Stylized Facts

Let us state some generalizations (stylized facts) about stock returns.

Random Walk

Stock returns are a random walk or almost a random walk.

The stock return during a period is uncorrelated with the stock return in any nonoverlapping period.

Consequently a graph of a stock price against time is jagged, not smooth, as a smooth graph would imply that one can forecast the future stock price.

Correlation with Macroeconomic Variables

Economic theory says that stock returns should be correlated with the business cycle, the interest rate, the exchange rate, etc.

For example, an increase in the interest rate should reduce the present value of the dividends on a stock, so one would expect a negative correlation between the interest rate and the stock price.

However in fact the correlation of stock returns with macroeconomic variables is small.

Cross Correlation of Stock Returns

There is a significant cross correlation of stock returns: stock prices tend to rise and fall together. Even though stock returns have little correlation with macroeconomic variables, they are correlated with one another.

Stock Index

Stock price indices (the Dow Jones Industrial Average, the NASDAQ index, etc.) fluctuate quite a bit.

This behavior would not be the case, if there were no cross correlation of stock returns. Without cross correlation, an increase in a stock price would not be related to the changes in other stock prices. When some prices rise, others would fall. The stock index would be quite stable.

Market Model

Typically the correlation of a stock return with a stock index is higher than the correlation with another stock return.

This behavior accords with the “market model:”

$$R_i = \beta_i R_m + F_i.$$

The market model expresses the return R_i on a stock as a scalar β_i times the return R_m on a stock index plus a firm-specific term F_i. Here F_i is uncorrelated both with R_m and with other firm-specific terms.

For firms i and j, that the correlation of F_i and F_j is zero means that these terms contribute nothing to the covariance between the two returns. That each term has a significant variance does, however, raise the variance of each return. Consequently the firm-specific terms make the correlation less.

Skewness and Kurtosis

Consider a random variable x with mean m and standard deviation s.

Definition 1 (Skewness) *The skewness is*

$$\mathrm{E}\left[(x-m)^3\right]/s^3.$$

For a normal distribution, the skewness is zero.

Definition 2 (Kurtosis) *The kurtosis of the distribution is*

$$\mathrm{E}\left[(x-m)^4\right]/s^4-3.$$

For a normal distribution, the kurtosis is zero.

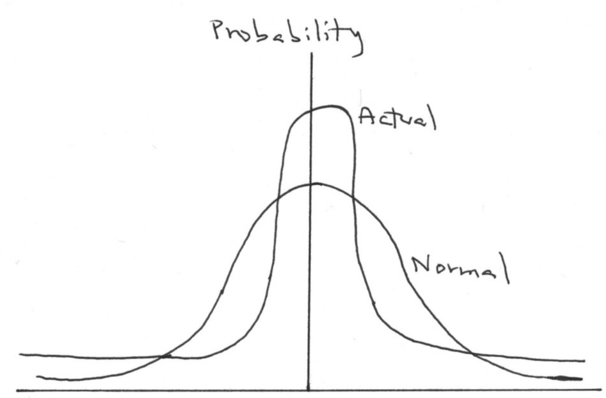
Fat Tails

The sample distribution for stock returns has “fat tails” (figure 1). Compared to a normal distribution with the same mean and variance, the sample distribution has more observations near the mean and in the tails (and hence fewer in between).

Fat tails imply that the kurtosis is positive.

All assets returns show fat tails: stocks, bonds, option prices, exchange rates, etc.

Figure 1: Fat Tails



Long-Run Variance

Let e^{R_t} denote the value at time t of one dollar invested at time $t-1$. Then

$$\mathrm{e}^{R_0}\mathrm{e}^{R_1}\cdots\mathrm{e}^{R_t}=\mathrm{e}^{\sum_{i=0}^{t}R_i}$$

is the value at time t of one dollar invested at time 0.

A random walk implies that the R_i are uncorrelated for different i, so the variance of the sum is the sum of the variances,

$$\mathrm{Var}\left(\sum_{i=0}^{t}R_i\right)=\sum_{i=0}^{t}\mathrm{Var}\left(R_i\right).$$

Thus the variance of the return rises in proportion to the total time. Risk does not disappear for long-run investment but keeps increasing.

This relationship does hold roughly.

Volatility

There is some persistence of volatility in stock returns.

Large returns tend to be followed by other large returns, but the sign is unpredictable, in accord with the random-walk theory. For example, a large drop in a stock price one day is often followed by a large change the next day. However the change may be either up or down, and the direction of change is unpredictable.

High or low volatility may persist for months.