Stocks and Bonds

1. Present Value

2. Bonds

3. Stocks

1 Present Value = today's value of income at a future date

- Income at one future date
  - value today of X dollars in one year

$$V_t = \frac{X_{t+1}}{(1 + i_t)}$$

where i_t is the nominal interest rate on assets held during period t

  - value today of X dollars in two years

$$V_t = \frac{X_{t+2}}{(1 + i_t)\left(1 + i^e_{t+1}\right)}$$

where i^e_{t+1} is the interest rate expected to prevail in one year

- Stream of income in the future, z_t today, z_{t+1} in one year, z_{t+2} in two years and z_{t+3} in three years
  - known stream

$$V_t = z_t + \frac{z_{t+1}}{1+i_t} + \frac{z_{t+2}}{(1+i_t)\left(1+i^e_{t+1}\right)} + \frac{z_{t+3}}{(1+i_t)\left(1+i^e_{t+1}\right)\left(1+i^e_{t+2}\right)}$$

  - unknown stream - replace known values of the future income with expectations

$$V_t = z_t + \frac{z^e_{t+1}}{1+i_t} + \frac{z^e_{t+2}}{(1+i_t)\left(1+i^e_{t+1}\right)} + \frac{z^e_{t+3}}{(1+i_t)\left(1+i^e_{t+1}\right)\left(1+i^e_{t+2}\right)}$$

– stream of income with interest rates constant

$$V_t = z_t + \frac{z^e_{t+1}}{1+i_t} + \frac{z^e_{t+2}}{(1+i_t)^2} + \frac{z^e_{t+3}}{(1+i_t)^3}$$

– constant interest rates and constant payments of z forever beginning one year from now

$$V_t = z\left(\frac{1}{1+i} + \left(\frac{1}{1+i}\right)^2 + \left(\frac{1}{1+i}\right)^3 + \ldots\ldots\right) = \frac{z}{i}$$

note that the value of paying interest on debt forever and the value of paying off the debt today are equal!

- Real stream of income - use real interest to take present-value
  - Divide both sides of PV equation by current price to express real present value

$$\frac{V_t}{P\,t} = \frac{z_t}{P_t} + \frac{z^e_{t+1}}{1+i_t}\frac{1}{P_t} + \frac{z^e_{t+2}}{(1+i_t)\left(1+i^e_{t+1}\right)}\frac{1}{P_t} + \frac{z^e_{t+3}}{(1+i_t)\left(1+i^e_{t+1}\right)\left(1+i^e_{t+2}\right)}\frac{1}{P_t}$$

  - Note that

$$(1+i_t)\frac{P_t}{P^e_{t+1}} = 1+r$$

Using this, real present value is stated as:

$$\frac{V_t}{P\,t} = \frac{z_t}{P_t} + \frac{z^e_{t+1}}{1+r_t}\frac{1}{P^e_{t+1}} + \frac{z^e_{t+2}}{(1+r_t)(1+r_{t+1})}\frac{1}{P^e_{t+2}} + \frac{z^e_{t+3}}{(1+r_t)(1+r_{t+1})(1+r_{t+2})}\frac{1}{P^e_{t+3}}$$

- Definitions
  - discount rate $= i$
  - discount factor $= \frac{1}{1+i}$

2 Bonds

2.1 Definitions

- Maturity - the length of time the bond promises payments to its holder
- Coupon bonds - bonds that promise multiple payments before maturity and one payment at maturity
- Discount bonds - bonds that promise a single payment at maturity
- Face value - the payment at maturity

2.2 Price of a discount bond

- Assume the bond will pay \$50 at the end of two years and that the interest rate is 4% and is expected to rise to 4.5% after one year. The price of the bond is the present value of that final payment.

$$P_{2t} = \frac{\$50}{(1+i_t)\left(1+i^e_{t+1}\right)} = \frac{\$50}{(1.04)\,(1.045)} = \$46.01$$

- Arbitrage - Will the investor prefer the two year bond above or two one year bonds?
  - For every dollar you put in a two year bond, get $\frac{1}{P_{2t}}$ two year bonds. At the end of the first year, now you have $\frac{1}{P_{2t}}$ one year bonds valued at P^e_{1t+1} for a total value of $\frac{P^e_{1t+1}}{P_{2t}}$.

– For every dollar you put in a one year bond, get $1 + i_t$.

– Which do you prefer?

$$\frac{P^e_{1t+1}}{P_{2t}} = 1 + i_t$$

or equivalently

$$P_{2t} = \frac{P^e_{1t+1}}{1 + i_t}$$

– The price of a two year bond must be the present-value of the expected price of the one-year bond. Note this ignores risk. Which option has risk in the first period?

- Yield to maturity on an n-year bond or equivalently the n-year interest rate is defined as that constant annual interest rate that makes the bond price today equal to the present value of future payments on the bond.

  - What is the yield to maturity on the two year bond above?

$$\$46.01 = \frac{\$50}{(1 + i_{2t})^2}$$

    solving for i_{2t} yields 4.25%.

  - Note that

$$\frac{\$50}{(1 + i_t)\left(1 + i^e_{t+1}\right)} = \frac{\$50}{(1 + i_{2t})^2}$$

    Rearranging yields

$$1 + 2i_{2t} + i^2_{2t} = 1 + i_t + i^e_{t+1} + i_t i^e_{t+1}$$

$$i_{2t} \approx \frac{1}{2}\left(i_t + i^e_{t+1}\right)$$

- Long-term rates are the average of expected future short-term rates (ignoring risk)
  - Why are long-term interest rates currently so low?

- Risk - Since the two-year bond has a risky value in period 1 (why?) the alternatives of a two-year bond and two one year bond are not equally attractive if you don't like risk. Therefore, the two-year bond alternative must pay a slightly higher return, given by the risk premium (rp).

$$\frac{P^e_{1t+1}}{P_{2t}} = 1 + i_t + (rp)_t$$

performing the same steps as before, we find that with risk:

$$i_{2t} \approx \frac{1}{2}\left(i_t + i^e_{t+1} + rp_t\right)$$

- With risk, long-term rates are a little above the average of expected future short-term rates.

- Yield curve = interest rates on n-period bonds as n (maturity) increases. Due to risk, the yield curve generally slopes upward, implying that longer term bonds have higher interest rates. Under what circumstances might we have an inverted (slopes downward) yield curve?

3 Stock Prices

- The value of a stock (and hence its price) is determined by the present value of its dividends. The ex-dividend price (after the current dividend has been paid) is therefore:

$$
\begin{aligned}
Q_t &= \frac{D^e_{t+1}}{1+i_t+rp} + \frac{D^e_{t+2}}{(1+i_t+rp)\left(1+i^e_{t+1}+rp\right)} \\
&\quad + \frac{D^e_{t+3}}{(1+i_t+rp)\left(1+i^e_{t+1}+rp\right)\left(1+i^e_{t+2}+rp\right)} + \dots\dots\dots
\end{aligned}
$$

- This expression is based on the *fundamental* value of the stock (the determinates of its true value).

- Dividends tend to move with profits, implying that higher expected future profits raise a stock's price.

- Higher interest rates reduce the present-value of the stream of dividends, thereby reducing the stock's price.

- An alternative expression:

- Note that the price expected to prevail in one period is given by:

$$Q^e_{t+1} = \frac{D^e_{t+2}}{\left(1 + i^e_{t+1} + rp\right)} + \frac{D^e_{t+3}}{\left(1 + i^e_{t+1} + rp\right)\left(1 + i^e_{t+2} + rp\right)} + \ldots\ldots..$$

Substituting this above yields:

$$Q_t = \frac{D^e_{t+1}}{1 + i_t + rp} + \frac{Q^e_{t+1}}{(1 + i_t + rp)}$$

- Rational speculative bubbles - occur when asset prices are driven from fundamental values by expectations of every-rising asset prices