A Two-Period Model of the Current Account
Obstfeld and Rogoff, Chapter 1
1 Small Open Endowment Economy

1.1 Consumption

- Optimization problem
  
  - maximize
    
    $$U^i_1 = u(c^i_1) + \beta u(c^i_2) \quad \beta < 1$$

  - subject to the budget constraint
    
    $$c^i_1 + \frac{c^i_2}{1 + r} = y^i_1 + \frac{y^i_2}{1 + r}.$$
- Euler equation

\[ u' \left( c^i_1 \right) = \beta (1 + r) u' \left( c^i_2 \right) \]

- When \( \beta (1 + r) = 1 \),

\[ c^i_1 = c^i_2 = c^i \]

- Use the budget constraint to solve for consumption

\[ \bar{c}^i = \frac{y^i_1 (1 + r) + y^i_2}{2 + r} \]
1.2 Current Account

- Increase in claims on ROW (rest of world)
  - Equivalently, increase in net foreign assets

\[ CA_t = B_{t+1} - B_t = Y_t + r_t B_t - C_t \]

\[ GNP_t = Y_t + r_t B_t \]

\[ GDP_t = Y_t \]

- Current account in period 1: assume \( B_1 = 0 \)

\[ CA_1 = B_2 - B_1 = Y_1 + r_1 B_1 - C_1 = Y_1 - C_1 = B_2 \]
• Current account in period 2: note $B_3 = 0$ since there are only two periods

\[
CA_2 = B_3 - B_2 = Y_2 + r_2 B_2 - C_2 = Y_2 + r_2 (Y_1 - C_1) - C_2 = -B_2
\]

• Sum current accounts in periods 1 and 2 to yield intertemporal budget constraint

\[
CA_1 + CA_2 = (Y_1 - C_1) (1 + r_2) + Y_2 - C_2 = 0
\]
1.3 Current Account as Intertemporal Trade

- Graph optimization problem with future consumption on vertical axis and current consumption on horizontal
  - Slope of budget constraint is $-(1 + r)$
  - Autarky: country must consume endowment
  - Trade can increase welfare since $CA_1 = Y_1 - C_1$, particularly for a country with unequal current and future income
  - A current account deficit today must be offset by a current account surplus in the future
• Comparative advantage

  – A country has a comparative advantage in the good it produces relatively cheaply in autarky

  – In autarky, consumption equals production

  – Solve for the autarkic interest rate, which yields goods market equilibrium \((Y_i = C_i)\) using the Euler equation

\[
\frac{\beta u'(Y_2)}{u'(Y_1)} = \frac{1}{1 + r^A} = \text{current consumption price of future consumption}
\]
If 
\[
\frac{1}{1 + r} < \frac{1}{1 + r^A},
\]
domestic price of future consumption is relatively high, implying that the domestic country has a comparative advantage in current consumption.

* Comparative advantage requires that the domestic country export the good for which its autarkic price is relatively low, that is export current consumption.

* Domestic country will run a current account surplus in period 1 and a current account deficit in period 2.
Consider an exogenous increase in $Y_1$

* $u'(Y_1)$ falls and $\frac{1}{1+rA}$ rises as current consumption becomes relatively cheaper compared to future consumption in autarky.

* Export even more current consumption, so period 1 current account surplus increases.
Welfare and the world interest rate

- Assume that $Y_2 > Y_1$ for home and that
  $$\frac{1}{1 + r} > \frac{1}{1 + r^A}$$

- Implies that the domestic price of future consumption is relatively low such that the domestic country has a comparative advantage in future consumption and a period-1 current account deficit as import current consumption

- Let world interest rate rise such that countries become more similar and comparative advantage shrinks

- Use graph to show that the domestic country becomes worse off
Effect of temporary versus permanent output change on current account

- Assume

$$\beta (1 + r) = 1$$

- If $Y_1 = Y_2$, then there is no current account imbalance

- If $Y_1 > Y_2$, current account surplus in period 1 and deficit in period 2
- Government consumption

  - Now, agents have only $Y_1 - G_1$ and $Y_2 - G_2$ available for consumption in periods 1 and 2

  - An increase in $G_1$, reduces output available for consumption in period 1 and creates a period-1 current account deficit

  - An increase in both $G_1$ and $G_2$ (permanent increase in G) has no current account effect
2 Small Open Production Economy with Investment

2.1 Current Account with Investment

- Investment with no depreciation

\[ I_t = K_{t+1} - K_t \]

- Change in total domestic wealth

\[ B_{t+1} + K_{t+1} - (B_t + K_t) = Y_t + r_t B_t - C_t - G_t \]
• Current account is increase in net foreign assets

\[ CA_t = B_{t+1} - B_t = Y_t + r_t B_t - (C_t + I_t + G_t) \]
\[ = \text{income - expenditure} \]

• National Savings is defined as

\[ S_t = Y_t + r_t B_t - C_t - G_t \]

• Current account is difference between savings and investment

\[ CA_t = S_t - I_t \]
2.2 Intertemporal Budget Constraint

- Assume

\[ B_1 = B_3 = 0 \]

- Current account in period 1

\[ B_2 = Y_1 - (C_1 + I_1 + G_1) \]

- Current account in period 2

\[-B_2 = Y_2 + rB_2 - (C_2 + I_2 + G_2)\]

\[ B_2 = \frac{C_2 + I_2 + G_2 - Y_2}{1 + r} \]
Equating the two expressions for $B_2$ yields the country’s intertemporal budget constraint

$$C_1 + I_1 + G_1 + \frac{C_2 + I_2 + G_2}{1 + r} = Y_1 + \frac{Y_2}{1 + r}$$
2.3 Optimal Consumption and Investment

- Production function

\[ Y_t = F (K_t) \]

- Intertemporal budget constraint solved for \( C_2 \)

\[ C_2 = (1 + r) [F (K_1) - C_1 - G_1 - I_1] + F (K_2) - I_2 - G_2 \]

  – Substitute using

\[ K_2 = K_1 + I_1 \]

\[ -K_2 = I_2 \]

\[ C_2 = (1 + r) [F (K_1) - C_1 - G_1 - I_1] + F (K_1 + I_1) + K_1 + I_1 - G_2 \]
Maximize present value of utility with respect to \( C_1 \) and \( I_1 \)

\[
u (C_1) + \beta u \{(1 + r) [F(K_1) - C_1 - G_1 - I_1] + F(K_1 + I_1) + K_1 + I_1 - G_2\}\]

First order condition with respect to \( C_1 \) is the standard Euler equation

\[
u' (C_1) = \beta (1 + r) u' (C_2)\]

Derivative with respect to \( I_1 \)

\[
\beta u' (C_2) \left[-(1 + r) + F' (K_2) + 1\right] = 0
\]

\[
F' (K_2) = r
\]

Investment is independent of preferences toward consumption and savings
– Can borrow at world interest rate to equate marginal product of capital with the interest rate
2.4 Production Possibility Frontier with Investment

- Two goods are future and current consumption

- PPF in autarchy

\[ C_2 = F(K_2) + K_2 = F(K_1 + F(K_1) - C_1) + K_1 + F(K_1) - C_1 \]

- Horizontal intercept sets \( C_2 = K_2 = 0 \) to maximize \( C_1 \)

\[ C_1 = K_1 + F(K_1) \]

- Vertical intercept sets \( C_1 = 0 \) to maximize \( C_2 \)

\[ C_2 = F(K_1 + F(K_1)) + K_1 + F(K_1) \]
– Slope of PPF
\[
\frac{dC_2}{dC_1} = -F'(K_2) - 1 < 0
\]

– Capital’s diminishing marginal product makes the PPF concave
\[
\frac{d^2C_2}{dC_1^2} = -F''(K_2) \frac{\partial K_2}{\partial C_1} < 0
\]

– Consider point where \( C_1 \) is high and \( C_2 \) and \( K_2 \) are low
  * \( F'(K_2) \) is high and falls as \( C_1 \) falls
  * therefore, as \( C_1 \) falls, slope increases (becomes flatter since slope is negative)
  * with high \( C_1 \) and low \( C_2 \), diminishing marginal productivity implies that give up little \( C_1 \) to get a lot of \( C_2 \)
* as continue to substitute get less and less $C_2$ for each unit of $C_1$
2.5 Current Account as Intertemporal Trade

- Graph with social optimum in autarky
  - Indifference curve tangent to PPF at point A determines $C_1, C_2,$ and $I_1$
  - Slope of indifference curve and PPF at optimum is $1 + F'(K_2) = 1 + r^A$

- Let the world interest rate be lower $r < r^A$, such that future goods are relatively expensive abroad and cheap at home

$$\frac{1}{1 + r} > \frac{1}{1 + r^A}$$
• Production occurs at point B where world relative price of future to current goods is tangent to PPF – since \( r < r^A \), slope is flatter

• Consumption occurs at point C where relative price of future to current goods is tangent to indifference curve

• Domestic country has a comparative advantage in future goods, so it imports current goods and exports future goods

• Higher indifference curve with trade illustrates gains from trade
• Horizontal distances
  – Between A and B is extra investment generated by opening trade
  – Between A and C is extra consumption of $C_1$
  – Between B and C is period 1 current account deficit

• Government consumption shifts PPF inward
3 Two-Country Endowment Economy

- World goods market equilibrium
  \[ Y_t + Y_t^* = C_t + C_t^* \]

- World savings must equal zero
  \[ S_t = Y_t - C_t = -S_t^* = C_t^* - Y_t^* \]

- Assume savings is increasing in the interest rate
  - and that \( r^A < r^{A*} \)
  - home country has a comparative advantage in current goods
- with opening of trade, home interest rate rises, increasing savings and CA surplus while foreign interest rate falls, reducing foreign savings and foreign CA

• Welfare is positively related to the term of trade (TOT)

\[ TOT = \frac{\text{price of exports}}{\text{price of imports}} \]

- Home country exports current goods whose price relative to future goods is \(1 + r\)

- Therefore, home \( TOT = 1 + r \)

- Let \( Y_2 \) increase, reducing savings and increasing \( r^A \)

  * Equilibrium \( r \) increases to return world savings to zero
* Domestic TOT increase and foreign TOT fall, increasing home welfare at the expense of foreign welfare

* Immiserizing Growth:
  - An increase in current output (growth) increases savings and reduces the equilibrium interest rate.
  - When home country exports current goods, the increase in current income can make it worse off

- Justify savings as an increasing function of the interest rate
  - Assume CRRA utility

\[ u(C) = \frac{C^{1-1/\sigma}}{1 - 1/\sigma} \]
– Use Euler equation

\[ C_1 = [\beta (1 + r)]^{-\sigma} C_2 \]

– and budget constraint

\[ C_2 = (Y_1 - C_1)(1 + r) + Y_2 \]

to solve for \( C_1 \)
The effect of an increase in interest on savings is the negative of its effect on $C_1$

* Substitution effect $[(1 + r)^\sigma]$ Higher interest makes current goods relatively more expensive causing agents to substitute out of current goods into future goods, raising savings

* Income effect $[(1 + r)^{-1}]$ Given PVLR, higher interest allows more future consumption, reducing savings. If $\sigma > 1$, substitution effect dominates and higher interest reduces consumption and raises savings.
* Wealth effect: Reduces PVLR, reducing current and future consumption and raising savings
– Alternative expression

\[ C_1 = [\beta (1 + r)]^{-\sigma} [(Y_1 - C_1)(1 + r) + Y_2] \]

* Totally differentiate with respect to \( C_1 \) and \( r \)

\[
\frac{dC_1}{dr} = \frac{(Y_1 - C_1) - \sigma \left( \frac{C_2}{1+r} \right)}{1 + r + C_2/C_1}
\]

- Second term is negative due to substitution effect

- First term is ambiguous TOT effect and depends on whether agent is a borrower (negative effect) or a lender (positive effect)

- If lender \((Y_1 - C_1 > 0)\), comparative advantage in current goods, \(TOT = 1 + r\), and increase in interest increases TOT and real income
4 Two-Country Production Economy with Investment

- Savings

\[
\frac{dC_1}{dr} = Y_1 - C_1 - I_1 - \sigma \left( \frac{C_2}{1+r} \right) \frac{1 + r + C_2/C_1}{1 + r + C_2/C_1}
\]

- Investment

\[
AF'(K) = r \quad A^*F'^*(K^*) = r^*
\]

where output is

\[
Y = AF(K) \quad Y^* = A^*F^*(K^*)
\]
• World equilibrium

\[ Y_1 + Y_1^* = C_1 + C_1^* + I_1 + I_1^* \]

\[ S_1 + S_1^* = I_1 + I_1^* \]

• Metzler diagrams and shocks

  – increase in home productivity in period 1

  – Increase in home productivity in period 2
• Capital market restrictions

  – A country with a current account deficit has a comparative advantage in future goods and its $TOT = \frac{1}{1+r}$

  – Can improve home welfare (at expense of foreign welfare) by increasing TOT (lowering $r$)

  – Lower world interest rate if tax home consumption, raising home saving

  – Both countries have the incentive to manipulate the TOT to their advantage, and the result can be reduced intertemporal trade

  – Beggar thy neighbor policy
• Oil prices and world interest rates
  – in 1970’s, severe increase in world oil prices and redistribution of income toward oil exporters
    * world savings increased and world interest rates fell
  – in 1980’s world savings fell with very large US government budget deficits and larger OPEC spending
    * world interest rates increased
  – currently very high savings rates in emerging market countries
    * Bernanke’s savings glut
    * low interest rates