Interest Rate Parity
Chapter 14 (second part)
1. Nominal Returns

2. Real Returns

3. Interest rate parity

4. Exogenous shocks

5. Covered interest rate parity
1 Nominal Returns

• Nominal return (dollar values, gross returns)
  
  - Dollar value of asset in period $t + 1$ relative to its value in period $t$
    \[
    \text{nominal rate of return} = \frac{\text{\$ value (t + 1)}}{\text{\$ value (t)}}
    \]
  
  - For a dollar bond, nominal dollar return is $1 + \text{nominal dollar interest rate}$
    \[
    \text{nominal dollar return on dollar bond} = 1 + R$
    \]
  
  - For a euro bond, nominal euro return is $1 + \text{nominal euro interest rate}$
    \[
    \text{nominal euro return on euro bond} = 1 + R$
    \]
• Compare dollar returns on dollar bond and on Euro bond

  – Take $1 and buy either a dollar bond or a euro bond

  – if buy a dollar bond, at the end of the year will have dollars equal to

    \[ 1 + R_s \]

  – if buy a euro bond, can buy \( \frac{1}{E_t} \) euro bonds with $1

    * those euro bonds earn \( 1 + R_€ \), yielding total euros of

    \[ \frac{1}{E_t} \times (1 + R_€) \]

    where \( E \) is \( E$/€ \)

    * converting these euros back to dollars next periods yields dollars
generated by purchasing euro bond of

$$\frac{E^e_{t+1}}{E_t} \times (1 + R_\epsilon)$$

– since the denominator is $1$, these expressions represent dollar returns
on a dollar bond and a euro bond, respectively

• Approximations

– fact: for small $X$,

$$\ln(1 + X) \approx X$$

– dollar return on dollar bond

$$1 + R_\$$$
- net returns equal gross returns minus 1

\[ R_\\$ = \ln (1 + R_\\$) \]

- equivalently, net returns equal the logarithm of gross returns

* dollar bond

\[ \ln (1 + R_\\$) = R_\\$ \]

* euro bond

\[
\ln \left( \frac{E_{t+1}^e}{E_t} \times (1 + R_€) \right) = \ln \left( \frac{E_{t+1}^e}{E_t} \right) + \ln (1 + R_€)
\]

\[
\ln \frac{E_{t+1}^e}{E_t} = \ln \frac{E_{t+1}^e - E_t + E_t}{E_t} = \ln \left( \frac{E_{t+1}^e - E_t}{E_t} + 1 \right) \approx \frac{E_{t+1}^e - E_t}{E_t}
\]
\[ \ln \left[ \frac{E_{t+1}^e}{E_t} \times (1 + R_\epsilon) \right] \approx \frac{E_{t+1}^e - E_t}{E_t} + R_\epsilon \]
1.1 Real Returns

- Real returns are values in terms of purchasing power (gross returns)

- Real value of asset in period $t + 1$ relative to its real value in period $t$

  \[
  \text{real rate of return} = \frac{\text{expected real value} \ (t + 1)}{\text{real value} \ (t)} = \frac{\text{dollar value in } t + 1}{P_{t+1}^e} \div \frac{\text{dollar value in } t}{P_t}
  \]

- Dollar bond

  \[
  \frac{\text{dollar value in } t + 1}{P_{t+1}^e} \div \frac{\text{dollar value in } t}{P_t} = \frac{1 + R_s}{\frac{P_t}{P_{t+1}^e}} = \frac{1 + R_s}{P_t} \times \frac{P_t}{1} = \frac{P_t (1 + R_s)}{P_{t+1}^e}
  \]
− If prices rise, $\frac{P_t}{P_{t+1}} < 1$, and the real return is less than the nominal return

− Net return - take logarithm

$$\ln \frac{P_t (1 + R$_t$)}{P_{t+1}^e} = \ln (1 + R$_t$) + \ln \frac{P_t}{P_{t+1}^e}$$

$$\ln (1 + R$_t$) \approx R$_t$$$

$$\ln \frac{P_t}{P_{t+1}^e} = - \ln \frac{P_{t+1}^e}{P_t} = - \ln \frac{(P_{t+1}^e - P_t) + P_t}{P_t}$$

$$= - \ln \left[ \frac{(P_{t+1}^e - P_t)}{P_t} + 1 \right] \approx - \frac{(P_{t+1}^e - P_t)}{P_t}$$
* therefore

\[
\ln \frac{P_t (1 + R_\$$)}{P^e_{t+1}} \approx R_\$$ - \frac{(P^e_{t+1} - P_t)}{P_t}
\]

• Euro bond

\[
\frac{E^e_{t+1}}{E_t} \times (1 + R_\£) = \frac{E^e_{t+1}}{E_t} \times \left(1 + \frac{1}{P^e_t} \right) = \frac{E^e_{t+1}}{P^e_{t+1}} \times \frac{P^e_t}{P^e_{t+1}} \times \frac{P_t}{1}
\]

\[
= P_t \times \frac{E^e_{t+1}}{E_t} \times \left(1 + R_\£\right) = \frac{E^e_{t+1}}{E_t} \times \frac{P_t}{P^e_{t+1}} \left(1 + R_\£\right)
\]

– If the dollar is expected to depreciate \(\frac{E^e_{t+1}}{E_t} > 1\), return on the euro bond is higher
- Net return - take logarithm

\[
\ln \left[ \frac{E_{t+1}^e P_t}{E_t P_{t+1}^e} (1 + R_\epsilon) \right] = \ln \frac{E_{t+1}^e}{E_t} - \ln \frac{P_{t+1}^e}{P_t} + \ln (1 + R_\epsilon)
\]

\[
\approx \frac{E_{t+1}^e - E_t}{E_t} - \frac{(P_{t+1}^e - P_t)}{P_t} + R_\epsilon
\]
2 Equilibrium in the Foreign Exchange Market

2.1 Demand for assets depends upon

- Nominal rates of return

- Risk

- Liquidity
2.2 Interest rate parity

- Interest rate parity equates dollar returns in two currencies

\[ R_\$ = R_€ + \frac{E_{t+1}^e - E_t}{E_t} \]

- To begin

  - Exogenous variables include

    \[ R_\$, R_€, E_{t+1}^e \]

  - Endogenous variable

    \[ E_t \]
• Graph

  – $R_\$ as a function of $E_t$

  – $R_\€ + \frac{E_t^e - E_t}{E_t}$ as a function of $E_t$
2.3 Changes in exogenous variables

- $R_\$ increases
- $R_\$ increases
- $E^{e}_{t+1}$ increases
3 Covered Interest Rate Parity

3.1 Use forward market

- Replace $E_{t+1}^e$ with $F_t$

- Covered interest rate parity

\[ R_s = R_e + \frac{F_t - E_t}{E_t} \]