Growth 2

Chapter 6 (continued)

1. Solow growth model continued
2. Use the model to understand growth
3. Endogenous growth
4. Labor and goods markets with growth

1 Solow Model with Exogenous Labor-Augmenting Technological Progress

1.1 Summary of model to date

- Determines long-run equilibrium values of per worker capital k and per worker output $y = Ak^{\alpha}$
- No growth in output per worker over time - counterfactual
- Growth in output only as number of workers grow - output growth equals population growth if participation rates unchanging

1.2 Amend the model to allow growth in the long run

- Measure labor in effective units given by the number of workers multiplied by their quality. - Better technology leads to higher effective of labor. This includes the skills and education of the labor force.

$$E = N \times Q$$

- Measure variables per effective unit of labor instead of per unit of labor (divide by E instead of by N) and let small letters denote per effective unit of labor

- Let

$$\frac{Q_{t+1}N_{t+1}}{Q_t N_t} = \frac{E_{t+1}}{E_t} = 1 + g_E = (1+n)\left(1+g_Q\right) \approx 1 + n + g_Q.$$

- Derive the Neoclassical growth equation:

$$K_{t+1} = K_t\,(1-d) + sY_t = K_t(1-d) + sAK_t^{\alpha}\,(Q_tN_t)^{1-\alpha}$$

$$\frac{K_{t+1}Q_{t+1}N_{t+1}}{Q_tN_tQ_{t+1}N_{t+1}} = \frac{K_t}{Q_tN_t}(1-d) + sA\frac{K_t^{\alpha}\,(Q_tN_t)^{1-\alpha}}{(Q_tN_t)^{\alpha}\,(Q_tN_t)^{1-\alpha}}$$

$$k_{t+1}\,(1+g_E) = k_t\,(1-d) + sAk_t^{\alpha}$$

- Long-run equilibrium level of capital per effective unit of labor - identical to earlier model if we replace n with $g_E = n + g_Q$.

$$k\,(g_E + d) = sAk^{\alpha}$$

investment per effective unit of labor equals savings per effective unit of labor - note that long-run equilibrium investment per effective worker is that necessary to replace capital per effective worker as it erodes due to growth in effective labor and depreciation

2 Long-run equilibrium growth

- What determines rate of growth of output?
  - Output per effective unit of labor must be constant over time

$$y = \frac{Y}{NQ}$$

  - Why?

– Take logs

$$\ln y = \ln Y - \ln N - \ln Q$$

Totally differentiate with respect to all variables and set equal to zero since y cannot change in long-run equilibrium.

$$\frac{dy}{y} = \frac{dY}{Y} - \frac{dN}{N} - \frac{dQ}{Q} = 0$$

$$\frac{dY}{Y} = \frac{dN}{N} + \frac{dQ}{Q}$$

The rate of growth of output equals the rate of growth of labor plus the rate of growth of labor-augmenting technological progress.

- What determines rate of growth of output per worker?

$$\frac{dY}{Y} - \frac{dN}{N} = \frac{dQ}{Q}$$

output per worker can grow continually at the rate at rate of growth of labor-augmenting technological progress.

- What determines the rate of growth of savings, consumption, and capital? of saving, consumption, and capital per worker?

2.1 Why do some countries grow faster than others?

- If countries have acquired their long-run equilibrium level of capital, then output growth depends only on population growth and technology growth. Faster growing countries have faster growth in one or both of these. Faster growth in per worker GDP requires faster technological progress.

- Why do some countries have faster technological progress?

- Countries which have not reached their long-run equilibrium level of capital are growing both due to technological progress and population, but also due to a catch-up as the capital stock increases.

- Convergence Hypothesis: If all countries have the same rate of technological progress, then per worker output growth should converge over time.
  - Why?
  - Should per worker output also converge?

2.2 Solow Residual and labor-augmenting technological progress

- Production function

$$Y_t = AK_t^{\alpha}\left(Q_t N_t\right)^{1-\alpha}$$

- Growth accounting with A constant

$$\frac{dY_t}{Y_t} = \alpha\frac{dK_t}{K_t} + (1-\alpha)\left(\frac{dQ_t}{Q_t} + \frac{dN_t}{N_t}\right)$$

- The measure of productivity - Solow residual - is given by

$$\frac{dY_t}{Y_t} - \alpha\frac{dK_t}{K_t} - (1-\alpha)\frac{dN_t}{N_t} = (1-\alpha)\frac{dQ_t}{Q_t}$$

  Labor-augmenting technological progress is the growth in technology. It is a residual after measuring output growth, capital growth and labor growth.

2.3 Growth is determined by technological progress, but what determines technological progress?

- Capital accumulates only in response to technological progress.

- A higher saving rate increases equilibrium capital per effective labor unit, but does not raise long-run equilibrium growth.

3 Endogenous growth

- Objective: endogenize the rate of growth of technological progress

- Learning-by-doing model
  - Assumptions
    - Knowledge is accumulated as an accidenta by-product of production. Each firm takes the level of existing knowledge as given, but existing knowledge depends on the total amount of output produced.
    - Knowledge (B for by-product) is produced according to:

$$B_t = AK_t^{1-\alpha}$$

where A is constant. Knowledge is a positive externality of production.

    - Output is given by

$$Y_t = B_t K_t^{\alpha} N_t^{1-\alpha}.$$

* Substituting for knowledge yields

$$Y_t = AK_t N_t^{1-\alpha}$$

Normalize the stock of labor to one and assume it does not grow ($n = 0$),

$$Y_t = AK_t$$

with output per worker given by

$$y_t = Ak_t$$

Note that capital does not have a diminishing marginal product!

– Capital accumulation equation

$$k_{t+1} = (1-d)\,k_t + sAk_t$$

– Growth rate of capital per worker

$$\frac{k_{t+1}}{k_t} - 1 = (1 - d) + sA - 1 = sA - d$$

– Capital per worker (and output per worker) grows if $sA > d$. A higher saving rate implies higher capital accumulation which increases the rate of technological progress increasing per worker growth rate.

4 Market Equilibrium with Growth

4.1 Labor Market

- Labor supply

- Labor demand

- Equilibrium

4.2 Goods Market

- Investment

- Saving

- Equilibrium