Investment and Goods Market Equilibrium
1. Investment

2. Goods market equilibrium
1 MPK Theory of Investment

1.1 Desired capital stock

- Assumptions
  - Capital stock in the current period is fixed
  - Acquisition of new capital takes at least one period
  - Firms choose next period’s capital stock to maximize the value of real profits in the future
  - Real profits are determined by
* real output

* real labor costs

* real capital costs

  - user cost of capital = real cost of using one unit of capital for a period = \( rp_k + dp_k \) where \( p_k \) is the price of capital relative to the price of the firm’s production good and \( d \) is depreciation, that is the fraction of the good that will wear out over the period

\[
AF(K, N) - \varpi N - ucK
\]

- Maximization
– mathematically - take the partial derivative of profits with respect to capital and set equal to zero.

\[ A \frac{\partial F(K, N)}{\partial K} - uc = 0 \]

marginal product of capital (MPK) must equal the user cost

– graphically - plot MPK and user cost as a function of capital

• Theory of demand for capital (desired capital stock)

• Demand for capital for a given user cost is the marginal product of capital (MPK)

  – higher user cost (interest, price of capital, depreciation) reduces desired capital stock
Distortionary taxes - tax value of production so that firm only receives the after-tax value of sales

- Maximization problem becomes

$$\max_K \left[(1 - t) AF (K, N) - \omega N - ucK\right]$$

yielding

$$(1 - t) A \frac{\partial F (K, N)}{\partial K} - uc = 0$$

- after-tax MPK must equal user cost

- equivalently MPK = tax-adjusted user cost

$$A \frac{\partial F (K, N)}{\partial K} = \frac{uc}{1 - t}$$
graphically, after-tax user cost is higher with the tax, implying less desired capital with the tax

– need to consider effect on other components of government budget constraint and their effects on desired capital to get full answer

1.2 Investment and the desired capital stock

• Investment is the means of raising capital to its desired level

  – Capital stock increases as firms purchase new capital stock (gross investment = $I$)

  – Capital stock decreases as existing capital depreciates ($dK_t$)
- net investment is the increase in the capital stock

\[ K_{t+1} - K_t = I_t - dK_t \]

- rewriting, gross investment \((I_t)\) equals net investment \((K_{t+1} - K_t)\) plus depreciation \((dK_t)\)

\[ I_t = K_{t+1} - K_t + dK_t \]
• Time to adjust

– One period: $K_{t+1}^* = K_{t+1}$, implying that investment increases whenever $K_{t+1}^*$ increases above $K_t$

$$I_t = K_{t+1}^* - K_t + dK_t.$$ 

– Multiple periods: an increase in $K_{t+1}^*$ creates a smaller increase in $I_t$, but it is sustained over more periods

– Once capital reaches its desired level, investment is necessary only to replace capital which wears out.
2 Tobin’s Q Theory of Investment

- When capital is at its desired level, the market value of the firm’s capital stock should equal its replacement cost.

- If the market value of the firm’s capital is higher than its replacement cost, invest.
  
  - Market value of capital is represented by the firm’s stock market value $V$
  
  - Replacement cost of capital is its price ($p_k$) times the quantity of capital ($K$)

\[ q = \frac{V}{p_k K}. \]
– When $q > 1$, the firm should invest to increase the capital stock

• Implies that investment and the stock market should move together

• Check to make sure that this theory of investment is consistent with the MPK theory

  – effect of interest rate on $q$?

  – effect of MPK on $q$?
3 Aggregate Desired Investment

- Add up all firms’ desired investment

- Aggregate investment is decreasing in the real interest rate

- Other determinates of investment
  - MPK (expected future)
  - price of capital
  - tax rate
  - depreciation rate
4 Goods Market Equilibrium

- Output equals desired spending on goods

\[ Y = C^d + I^d + G \]

or rearranging

\[ Y - T - C^d + T - G = I^d \]

\[ S^d = I^d \]

- Real interest adjust to equate desired savings and investment (graph)

- Anything which shifts \( I^d \) or \( S^d \) will change the equilibrium real interest rate
• Why are real interest rates so low now?