beta}{(1 + \beta)} (Y_1 + D_2) \]

- If plan to default, reduce capital accumulation because no need to accumulate enough to satisfy consumption smoothing and repay

- As debt increases beyond \( \bar{D} \), investment would crash discontinuously as country prefers sanctions on smaller output than repaying debt
- Optimal investment in original problem with debt ceiling $D_2 \leq \bar{D}$

$$\max_{K_2, D_2} \left\{ u \left( Y_1 + D_2 - K_2 \right) + \beta u \left( F \left( K_2 \right) + K_2 - (1 + r) D_2 \right) \right\}$$

subject to

$$D_2 \leq \bar{D}$$

* Lagrangian

$$L = u \left( Y_1 + D_2 - K_2 \right) + \beta u \left( F \left( K_2 \right) + K_2 - (1 + r) D_2 \right) - \lambda \left( D_2 - \bar{D} \right)$$

* FO conditions

  - $K_2$

    $$u' \left( C_1 \right) = \beta u' \left( C_2 \right) \left[ F' \left( K_2 \right) + 1 \right]$$
\[ D_2 \]

\[ u'(C_1) = \beta u'(C_2) [1+r] + \lambda \]

\[ \text{Kuhn-Tucker Condition} \]

\[ \lambda (D_2 - \bar{D}) = 0 \]

* Inequality constraint not binding so that \( \lambda = 0 \)

* equilibrium is standard neo-classical equilibrium

* Inequality constraint is binding such that \( \lambda > 0 \)

\[ \beta u'(C_2) \left[ F'(K_2) + 1 \right] = \beta u'(C_2) [1 + r] + \lambda \]

\[ F'(K_2) = r + \frac{\lambda}{\beta u'(C_2)} \]
- MPK exceeds interest rate, but cannot borrow enough to get equality

- Consider $\beta [1 + r] = 1$

  $$u'(C_1) = u'(C_2) + \lambda$$

  $$u'(C_1) > u'(C_2)$$

  $$C_1 < C_2$$

- Consumption is tilted upwards

- Reflects a "shadow" rate of interest where MPK exceeds world interest rate

- Even with upward tilt, since $K_2$ is below full optimum, $C_2$ could be below full optimum
● When country can pre-commit to investment level

   – Debt ceiling equals the present-value of value of sanctions

\[ \bar{D} = \frac{\eta (F(K_2) + K_2)}{(1 + r)} \]

   * If the country commits to invest more, will have more for sanctions and higher debt ceiling
- Lagrangian with pre-commitment

\[
L = u(Y_1 + D_2 - K_2) + \beta u(F(K_2) + K_2 - (1 + r)D_2) \\
- \lambda [(1 + r)D_2 - \eta (F(K_2) + K_2)]
\]

* FO conditions

  - \(K_2\)

\[
u'(C_1) = \beta [u'(C_2) + \lambda \eta] [F'(K_2) + 1]
\]

  - \(D_2\)

\[
u'(C_1) = [\beta u'(C_2) + \lambda] [1 + r]
\]

  - Kuhn-Tucker Condition

\[
\lambda [(1 + r)D_2 - \eta (F(K_2) + K_2)] = 0
\]
Together equations imply MPK exceeds interest rate when the debt ceiling is binding and \( \lambda > 0 \)

\[
\beta \left[ u'(C_2) + \lambda \eta \right] \left[ F'(K_2) + 1 \right] = \beta u'(C_2) [1 + r + \lambda]
\]

\[
F'(K_2) + 1 = (1 + r) \frac{u'(C_2) + \lambda}{u'(C_2) + \lambda \eta}
\]

When the debt ceiling is binding, consumption is tilted upwards because the marginal benefits to investing include the benefits of relaxing the debt ceiling.
Dynamic inconsistency

* Once country has promised an investment level, can benefit from reneging on that promise and actually investing less, as in previous problem

* Need some commitment device to assure compliance
4.2 Reputation and Investment in a Deterministic Model (Obstfeld and Rogoff)

\[ Y = AF(K) \]

- Optimum

\[ AF'(\bar{K}) = r \]

- Purpose of international financing is to allow capital to reach optimum
  - Once capital is at the optimum, with no uncertainty, there is no need for world capital market
– Therefore, no cost to being placed in financial autarky forever

– Countries will not repay in period in which capital reaches its optimum

– Therefore agents will not lend the period before, and problem unravels backwards
5 Debt Overhang (Obstfeld and Rogoff)

- Outstanding debt can reduce investment, reducing output and consumption below optimal levels

- Model in which default happens in equilibrium

- Never ask why agents were willing to lend debt large enough to generate risks of default
5.1 Model

- Assumptions
  - 2 periods
  - inherited debt of $D$ due in period 2
  - Income in period 1 is $Y_1$ and in period 2 is $AF(K_2)$
    * $A$ is random and bounded with mean unity
    * $\pi(A)$ is probability function
  - capital in use depreciates by 100%
    * $AF(K_2)$ is total resources for economy in period 2
* $K_2 = I_1$

- agents are risk neutral

\[ U_1 = C_1 + EC_2 \]

- interest rate is zero ($r = 0$) and discount factor is unity ($\beta = 1$)

- Sanctions in the event of non-repayment of debt

\[ \eta AF(K_2) \]
- Budget constraints
  
  * period 1

  \[ C_1 = Y_1 - K_2 \]

  * period 2

  \[ C_2 = AF(K_2) - \min(\eta AF(K_2), D) \]
• Optimization problem

  – Substitute budget constraints into utility function

\[
U_1 = Y_1 - K_2 + E \left[ AF(K_2) - \min(\eta AF(K_2), D) \right]
\]

  – Take expectations and define \( V(D, K_2) \) as expected debt repayment, equivalently debt’s market value

\[
U_1 = Y_1 - K_2 + F(K_2) - V(D, K_2)
\]

* Borrower will default when

\[
\eta AF(K_2) < D
\]

* Critical value for \( A \)

\[
A^* = \frac{D}{\eta F(K_2)}
\]
· When $A > A^*$, borrower repays

· When $A < A^*$, borrower defaults

* Compute the value of debt

$$V(D, K_2) = \eta F(K_2) \int_{\tilde{A}}^{D} \frac{D}{\eta F(K_2)} A \pi(A) dA + D \int_{D}^{\tilde{A}} \frac{\tilde{A}}{\eta F(K_2)} \pi(A) dA$$

- Optimal investment maximizes utility with respect to $K_2$

$$-1 + F'(K_2) - \eta F'(K_2) \int_{\tilde{A}}^{D} \frac{D}{\eta F(K_2)} A \pi(A) dA = 0$$

$$F'(K_2) \left[ 1 - \eta \int_{\tilde{A}}^{D} \frac{D}{\eta F(K_2)} A \pi(A) dA \right] = 1$$
* marginal product of investing, net of the penalty payment to creditors, equals the consumption cost of investing

* as $D$ increases, default range increases raising the penalty cost reducing the gains to investing

* "debt overhang" problem where need to repay debt reduces investment

* when have debt overhang, $\frac{\partial K_2}{\partial D} < 0$
Debt Laffer Curve

\[ V(D, K(D)) = \eta F(K_2) \int_{\tilde{A}}^{\frac{D}{\eta F(K_2)}} A\pi(A) dA + D \int_{\frac{D}{\eta F(K_2)}}^{\tilde{A}} \pi(A) dA \]

- Differentiate with respect to \( D \)

\[
\frac{dV(D, K(D))}{dD} = \int_{\tilde{A}}^{\frac{D}{\eta F(K_2)}} \pi(A) dA + \eta F'(K_2) K'(D) \int_{\tilde{A}}^{\frac{D}{\eta F(K_2)}} A\pi(A) dA
\]

* First term is probability of full repayment

* Second is negative since \( K'(D) < 0 \), and reflects reduction in investment as debt increases

* An increase in \( D \) depresses investment raising the probability of default
* Value of debt, \( V \), rises less than in proportion to \( D \) due to an increasing probability of non-repayment and sanctions

* For large values of debt, second term could dominate the first

– Yields a Debt Laffer Curve, whereby the value of debt is initially increasing in debt, eventually peaks, and then begins decreasing

* If on the wrong side of the Debt Laffer Curve, creditors could gain value by forgiving some debt

* Free-rider problem: let others forgive
• Debt buy-backs

  – Proposal: let countries buy back their own debt at bargain-basement prices

  – Market price of debt

    \[ p = \frac{V(D, K_2)}{D} \]

  – Country uses some \( Y_1 \) to buy back \( Q \) of its debt on date 1 at market price \( p \), where \( p \) is the post-buy-back price and incorporates rational expectations of the effect of debt reduction on investment and the value of debt
Utility after buy-back

\[ U_1 = C_1 + C_2 \]
\[ = Y_1 - pQ - K_2 + F(K_2) - V(D - Q, K_2) \]

* Substitute

\[ pQ = \frac{V(D - Q, K_2)Q}{D - Q} \]

\[ U_1 = Y_1 - K(D - Q) + F[K(D - Q)] \]
\[ - V[D - Q, K(D - Q)] \left(1 + \frac{Q}{D - Q}\right) \]
\[ = Y_1 - K(D - Q) + F[K(D - Q)] \]
\[ - V[D - Q, K(D - Q)] \left(\frac{D}{D - Q}\right) \]

- Differentiate utility with respect to \( Q \) for \( Q = 0 \)
simplify after taking derivative since $Q = 0$

* note $\frac{dV[D-Q, K(D-Q)]}{dQ} = -\frac{dV[D, K(D)]}{dD}$

$$\frac{dU_1}{dQ} = - \left[1 + F'(K(D))\right] K'(D)\frac{V(D, K(D))}{D} + \frac{dV[D, K(D)]}{dD}$$

* first term is positive and an unambiguous gain since debt reduction spurs investment, moving the country toward the first best

* second term equals the negative of the average price plus the marginal price

- since $V(D)$ is concave, the marginal price is less than the average price, representing a net loss
- the country pays the average price and the reduction in debt liability
is only the marginal price as value of debt increases due to the debt reduction

– Debt buy-back increases utility only if the investment stimulus is strong enough

* If we assume that investment is at the optimum and use FO conditions and earlier expressions to substitute, get

\[
\frac{dU_1}{dQ} = -\frac{\eta F(K)}{D} \int_D^{\bar{A}} \frac{\pi(A)}{\eta F(K_2)} dA < 0,
\]

implying that at the optimal level of investment, debt buy-backs hurt a country

* occurs because investment is at the optimum, so that it has only second-order effects
– In practice, debt buy-backs in the 1980’s and early 1990’s had very small effects on the market value of government debt compared with the cost of the buy-back

* To significantly reduce debt, buy-backs need to be accompanied by concessions from creditors such as interest rate reductions
6 Default with Standard Debt Contracts (Schmitt-Grohe and Uribe, 13)

- Replace state contingent contracts with debt
- Reverse the result that countries will be tempted to default in good times
- Default can occur in equilibrium
6.1 Model

- Assumptions
  - small open economy
  - endowment each period is stochastic and bounded
  - utility

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \]

- beginning of period household is either in good or bad financial standing
- bad standing, household consumes endowment

\[ c = y \]

* bad financial standing is an absorbing state

* value function associated with bad financial standing

\[ v^b(y) = u(y) + \beta E v^b(y') \]
good standing, household can choose to repay or default on its debt

* if choose to repay, budget constraint becomes

\[ c + d = y + q(d')d' \]

\[ \text{where } q(d') \text{ is the market price of debt} \]

* value function associated with continuing to pay

\[ v^c(d, y) = \max_{d'} \left\{ u\left( y + q(d')d' - d \right) + \beta E v^g(d', y') \right\} \]

\[ \text{subject to debt limit to prevent Ponzi schemes} \]

\[ d' \leq \bar{d} \]

where

\[ v^g(d, y) = \max \left\{ v^b(y), v^c(d, y) \right\} \]
6.2 Decision to default

- Default set contains all endowment levels at which a household chooses to default given a particular level of debt

\[ D(d) = \{ y \in Y \text{ such that } v^b(y) > v^c(d, y) \} \]

- Default set is empty when \( d < 0 \), because never in household’s interest to default when debt is negative
• For debt levels for which the default set is not empty, the economy, which chooses not to default, will run a trade surplus

– Proof: Suppose to the contrary that $q \left( \hat{d} \right) \hat{d} - d \geq 0$ for some $\hat{d} < \bar{d}$. Then

$$v^c(d, y) \equiv \max_{d' < \hat{d}} \left\{ u \left( y + q \left( d' \right) d' - d \right) + \beta Ev^g \left( d', y' \right) \right\}$$

$$\geq u \left( y + q \left( \hat{d} \right) \hat{d} - d \right) + \beta Ev^g \left( \hat{d}, y' \right)$$

because $\hat{d}$ is a feasible point over which utility is maximized

$$\geq u \left( y \right) + \beta Ev^b \left( y' \right)$$

because utility of output plus something positive exceeds utility of output and because utility of the good financial status exceeds utility of the bad

$$\equiv v^b(y)$$
because if the agent is not borrowing and will have the value function associated with the bad financial state next period, then he must have the bad financial state today

* If the default set next period is empty, then the country can run a trade deficit without risking default next period

* Equivalently, the country will run a trade deficit only if there is no income realization next period for which it could choose to default

* If the default set is not empty and the country continues to repay, then it will run a surplus allowing it to reduce its debt level
- Economy tends to default in bad times
  - Show that if a household with a certain level of debt and income chooses to default then it will also choose to default at the same level of debt and a lower income
  - The country has to run a trade surplus if it is to continue to pay
  - Let there be a level of debt and income for which value function with continued repayments (consumption today less than the endowment) is less than the value of autarky such that the country defaults
  - An even lower level of income would make utility with repayment even lower and with diminishing marginal utility the fall in utility with repayment exceeds the fall in utility with autarky and the country would default at lower levels of income
• The default set is a larger interval the larger the value of debt

  – Equivalently, the probability of default is larger the higher is debt

  – For a particular level of debt, if the default set is not empty, agents will default for all values of output below \( y^* (d) \)

  – The value of \( y^* (d) \) is given implicitly by equating the value function for bad financial status with the value function for continuing repayments

  \[ v^b [y^* (d)] = v^c [d, y^* (d)] \]

* Differentiating

  \[ \frac{dy^* (d)}{dd} = \frac{v^c_d [d, y^* (d)]}{v^b_y [y^* (d)] - v^c_y [d, y^* (d)]} \]

* The denominator is negative due to diminishing marginal utility
The numerator is negative since higher debt reduces consumption

Therefore, $y^*(d)$ is increasing in debt

- Get default in equilibrium
  - Debt contracts
  - Uncertainty
6.3 Risk Premium

• Assumptions
  
  – Foreign lenders are risk neutral and require that the expected return on domestic debt equal $r^*$

  If the country does not default, lenders will receive $1/q(d')$ per unit lent

• Arbitrage requires

$$1 + r^* = \frac{\text{Prob} \{ y' \geq y^* (d') \}}{q(d')}$$

  – The world interest rate must equal the probability that the country does not default times the payout $1/q(d')$
- Letting $F(y)$ denote the cumulative density function of the endowment shock

$$q(d') = \frac{1 - F(y^*(d'))}{1 + r^*}$$

- Taking the derivative

$$\frac{dq(d')}{dd'} = \frac{-F'(y^*(d')) y'^*(d')}{1 + r^*} \leq 0$$

since both derivatives are positive

- Since $q$ is decreasing in $d'$, its inverse is increasing, and the country spread is increasing in debt

$$\frac{1}{q(d')} - (1 + r^*)$$
7 Quantitative Analysis of Model with Debt Contracts

7.1 Additional Assumptions Necessary to Fit Data

7.1.1 Serially correlated endowment shocks

\[ \ln y_t = \rho \ln y_{t-1} + \sigma \epsilon_t \]

- Period $t$ price of debt due in $t+1$ also depends on current endowment, $y_t$
  - Lower output today raises the probability of lower output in the future raising the probability of default
Therefore $q(y_t, d_{t+1})$

7.1.2 Reentry into credit markets

- After default, a country regains entry into credit markets with probability $\theta$

- Implies that the average exclusion period is $\frac{1}{\theta}$
  - Probability that excluded for exactly one period is probability that gains reentry after one period and therefore $\theta$
  - Probability that excluded for exactly two periods is probability that does not gain reentry after one period $(1 - \theta)$ multiplied by the probability that gains entry in next period $\theta$
– Probability excluded for exactly \( j \) periods is \((1 - \theta)^{j-1} \theta\)

– Expected number of periods excluded is

\[
1\theta + 2(1 - \theta) \theta + 3(1 - \theta)^2 \theta + \ldots
\]

\[
= \theta \sum_{j=1}^{\infty} j (1 - \theta)^{j-1}
\]

\[
= \frac{1}{\theta}
\]

### 7.1.3 Output loss

- Default causes countries to lose some of their endowment for each period they are in bad standing
– Output loss is higher in good states than in bad, discouraging default in good states

– Ad hoc assumption and not microfounded

– Output loss does occur with default but direction is causation is not established

• Output loss raises the cost of default allowing the country to accumulate more debt in equilibrium

7.1.4 Calibrate to Argentina

• Calibration choices
– Low discount factor \( (\beta = 0.85) \) implies the country wants to accumulate debt

– Reduce targeted debt/output ratio since countries do not default 100%
  * Assuming haircut of 50%, set "unsecured" debt at half of its actual value
  * Assumption is that country must repay half of its debt

• Results

  – Generally good

  – Positive correlation between risk premium and trade balance implies that countries do raise their debt repayments when the probability of default increases
– Explains only half of the average country risk premium (due to fact that model presupposes that default frequency equals the average country risk premium)

– Consumption is more volatile than output
  * periods of good financial standing – negative output shock raises probability of default raising interest rate reducing consumption even more to pay back some debt
  * do not use financial markets to smooth consumption when possibility of default
  * periods of bad financial standing consumption and output equally volatile

– Countercyclicality of trade balance
* negative output shock reduces consumption even more than output due to increase in interest rate based on higher probability of default reducing consumption more

- Costs due to lost output are essential for results
  - Without output loss, can support virtually no debt in equilibrium with only exclusion from credit markets

- Costs due to exclusion from credit markets are not important as omitting them has virtually no quantitative effects