1. Assume that an open economy is populated by representative agents with perfect foresight and with utility given by:

\[ U = \sum_{t=0}^{\infty} \beta^t \left( \gamma \ln C_t + (1 - \gamma) \ln \left( \frac{M_t}{P_t} \right) \right), \]

where \( \beta < 1 \) represents the discount factor, \( C \) represents real consumption, \( M \) is nominal money balances, and \( P \) is the price level. The agent receives exogenous real endowment income \((Y)\) each period and pays taxes \((\tau)\). He has access to an international real bond \((B)\). His flow budget constraint is given by:

\[ B_{t+1} + \frac{M_t}{P_t} = (1 + r) B_t + \frac{M_{t-1}}{P_t} + Y_t - C_t - \tau_t. \]

He also must satisfy:

\[ \lim_{t \to \infty} \left( B_{t+1} + \frac{M_t}{P_t} \right) \left( \frac{1}{1 + r} \right)^t = 0. \]

(a) Compute the first order conditions and solve for an expression for expenditures on money as a function of consumption expenditures. Consider this to be a money demand function and take natural logarithms, letting small letters denote the natural logarithm of capital letters.

(b) You end up with an interest rate expression which does not simplify. Linearize this about \( i \), where \( i \) represents a long-run equilibrium interest rate. Now, write the money demand function in the logs of money, price, consumption and in the interest rate (not its log). Collect the constants into a term \( h \), and the coefficient on the nominal interest rate into the term \( \alpha \).

(c) Assume that the foreign country has the same money demand function with the same parameters. They can have different values for variables. Write the foreign money demand function, denoting foreign variables by an asterisk.

(d) Impose purchasing power parity and solve for the exchange rate.

(e) Write the expression for interest rate parity. Take logs and write an approximation for interest rate parity in terms of interest rates and log exchange rates.

(f) Substitute the approximation for interest rate parity into the expression for the exchange rate and solve for the exchange rate as a function of the future exchange rate.

(g) Now, solve this equation forward to get the exchange rate as a function of expected future relative money and relative consumption. Use this equation to explain what causes sustained exchange rate depreciation. Compare this to the causes of sustained inflation.
(h) Now, return to the equation in which the exchange rate is expressed as a function of relative money, relative consumption, and relative interest rates. Assume that \( \beta (1 + r) = 1 \) so that consumption is constant. Assume that the country pegs the exchange rate. What does this imply about the interest rate differential? Now, let the log of the peg be \( \bar{e} \). Solve for the value of \( \ln M \) which will assure this pegged rate is an equilibrium.

(i) Let:

\[ M_t = D_t + R_t, \]

where \( D \) is domestic credit and \( R \) is reserves. Assume that domestic credit is growing at rate \( g \) such that

\[ D_t = D_0 e^{gt}. \]

Solve for the rate of growth of reserves under the pegged rate regime.

(j) Define the shadow exchange rate as the equilibrium exchange rate once reserves have been exhausted. Write an equation for the logarithm of the shadow exchange rate.

(k) What is the criterion for determining the date of exchange rate collapse? Compute collapse time, \( T \). Explain intuitively its determinants. [Hint: Substitute \( d_t = \log (D_0 e^{gt}) \)]

(l) Draw graphs for time paths of the log exchange rate, log shadow rate, log money and log reserves. Explain the time paths.

(m) Explain intuitively why a speculative attack is a necessary component of equilibrium.